A New Approach Based on Genetic Algorithm for Optimal Reactive Power Flow Solution in Multi-terminal AC-DC Systems

Abstract. This paper presents a new approach based on Genetic Algorithm (GA) for the solution of Optimal Reactive Power Flow (ORPF) in multi-terminal ac-dc systems. Active power loss optimization is implemented through GA considering the constraints of entire multi-terminal ac-dc system. Ac-dc power flow is solved by sequential method. So, any ac and dc power flow algorithm can be used effectively in the proposed approach. Unlike the similar studies, active powers of all converters except at least one are selected as control variables for ORPF in this study to achieve most suitable converter active powers and converter types that improve active power loss minimization. This paper presents the first study that utilizes GA for the solution of optimal reactive power flow in multi-terminal ac-dc systems in the literature. The proposed approach is tested on the modified IEEE 14-bus ac-dc test system. The obtained results by this study and another alternative study are also given. Comparative results showed that the proposed approach is more efficient and reliable in reaching to a global optimum while satisfying entire system constraints.

Streszczenie. W artykule przedstawiono propozycję metody zarządzania rozpływem mocy biernej w sieci AC-DC, opartej na algorytmie genetycznym. Optymalizacja strat mocy czynnej została dokonana przez uwzględnienie w algorytmie ograniczeń w całym systemie AC-DC. Proponowane rozwiązanie jest pierwszym tego typu podejściem do optymalizacji rozplywu mocy biernej w wielokomercjalowej sieci. Testy wykonane zostały na zwykłodanym 14-szynowym systemie w standardzie IEEE. Analiza porównawcza z drugimi metodami wskazuje, na większą skuteczność i niezawodność przedstawionej metody. (Optymalizacja rozpływ mocy biernej w wielokomercjalnym systemie AC-DC w oparciu o algorytm genetyczny).

Keywords: Optimal Reactive Power Flow, AC-DC System, Genetic Algorithm
Słowa kluczowe: Optymalny rozpływ mocy biernej, system AC-DC, algorytm genetyczny.

Introduction
In electrical power systems, system over-loading related to constantly increasing demand is extremely important for both the supply of existing and future demand and the reliability of power system. Due to high costs and inconvenient terrain circumstances economically, there are difficulties for allocating new energy transmission lines. Hence, the existing power systems must be used efficiently. Therefore, active power loss minimization in power systems is vastly important for system efficiency and reliability. Active power loss minimization is provided by Optimal Reactive Power Flow (ORPF). Dynamic and static reactive power compensators, shunt capacitors and reactors, generators’ voltage amplitude values and tap changers in power systems are the main factors of ORPF because of having the ability of changing reactive power flow [1].

Even though the investment costs related to High Voltage Direct Current (HVDC) systems are high, they are more economic than ac transmission lines for very long distances. Additionally, efficient conductor intersection, flexible control, system reliability and consistency, efficient implementation, not having reactive power problem, and continuously increasing development in semiconductor technology are the advantageous characteristics for HVDC systems [2]. For these reasons, researchers have been working on integrated ac-dc systems for a long time. Many methods have been proposed for ac-dc power flow. These methods in the literature can be separated into two main parts: sequential method and simultaneous method. In sequential method, ac and dc power flow are implemented separately and convergence can be provided by getting back and forward [3-4]. In simultaneous method, all equations regarding to ac-dc system are one within other and the equations are solved together [5-6].

Although there are many significant studies about ac-dc power flow in this area, there are not enough substantial studies on optimal power flow of two or multi-terminal ac-dc systems. Existing optimal power flow studies in ac-dc systems have been implemented successfully by using quadratic programming, linear programming, mixed-integer nonlinear programming, gradient-restoration algorithm and steepest descent algorithm [7-14]. However, there are convergence and dropping to local minimum problems in these methods [15].

Recently, heuristic methods like differential evolution [16], artificial bee colony algorithm [17], particle swarm optimization [18], and artificial ant colony [19] have been developed for the solution of global optimization problems and they have been implemented to those problems successfully. These methods are more efficient with respect to accurate and faster convergence and not dropping to local minimum than conventional calculation techniques mentioned above.

Genetic Algorithm (GA) is also one of the heuristic methods. It has been successfully applied to the solution of optimal active, reactive and active-reactive power flow in ac power systems as well as in other fields [20-21].

