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The error analysis of the three-pass differential interferometery

***Abstract**: The three pass differential interferometry technology is an important means of ground deformation monitoring, which has been successfully used in the large ground deformation monitoring and research such as the earthquake, volcano activity, glacial drift, landslides, and city settlement etc. However the technology is affected by multiple errors in practical application, which have serious influence on the deformation monitoring precision. For spaceborne radar, this paper derives the error propagation coefficients of the baseline error, the phase error, the atmospheric delay error and the earth curvature error and other types of errors on the three pass differential interferometry base on the [three pass differential interferometry] principle, and analyses the characteristics of these errors, finally discusses the influence rules of the errors on three pass differential interferometry.*

***Streszczenie.** Artykuł dotyczy zagadnienia trójpasmowej interferometrii różnicowej, jako narzędzia do monitorowania deformacji gruntu w przypadku trzęsień ziemi, aktywności wulkanicznej itp. Na potrzeby radarów znajdujących się na orbicie kosmicznej, wyznaczono współczynnik propagacji błędów, typowych w tego rodzaju pomiarach (błąd fazowy, opóźnienie w atmosferze, zakrzywienie powierzchni Ziemi). Opisano mechanizm wpływu obecności tych błędów i uchybów na wynik działania metody. (**Analiza błędów i uchybów w trójpasmowej interferometrii różnicowej**).*

Keywords: InSAR; three pass differential interferometry; atmospheric delay; error analysis
Słowa kluczowe: InSAR, trójpasmowa interferometria różnicowa, opóźnienie w atmosferze, analiza błędów.

1 Introduction

The differential interferometric synthetic aperture radar (D-InSAR) is the technology to obtain surface change information from the phase information of the complex image got by synthetic aperture radar [1]. As a new mean of earth observation from space, the current D-InSAR technology has been successfully used in the large ground deformation monitoring and research such as earthquake, volcano, glacial drift, landslides, and city settlement etc. According to the removal method of the topographic information, the D-InSAR technology can be divided into two-pass, three-pass and four-pass methods, and three pass method is most commonly used, which can mostly embody characteristics and advantages of the differential interferometry technology [2]. The ground deformation monitoring results got by the three pass differential interferometry were affected by the radar parameters, the earth curvature, the baseline, the interferometric phase, the atmospheric delay and other factors. Based on the above issues, this article discussed the related factors of measurement by the three pass differential interferometry.

2 The principle of the 3-pass differential interferometry

The Fig.1 is the geometric diagram of repeated track interferometry. S_1 , S_2 are respectively the satellite antenna position before terrain deformation and the satellite antenna position after the terrain deformation where the same area is twice imaged [3]. The baseline distance B_1 represents the spatial distance between the two antennas S_1 and S_2 , α_1 is the angle between the baseline B_1 and the horizontal direction. H is the satellite platform elevation, the distance of the target point P to the two antennas S_1 and S_2 is ρ_1 and ρ_2 , θ is the incidence angle, the elevation of the target point is represented by h [4].

Suppose the surface deformation does not occur during the observation period, the measuring phase difference of the antennas S_1 and S_2 on the target point P is:

$$\varphi_{12} = \frac{4\pi}{\lambda}\left(\rho_2 - \rho_1\right) \tag{1}$$

In the Fig.1, the geometrical relationship can be drawn.

$$B_{//} = B_1 \sin\left(\theta - \alpha_1\right) \tag{2}$$

$$B_{\perp} = B_1 \cos\left(\theta - \alpha_1\right) \tag{3}$$

The elevation h of the target point P is:

$$h = H - R_1 \cos\theta \tag{4}$$

where: $B_{//}$ is the component of the baseline distance B_1 along the line sight, and $B_{\perp}$ is the component of the baseline distance B_1 perpendicular to the line sight [6].

