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Development of Macromodels using the Evolutionary Algorithms of Optimization

Abstract. *A new technique for mathematical model development based on evolutionary algorithms and optimization approach to macromodels construction has been proposed in this paper. Examples of models constructed with the use of the proposed algorithm have been presented. A comparison of new models and models created using sole optimization approach has been done.*

Streszczenie. *Praca przedstawia nową metodę budowania makromodeli przy użyciu algorytmów ewolucyjnych optymalizacji. Proponowane podejście zostało zilustrowane przykładami konkretnych modeli, a wyniki porównane z tymi uzyskanymi przy zastosowaniu tradycyjnych deterministycznych metod optymalizacji (**Budowa makromodeli z użyciem algorytmów ewolucyjnych**).*

Keywords – mathematical model, macromodel, optimization, evolutionary algorithms
Słowa kluczowe: modelowanie matematyczne, optymalizacja, algorytmy ewolucyjne

Introduction

At present time there is a considerable progress in construction of macromodels of electrotechnical components designed for the transients' analysis. The permanent growth of macromodel complexity leads to the necessity of the macromodel construction process automation in case it is possible. A promising approach in this direction is the usage of optimization.

According to this technique the values of macromodel coefficients are found by the minimization of some goal function $Q(\lambda)$. This function represents the deviation of the object behavior calculated using the model being constructed and experimental data as a function of model coefficients. By finding the point where the mentioned function reaches its minimum we will find an optimal set of the model coefficients within a selected mathematical form of the model representation.

Described approach can be used for the construction of macromodels in any mathematical form, which is represented by a limited set of coefficients. Also it does not impose any restrictions on required input information except the obvious requirement to describe the object fully enough. Additionally, the usage of optimization eliminates the calculation problems related to the fact that mathematical model identification is often an ill-conditioned problem.

An important question in optimization approach is the selection of mathematical form of model representation and a corresponding set of model coefficients. Optimization approach allows us to build macromodels in any form but does not answer the question which of the possible model forms should be selected.

There are two common ways to answer this question:

- to rule out a "black box" approach and take underlying physics into account;
- to apply certain widely used model representation, for example, discrete state variables form with polynomial approximation of nonlinearities [1]:

$$(1)\qquad \begin{cases} \mathbf{x}^{(k+1)} = \mathbf{F}\mathbf{x}^{(k)} + \mathbf{G}\mathbf{v}^{(k)} + \mathbf{\Phi}\left(\mathbf{x}^{(k)}, \mathbf{v}^{(k)}\right) \\ \mathbf{y}^{(k+1)} = \mathbf{C}\mathbf{x}^{(k+1)} + \mathbf{D}\mathbf{v}^{(k+1)} \end{cases}$$

where $\mathbf{v}$ is a vector of input values; $\mathbf{y}$ is a vector of output values; $\mathbf{x}$ is a vector describing the internal state of the object; $\mathbf{F}$, $\mathbf{G}$, $\mathbf{C}$, $\mathbf{D}$ are matrices of the model coefficients; $\mathbf{\Phi}$ is some nonlinear vector-function of many arguments; k is a discrete index.

The discrete form of representation is best suited for computer calculations because it allows us to omit approximation of input data arrays. The state equations form is also determined by convenience of further usage of the model as a component of dynamic system containing a large number of elements.

Both mentioned approaches have significant disadvantages. Taking physics into account requires from a researcher to do the work which is not appropriate when we are trying to develop an automated procedure. Polynomial approximation leads to a big number of unnecessary model coefficients and thus to not efficient and often not adequate model.

A technique which allows us to select automatically an optimal mathematical form of macromodel representation based on optimization approach to macromodel construction has been proposed in this paper. This technique is based on evolutionary algorithms [2].

Evolutionary algorithm for model form selection

The idea of proposed algorithm comes from the "Occam's razor" principle, which states that the best model among the models of same accuracy is the simplest model. For practical usage of this criterion we need to modify our goal function by adding a term that takes a model complexity into account:

$$(2)\qquad Q_e(\boldsymbol{\lambda}) = Q(\boldsymbol{\lambda}) + Q_c$$

where Q_c represents a model complexity. In the simplest case it can be calculated as $Q_c = N \cdot \gamma$, where N is a number of non-zero coefficients and γ is a weight coefficient.