In this study, the newest approach is presented for solution of optimal reactive power flow problem in multi-terminal ac-dc systems by using GA. Sequential method is used for ac-dc power flow problem and GA is used for optimal reactive power flow problem. Furthermore, the constraints of control and state variables vectors are also included into the ORPF solution. Finally, the proposed approach’s accuracy and consistency are tested on the modified IEEE 14-bus ac-dc test system.

The Illustration of the Proposed Sequential AC-DC Power Flow
Power flow problem in multi-terminal ac-dc system is solved by using the sequential method as mentioned above. Dc power flow is performed by Gauss iteration method and ac power flow is performed by Newton method throughout ORPF solution. Because the variables belonging to dc system are not taken into account of calculating Jacobian matrix, less dimension is required and fast convergence rate is achieved. Active and reactive powers of converters are taken into account constant load for ac power flow in terms of the sequential method.

The converter model for dc system is shown in Fig. 1. In this model, main harmonic components for ac system and average values without ripples for dc system are taken as base values. This study is based on per-unit system.

Dc terminal voltage of the $i^{th}$ converter is
Reactive power for the $i^{th}$ converter is

$$q_{di} = [p_{di} \tan \theta_i]$$

If the dc terminal connected to the $1^{st}$ converter in dc system is chosen as reference node, the open circuit voltages of the $i^{th}$ converter are determined as follows:

$$e_i = e_1 - r_i \delta_{i1} + r_i \delta_{i0} + \sum_{j=2}^{n_e} r_{ij} \delta_{ij} \quad (i = 2 \ldots n_e)$$

where $e_1$, $r_{ij}$ and $n_e$ represent referenced dc terminal open circuit voltage, bus resistance matrix components obtained from $1^{st}$ dc terminal and converter number of dc system, respectively.

The algebraic sum of the dc currents flowing to the dc system must be zero:

$$\sum_{i=1}^{n_e} i_{di} = 0$$

The least square method and based current-balancing mode are used in order to balance the dc currents of converters regarding the multi-terminal dc system to Eq. (6) [22]. Active powers of all converters except at least one are selected as control variables for ORPF in the study to achieve most suitable converter active powers and converter types that improve active power loss minimization. The details of sequential ac-dc power flow algorithm used for ORPF solution are shown in Fig. 2.

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1. Input the control variables calculated in optimization algorithms.
2. Estimate reactive powers for all converters and active powers for converters which operate only in nonconstant active power mode and then apply ac power flow algorithms.
3. Estimate dc terminal voltages, $v_c = v_1 \cos \phi_{1m} \quad (i = 1 \ldots n_e)$
4. Input the active and reactive powers of converters obtained in step 1 and step 2 to dc power flow.
5. Calculate the dc currents of all the converters by current-balancing mode.
6. Find the sign of commutation resistance using $r_{ij} \frac{1}{c_{ni}} (i = 1 \ldots n_e)$
7. Calculate the open circuit voltages using dc terminal voltages estimated in step 3.
   $$e_i = e_1 - r_{i1} \delta_{i1} + r_{i0} \delta_{i0} + \sum_{j=2}^{n_e} r_{ij} \delta_{ij} \quad (i = 2 \ldots n_e)$$
8. Calculate the dc terminal voltages, $v_c = v_1 \cos \phi_{1m} \quad (i = 2 \ldots n_e)$
9. Is the difference between dc terminal voltages estimated in step 3 and in step 8 smaller than the predetermined tolerance?
10. Assign the dc terminal voltages obtained in step 9 as new values
11. Determine firing or extinction angles of converters
    $$\alpha_i \quad \gamma_i = \text{arccos} (\sqrt{1 - C_i}) \quad (i = 1 \ldots n_e)$$
12. Assign as $i = 1$.
13. Control ($\alpha_i$ or $\gamma_i$) angles whether imaginary or ($\alpha_i$ or $\gamma_i$) ($\alpha_{i,m}$ or $\gamma_{i,m}$).
14. If the $\alpha_i$ or $\gamma_i$ angles at step 11 imaginary or they have less than the minimum limit values being indicated in data file, update the $\alpha_i$ or $\gamma_i$ angles and tap changer's tap values of the converters.
    $$\alpha_i = \frac{i}{l_{\text{max}}} \quad \gamma_i = \frac{i}{l_{\text{max}}}$$
15. Is $i$ equal to $n_e$?
16. Assign that, $i = i + 1$.
17. Calculate the active and reactive power of converters.
    $$p_{di} = v_{di} i_{di}$$
    $$\theta_i = \text{arccos} (\frac{p_{di}}{v_{di} i_{di}})$$
    $$q_{di} = [p_{di} \tan \theta_i]$$
18. Is the difference between active and reactive powers of the converters in step 2 and in step 17 smaller than the predetermined tolerance?
19. Use the active and reactive powers of the converters obtained from step 17 as new values and return to ac power flow.
20. Ac power flow accomplished, Go to optimization algorithm with ac power flow results.
ORPF Problem