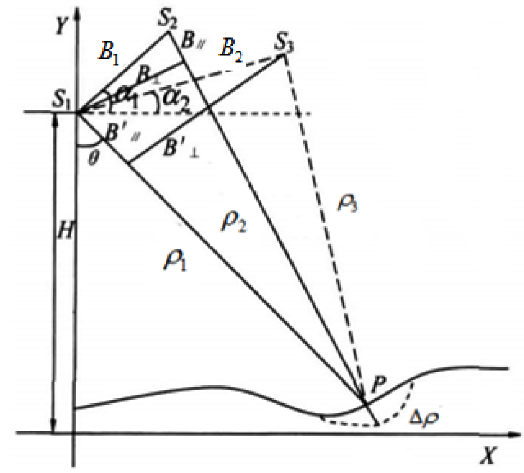

Fig .1.The sketch of the three pass differential interferometry

By the formula (2) and the formula (1), we can get:

$$\varphi_{12} = \frac{4\pi}{\lambda} B_1 \sin\left(\theta - \alpha_1\right) = \frac{4\pi}{\lambda} B_{//} \tag{5}$$

During the observation , S_3 is the antenna position of satellite imaging after the terrain changes if the surface deformation occurred, $\Delta\rho$ is the variation on the oblique distance direction caused by the change of the surface trace deformation [7], the measuring phase difference of the antenna S_1 and S_3 on the target point P after the deduction of earth surface effect is deducted from [8]:

$$\varphi_{13} = \frac{4\pi}{\lambda}\left(B_2 \sin\left(\theta - \alpha_2\right) + \Delta\rho\right) = \frac{4\pi}{\lambda}\left(B'_{//} + \Delta\rho\right) \tag{6}$$

where: $B'_{//} = B_2 \sin(\theta - \alpha_2)$.

Combined with the formula (5) and (6), it can be the ratio of φ_{12} and φ'_{13} .

(7) $$\frac{\varphi_{12}}{\varphi'_{13}} = \frac{B_{//}}{B'_{//}}$$

where: φ'_{13} is the phase difference contributed by terrain relief in the second interferometer, the deformation phase φ_d of the surface trace deformation place can be obtained based on the formula(7) [9].

(8) $$\varphi_d = \varphi_{13} - \frac{B'_{//}}{B_{//}}\varphi_{12} = \frac{4\pi}{\lambda}\Delta\rho$$

In the formula (8), if the value of $\Delta\rho$ is positive, it indicates that the ground deformation is elongated along the direction of the radar sight, on the contrary, is shrank along the radar line-of-sight direction [10]. Unwrapping the differential phase in formula (8) , the deformation variable of ground target can be obtained [11]. The differential interferogram will add a whole circumference stripe, if the ground deformation along the direction the radar sight gets to 2.8 cm by the formula (8).Therefore, the surface deformation of centimeter level can be monitored by the differential interferometry technique [12]. When the phase measurement error is less than 2.24 rad, the theory accuracy of the deformation observation is less than 1.0 cm, and will be the millimeter accuracy.

3 Analysis of the influencing factors

The three pass differential interferometry uses the radar echo phase of the ground target to obtain the ground deformation information, and the radar echo phase is often influenced by the terrain, terrain features, atmospheric conditions and other factors, therefore, these factors directly affect the accuracy of differential interferometry [14].

3.1 The influence of the phase measurement error

The relationship of the phase error generated by the ground deformation and the displacement in the line of sight direction of the ground deformation can be obtained by the formula (6), (7) and (8), if baseline calculation has no error, and the influence of the earth curvature and the atmosphere is not considered.

(9) $$\delta_{\Delta\rho} = \frac{\lambda}{4\pi}\left[\varphi_{13} - \frac{B_2\sin(\theta-\alpha_2)}{B_1\sin(\theta-\alpha_1)}\varphi_{13}\right]$$

Suppose $\sigma_{\varphi_{12}} = \sigma_{\varphi 13} = \sigma_\varphi$,the influence of phase error on the deformation can be obtained by the propagation law of error.