It is obvious that the number of alternative model forms is considerable thus conducting an optimization of all possible forms is not practical because it requires too much computational resources. An evolutionary algorithm can be used to make the selection procedure of the best form of the model being constructed more efficient. In general this algorithm is as follows.

Let's introduce the term "generation", which includes some set of intermediate models of the object (usually in different forms of model representation). Every iteration stage of the algorithm comprises following steps:

1. to form a new "generation" of model forms by selecting a random form from the previous "generation" and make a "mutation" by adding or removing some random coefficient. The number of model forms in this new "generation" is set by operator but generally should be commensurable with the total number of possible coefficients.

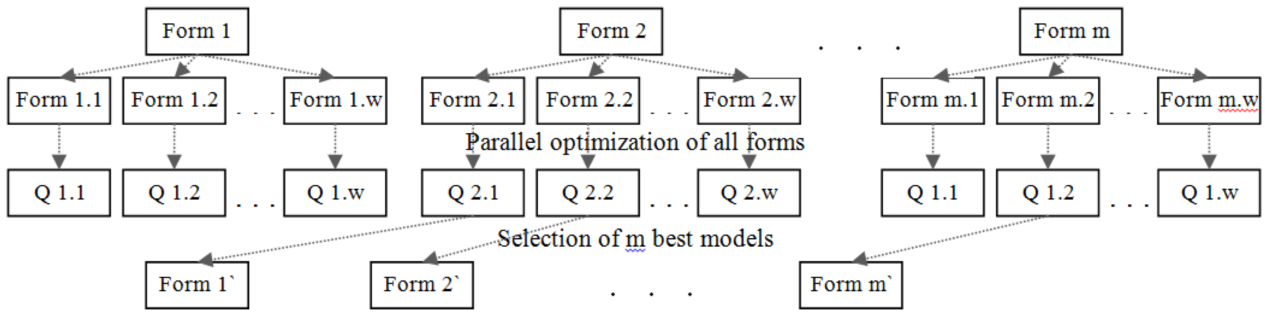

Fig.1. An evolutionary approach for model form selection

2. to execute a limited number of optimization algorithm steps for each form in new "generation". The number of iterations should be not very big but sufficient for a noticeable reduction of the goal function $Q(\lambda)$. Theoretically the number of iterations can be increased with each new "generation" because the effectiveness of optimization algorithm generally decreases when we come closer to the optimal solution. But authors used a constant number of iterations equal to 100.

3. to conduct a "selection": leave only limited number of best model forms according to the criterion (2)

4. to switch to the next generation and continue from step 1.

Schematically these steps are shown in Fig. 1.

A criterion for procedure completion in this case can be some number of iterations without better result found according to the expression (2). It should be pointed out that the best model does not need to be present in the last "generation". Because of "mutations" we can have a situation when all models in the last "generation" are worse than some model developed on previous steps of the algorithm execution. This should be taken into account in practical implementation of this algorithm.

Described approach allows us to automate the selection of optimal mathematical form of model representation, though it requires much more computational resources in comparison to a single model construction. The weight coefficient γ is used to specify the desired balance between the accuracy of the model and its complexity. Authors used next empirical rule to select the value of this coefficient:

(3) $$\gamma = Q_{\exp} / N_{\exp},$$

where $Q_{\exp}$ is expected value of goal function $Q(\lambda)$ at the end of the model construction, $N_{\exp}$ is expected number of corresponding model coefficients. The expression (3) has been received on the basis of the assumption that at the end of the model construction procedure both summands of the expression (2) should be of the same order.

The effectiveness of the proposed algorithm has been tested by construction of a macromodel of a single-phase asynchronous motor with starting capacitor in instantaneous values and an asynchronous motor with cage rotor A051A4 using RMS values based on experimental transients' data. These objects have been selected because there are models constructed for them using different variations of optimization approach [3], [4]. It gives us the possibility to compare the proposed approach with other approaches to model construction.