The general optimization formula can be shown below,
\[
\text{Minimize } f(x,u) \\
\text{Subjected to } g(x,u), h(x,u)
\]
where, \( f(x,u), g(x,u), h(x,u) \), \( x \) and \( u \) represent objective function, equality constraints inequality constraints, state variables and control variables, respectively.

The total active power loss in ac-dc system can be calculated as follows:
\[
P_{\text{loss}} = \sum_{i=1}^{n_a} p_{g_i} - \sum_{i=1}^{n_a} p_{\text{load}} - \sum_{i=1}^{n_d} p_{\text{di}}
\]
where \( n_a, n_w, p_g, p_{\text{load}} \) and \( p_{\text{di}} \) represent generator number, ac bus number, generator active power output and active powers for ac loads, respectively.

Equality constraints for ac system,
\[
P_{\text{g}} - p_{\text{load}} - p_{\text{di}} - p_{\text{load}} = 0
\]
\[
q_{\text{g}} + q_{\text{load}} - q_{\text{di}} - q_{\text{load}} = 0
\]
where \( q_{\text{g}}, q_{\text{load}}, q_{\text{di}}, p_{\text{load}} \) and \( q_{\text{load}} \) represent generator reactive power output, synchronous condenser reactive power, reactive powers of ac loads, active power flowing from \( i^{\text{th}} \) ac bus to the other ac buses and reactive power flowing from \( i^{\text{th}} \) ac bus to the other ac buses, respectively.

Equality constraints for dc system,
\[
\sum_{i=1}^{n_d} t_{\text{di}} = 0
\]
Equality constraints in Eq. (9-11) defined as \( g(x,u) \) are solved in the ac-dc power flow mentioned in the preceding section.

Inequality constraints for ac system,
\[
P_{\text{g}}^{\text{min}} \leq p_{\text{g}} \leq P_{\text{g}}^{\text{max}}
\]
\[
q_{\text{g}}^{\text{min}} \leq q_{\text{g}} \leq q_{\text{g}}^{\text{max}}
\]
\[
v_{\text{i}}^{\text{min}} \leq v_{\text{i}} \leq v_{\text{i}}^{\text{max}}
\]
\[
t_{\text{di}}^{\text{min}} \leq t_{\text{di}} \leq t_{\text{di}}^{\text{max}}
\]
where \( t_{\text{di}} \) represents tap values of the tap changers between the ac buses.

Inequality constraints for dc system,
\[
P_{\text{di}}^{\text{min}} \leq p_{\text{di}} \leq P_{\text{di}}^{\text{min}}
\]
\[
v_{\text{di}}^{\text{min}} \leq v_{\text{di}} \leq v_{\text{di}}^{\text{max}}
\]
\[
t_{\text{di}}^{\text{min}} \leq t_{\text{di}} \leq t_{\text{di}}^{\text{max}}
\]
where min and max superscripts represent lower and upper limits of associated variables, respectively.

State variables of ac-dc system,
\[
x = [x_a, x_d]
\]
where \( x_a \) and \( x_d \) represent state variables of ac and dc system, respectively.

Control variables of ac-dc system,
\[
x_a = [\delta, v_{\text{g}}, v_{\text{i}}, v_{\text{di}}, v_{\text{load}}]
\]
\[
x_d = [1, t_{\text{di}}, \phi_{\text{a}}, \phi_{\text{i}}, \phi_{\text{di}}, \phi_{\text{load}}]
\]
where \( \delta \) and \( n_i \) represent ac bus voltage angle and ac load bus number without synchronous condenser, respectively.

Initial population of the algorithm is determined as,
\[
w_i = \text{rand}(0,1) \times (w_{\text{max},i} - w_{\text{min},i})
\]
\[
(i = 1 \ldots n_a) \quad (j = 1 \ldots n_p)
\]
where \( n_a, n_p, w_{\text{min},i} \) and \( w_{\text{max},i} \) represent number of the individuals within the population, number of the parameters of the individuals, minimum and maximum values of the parameters, respectively.