(10) $$\sigma_{\Delta\rho} = \frac{\lambda}{4\pi}\left(1 - \frac{B'_{//}}{B_{//}}\right)\sigma_\varphi$$

By the formula (8), the sensitivity of the phase difference and the ground deformation in the direction of line sight is:

(11) $$\frac{\Delta\rho}{\varphi_d} = \frac{\lambda}{4\pi}$$

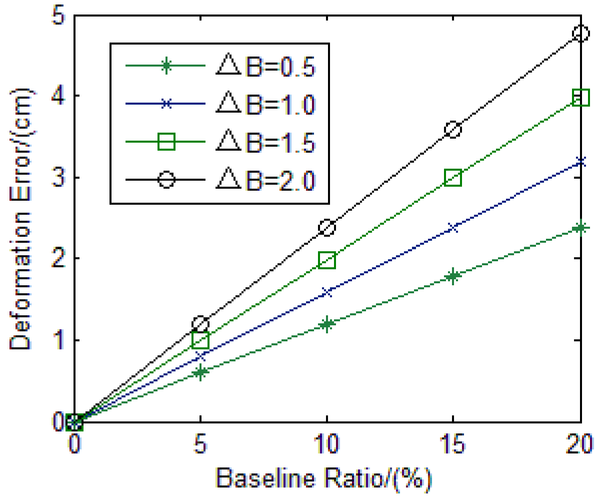

Fig. 2.The relationship of the Phase and the deformation error

Taking ERS-1 satellite for example, and $\lambda = 5.7$ cm, the Fig.2. shows impacts of three pass interferometry from four different baseline ratios on the phase [15-19]. From the Fig.2, we know that, the influence of phase error on the three pass interferometry is minimum, and the value is approximately 22 mm, when the baseline ratio is 0.5. It known that from shown in the formula (11), the phase is very sensitive to the terrain change, such as $\varphi_d = 2.8$ cm, which will cause the change of 2π in the three pass differential interferometry.

3.2 The influence of the baseline error

The baseline error is caused by the uncertainty of the baseline, which is related to the uncertainty of the satellite attitude and the baseline length. We analyses the influence on deformation error of the baseline length error and the baseline inclination error. If the phase measurement error and the influence of topographical factors are ignored, by the formula (6), formula (8) and formula (11), we can get:

(12) $$\Delta\rho = B_1\sin(\theta-\alpha_1) - B_2\sin(\theta-\alpha_2)$$

By the formula (10) we can get the effect of baseline error on deformation in the three pass interferometry:

(13) $$\sigma_{\Delta\rho} = \sqrt{\sin(\theta-\alpha_1)^2\sigma_{B_{12}}^2 + \sin(\theta-\alpha_2)^2\sigma_{B_{13}}^2}$$

Generally the baseline length is zero before the topographic change in the interference measurement, which can obtain the better result of removed topographic, considering the above, suppose that $\sigma_{B_{12}} = 0$, $\sigma_{B_{13}} = \sigma_B$, there is:

(14) $$\sigma_{\Delta\rho} = \sin(\theta-\alpha_2)\sigma_B$$

As we can be seen by the above formula, the deformation error in the line-of-sight direction is associated with baseline inclination [20]. The SAR satellite of ERS-1, JERS-1, and SIR-A are selected, whose wavelength are respectively 23°, 38°, and 50°, and we select the baseline inclination from zero to ninety degrees to conduct simulation experiments, the deformation error of baseline obliquity is shown in Fig.3.