In fitness scaling stage, the individuals that will be used in selection stage are determined.
where $f_{i, n_{av}}$ and $f_{i}$ represent average fitness value of the population and fitness value of the $i^{th}$ individual that equals to objective function defined in Eq. (26), respectively. The individuals whose fitness values are better (smaller) than the average fitness value are used in selection stage.

In selection stage, the parents to be crossed for producing children are selected within the determined individuals. For selection of the parents within these determined individuals, tournament method is performed and can be formulated as,

$$g_{i} = \frac{f_{i}}{\sum_{j=1}^{n_{pop}} f_{j}}$$

where $g_{i}$ is the weight of the $i^{th}$ individual. The weight of an individual defines the elective probability in this stage and the sum of the weights within population is 1.

$$\sum_{i=1}^{n_{pop}} g_{i} = 1$$

The number of the individuals for selection of the parents within the population is twice of the children number determined in the beginning of the algorithm. The parents in the same number of the children are selected within these selected individuals.

In crossover stage, the children are produced as new individuals by parents determined in selection stage. These new individuals in the same number of the children are produced through the crossing method. 0 and 1 values in the same number of individual’s gen number for crossing are randomly produced. If the value is 0, then gen is taken from father, if the value is 1, then gen is taken from mother and thus the child is produced. Crossing process can be illustrated as follows:

Cross: 0 1 1 0 1  
Mother: a b c d e  
Father: u w x y z  
Child: u b c y e

In mutation stage, new individuals are produced to be changed all or some gens of the selected individuals that undergo mutation within the population. The number of the selected individuals is determined in the beginning of the algorithm. These individuals are reproduced to be formed all the gens of the selected individuals within algorithm. Thus, new individuals in the same number of the selected individuals that undergo mutation are randomly produced by Eq. (28). Mutation process increases diversity of the population and prevents losing the individuals that provide good solutions.

There are many criterions for optimization that is performed by GA and similar heuristic methods in the literature. In this study, iteration number is selected for stopping the optimization by GA. The algorithm stops when it reaches the determined maximum iteration number. Additionally in this study, system constraints are included in the optimization stopping criterion so that all of the system constraints are provided at the end of the algorithm.

Results

The proposed approach’s accuracy and efficiency are tested on the modified IEEE 14-bus ac-dc test system shown in Fig. 4.

Total active power loss variation throughout the algorithm is shown graphically in Fig. 5. Total active power loss gained with proposed approach and another method, steepest descent algorithm (SDA), are compared in the same test system. In the literature, generally, 100 iterations are performed for GA. GA approximately reached to global optimum at 45th iteration for this study. For GA; 20 population sizes, 0.5 crossover rate and 0.1 mutation rate are used. These values used for GA are found at trials. The values used beyond current values didn’t change the global optimum for GA. The situation, using more sizes than the above values, decreased the number of iteration but increased optimization time in order to reach the global optimum. Penalty coefficient values $c$ used in Eq. (26) are found after several trials. 30 different optimization trials are done for GA method. The worst and the best total active power loss values for GA are 0.0697 per unit and 0.0664 per unit, respectively. Error deviation for GA is 4.96%. At the end of the optimization, it is observed that all control and state variables are in their limit values.
The proposed approach is better and more reliable than SDA [25] shown in Table 1 for reaching global optimum.

Table 1. Comparison of the results for the test system

<table>
<thead>
<tr>
<th>$p_{\text{min}}$(MW)</th>
<th>GA</th>
<th>SDA [25]</th>
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<tr>
<td>6.640</td>
<td></td>
<td>8.532</td>
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**Conclusion**

In this article, a new approach is proposed for optimal reactive power flow in multi-terminal ac-dc systems. This approach is one of the rare ones in optimal reactive power flow studies in multi-terminal ac-dc systems. GA is utilized for the first time in multi-terminal ac-dc system for ORPF solution in this study. Because of using sequential method in ac-dc power flow, any ac and dc power flow method can be utilized without any change in optimization algorithm. Unlike the other studies in the literature, converter active powers are used as control variables for optimization in the entire dc system in this study. So, both the most suitable converter active powers ($p_a^\text{min} \leq p_a \leq p_a^\text{max}$) and converter types (rectifier or inverter) are achieved at determined system conditions. Therefore, efficiency of the active power loss optimization is enhanced. The obtained results demonstrated that the proposed approach is better and more reliable for reaching global optimum than similar approaches not to drop to local minimum points. The proposed approach utilizes optimization technique to prevent the violation of system constraints.

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