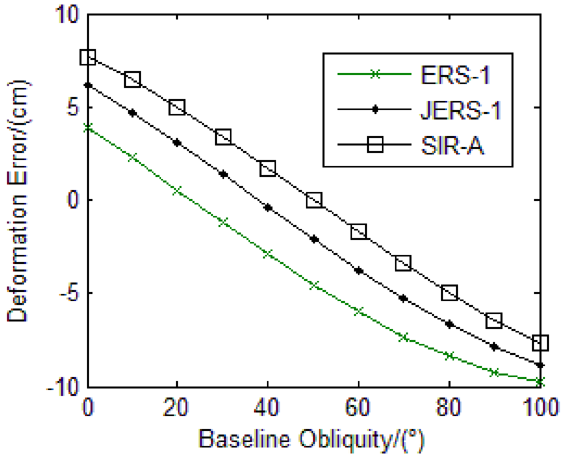

Fig. 3.The relationship of the baseline and the deformation error

As seen from Fig.3, the effect of the baseline length error on the three pass differential interferometry is decreasing with vertical baseline length increasing and the effect of the baseline inclination error on the three pass differential interferometry is increasing with vertical baseline length increased. Therefore, when using differential interferometry technique to extract the surface deformation information, we try to use the SAR image obtained by transmitter with small incident angle and the SAR image whose effective baseline is shorter to reduce the impact of baseline error on the three pass differential interferometry.

The satellite in flight is influenced by various factors, from which the baseline inclination will bring the errors; the error will affect the D-InSAR deformation measurement [21]. Supposed the phase measurement error is considered, and the errors of two baseline inclination α are equal, by the formula (12) based on the error propagation rate, we can get:

(15) $$\sigma_{\Delta\rho} = \sqrt{B_\perp^2 + B_\perp'^2}\,\sigma_\alpha$$

where: $B_\perp' = B_2 \cos\left(\theta - \alpha_2\right)$

Taking SAR satellite of ERS-1 (C band) that λ = 0.056 m , θ =23° ,H=758000 m and ρ =852000 m, for example to simulate experiments, we can get the influent of baseline inclination error on the three pass differential interferometry. The relationship of the inclination and the deformation error is shown in Fig.4.

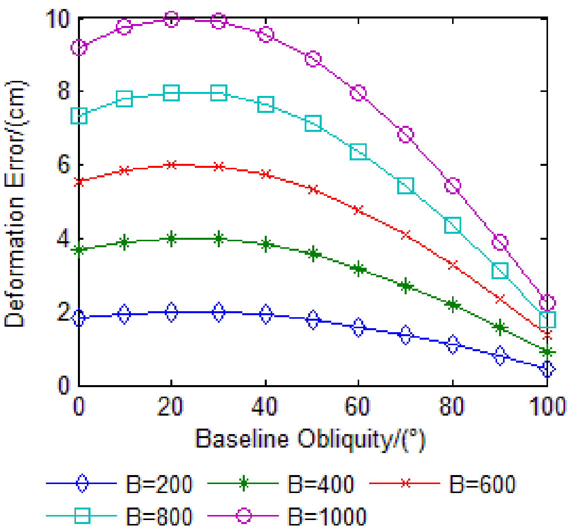

Fig .4.The relationship of the Inclination and the deformation error

As can be seen from the Fig.4, the longer the valid baseline is, the greater the impact of the baseline inclination on the three pass differential interferometry is, and the measurement error is decreasing with the baseline inclination increased. Compared the formula (14) and (15), we can see that the three pass differential interferometry is more sensitive to baseline inclination error than to the baseline length error [22]. Therefore, using the reasonable effective baseline is a key factor to weak the impact on the three pass differential interferometry.

3.3 The influence of the atmospheric delay

The deformation phase error formula (10) is in the ideal state, removing other factors, and considering atmospheric delay errors, the geometric model of the deformation phase error in the three-way differential interferometry can be expressed as:

(16) $$\varphi_d = \frac{4\pi}{\lambda}(\varphi_{13} - \varphi_{12} + \varphi_p^2 - \varphi_p^1)$$

where: φ_p^2 , φ_p^1 is respectively the ground atmospheric delay in one-way distance of the ground point P at time t_2 and t_1, the atmospheric delay errors of the absolute phase can be expressed as:

(17) $$\Delta\varphi = \frac{4\pi}{\lambda}(\Delta\varphi_p^2 - \Delta\varphi_p^1) = \frac{4\pi}{\lambda}\Delta\varphi_p^{2,1}$$

The covariance matrix of the phase $\varphi_d = \left[\varphi_{12} \quad \varphi_{13}\right]$ in three pass differential interferometry by the above formula is [23]:

(18) $$D_{\varphi_d\varphi_d} = \begin{bmatrix} \sigma_{\varphi_{12}} & \frac{1}{2}\sigma_{\varphi_{12,13}} \\ \frac{1}{2}\sigma_{\varphi_{13,12}} & \sigma_{\varphi_{13}} \end{bmatrix}$$

According to the error propagation law, if the tropospheric delay errors of absolute phase in three images are respectively σ_{φ_1} , σ_{φ_2} and σ_{φ_3} ,which are unrelated and equal, the effect of the phase errors caused by atmospheric effects in the three pass differential interferometric on the surface deformation is :

(19) $$\sigma_{\Delta\rho} = \frac{\lambda}{4\pi}\sqrt{1 - \left(\frac{B_\perp'}{B_\perp}\right) + \left(\frac{B_\perp'}{B_\perp}\right)^2}\,\sigma_{\Delta\varphi}$$

where: $\sigma_{\Delta\varphi}$ is the phase error of interferogram.

As the formula (19) shows, the deformation accuracy is related to the size of vertical baseline ratio ΔB ($\Delta B = B_\perp' / B_\perp$) .We should choose images with longer vertical baselines as topographic images, and images with shorter vertical baseline as deformation images in the three pass differential interferometric.

Taking ERS satellite for example, the fig.5 shows the influence of the wet delay of four different baseline ratios on the three pass differential interferometric. From the diagram, when the baseline ratio is 0.5, the tropospheric delay has the minimum impact on the deformation of the three pass differential interferometric, which is approximately 20mm; the relationship of the deformation accuracy of wet delay and the baseline ratio is shown in Fig.6, as seen from the Fig.6, we can know that the relationship of the baseline ratio and the deformation error is proportional, and the relative error is increasing with the increase of the baseline.

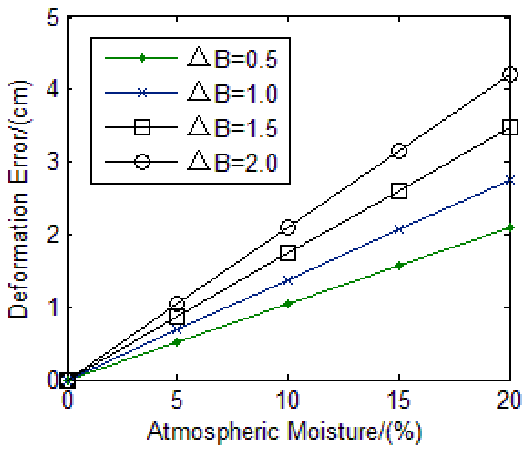

Fig. 5.The relationship of the atmospheric moisture and the deformation error

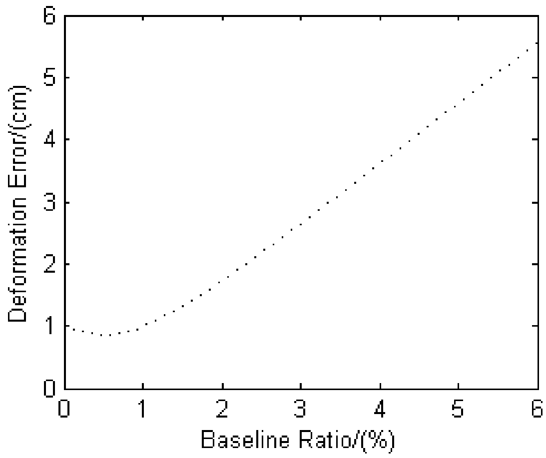

Fig .6.The relationship of the deformation accuracy of wet delay and the baseline ratio

3.4 The influence of the earth curvature

The spaceborne radar imaging area is about 100×100 km, the influence of the earth curvature in the three pass interference measurement must be considered [24]. Setting P_0 as the corresponding ground point P in the reference ellipsoid, the distance of the target point P_0 to the antenna S_1 is ρ_0 , and $\rho_1 = \rho_0$. The slant range difference is shown in Fig.1.

$$\delta_{\rho_{12}} = B_{\|} + \frac{B^2}{2\rho_1} = B_{\|} + \Delta \tag{20}$$

Considering the influence of the curvature of the Earth, the slant range difference is:

$$\delta'_{\rho_{12}} = B_{\|_0} + \frac{B^2}{2\rho_1} = B_{\|_0} + \Delta_0 \tag{21}$$

The phase difference whose curvature of the Earth is removed is:

$$\varphi_{flat} = \varphi_{12} - \varphi'_{12} = \frac{4\pi}{\lambda}\left(B_{\|} + \Delta - B_{\|_0} - \Delta_0\right) \tag{22}$$

where: $\rho_1 = \rho_0$, we can get:

$$\varphi_{flat} = \frac{4\pi}{\lambda}\left(B_{\|} - B_{\|_0}\right) = \frac{4\pi}{\lambda}\left(B_1 \sin(\theta - \alpha_1) - B_1 \sin(\theta_0 - \alpha_1)\right) \tag{23}$$

Letting $\delta_\theta = \theta - \theta_0$, because δ_θ is a small amount, which is caused by the elevation of the ground point P .So there is:

$$\varphi_{flat} = \frac{4\pi}{\lambda} B_1 \delta_\theta \cos\left(\theta_0 - \alpha_1\right) = \frac{4\pi}{\lambda}\delta_\theta B_{\perp_0} \tag{24}$$

where: $B_{\perp_0}$ is the direction component that the baseline B_1 is perpendicular to the satellite to the ground corresponding point P_0 . The interferometric phase after the earth curvature correction is affected by two factors: the terrain change relative to the reference ellipsoid and the surface deformation in the survey area [25].

Similarly, the phase difference after deformation is:

$$\varphi'_{flat} = \frac{4\pi}{\lambda}\delta_\theta B'_{\perp_0} + \frac{4\pi}{\lambda}\Delta\rho \tag{25}$$

where: $B'_{\perp_0}$ is the direction component that the baseline of the satellite S_1 and S_3 is perpendicular to the direction of the satellite to the ground point P_0 .

In summary, the influence of the curvature of the Earth is:

$$\Delta\rho = \frac{\lambda}{4\pi}\left(\frac{B'_{\perp_0}}{B_{\perp_0}}\varphi_{flat}\varphi'_{flat}\right) \tag{26}$$

As can be seen by the above formula (26), the deformation of the line-of-sight direction is related to the vertical component of the B_1, the direction component ratio on the vertical component of B_1 ,and the component of the satellite to the ground point P_0, and the phase difference before and after deformation. If there is no deformation in the observation period, the ratio of the interferometric phase after the ground phase removed is equal to the ratio of the vertical baseline component regardless of the terrain.

4 Conclusions

Based on the basic principles of the three pass differential interferometery， this paper deduced the formula of the baseline error, phase error, the Earth curvature error and the atmospheric delay error on the three pass differential interferometery, and analyzed the impact on the three pass interferometer measuring for the deformation. Through the above analysis, for space-borne radar, the impact of the phase measurement error on the deformation measuring is lower than the impact on the terrain; the influence on the deformation of the surface oblique distance direction of the baseline horizontal component is about 1 times more than that of baseline vertical component; the accuracy effect of atmospheric delay error on the deformation is related to the size of the vertical baseline ratio, we should choose the long vertical baseline images as the terrain images, and the short vertical baseline images as the deformation images in the three pass differential interferometry.

In spaceborne radar measurement, the deformation effect of the curvature of the Earth cannot be ignored. In addition, the longer the effective baseline is, the smaller the impact of the baseline component error on the ground point elevation. We should select the shorter baseline images in order to control or reduce the impact on the terrain when we use interference technique to extract the ground deformation information.

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