Base information for mathematical model construction has been taken from experimental measurements of current consumption and rotor speed during the motor's starting. These characteristics are shown in Fig. 2 and 3. These oscillograms were obtained using AD/DA converter 'ADA-1406'. The sampling frequency is of 2 kHz.

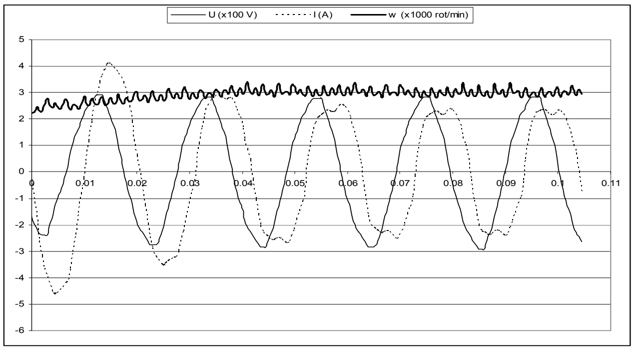

Fig.2. Voltage, current and rotor speed during start of the single-phase asynchronous motor with starting capacitor

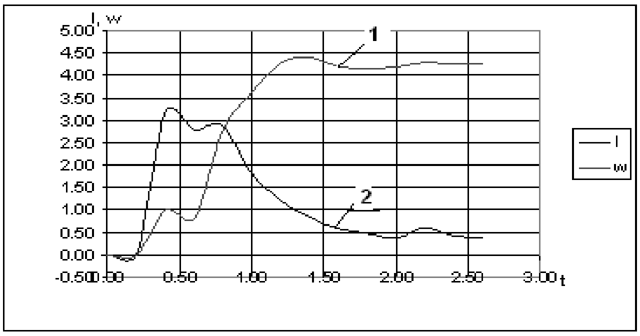

Fig.3 Rotor speed (1) and RMS of current (2) in relative values during start of the motor with cage rotor A051A4

Because the motor A051A4 is symmetrical with respect to phases only the values for the phase *A* have been used.

The instantaneous value of the voltage u applied to the motor has been selected as an input variable for the model of the single-phase asynchronous motor with starting capacitor in instantaneous values. The instantaneous value of current i and rotor speed ω have been selected as output variables. Thus vectors of input and output variables have the next form:

(4) $$\mathbf{v}^{(k)} = \left(u^{(k)}\right), \quad \mathbf{y}^{(k)} = \begin{pmatrix} i^{(k)} \\ \omega^{(k)} \end{pmatrix}$$

For the model of the motor A051A4 in RMS values input variables are: RMS of phase *A* voltage U applied to the motor and moment of force M applied to the rotor. The output variables for this motor are RMS of the current I in the same phase and rotor speed ω. Thus vectors of input and output variables have the next form:

$$\mathbf{v}^{(k)} = \begin{pmatrix} U^{(k)} \\ M^{(k)} \end{pmatrix}, \ \mathbf{y}^{(k)} = \begin{pmatrix} I^{(k)} \\ \omega^{(k)} \end{pmatrix} \quad (5)$$

In both cases the models have been constructed in the form of discrete state variables (1) using optimization algorithm based on the Rastrigin's direct cone method. Macromodels have been built using next approaches:

1. In the general form (1) with cubic approximation of nonlinearities. Splitting against output variables [1] has been used for the model of the single-phase asynchronous motor with starting capacitor in instantaneous values because of complexity of the optimization task.
2. A model form has been selected based on the analysis of physical processes in the motor [4]. It should be noted that the coefficients set goes beyond the range of cubic approximation, though the total number of the model coefficients is relatively small. This approach has been used only for the model of the single-phase asynchronous motor with starting capacitor in instantaneous values because this model is more complex.
3. A model form has been found using the algorithm proposed in this article. The selection of the coefficients set has been done within the form (1) with cubic approximation of nonlinearities. Thus within the option #1.

The results of construction of the models of the single-phase asynchronous motor with starting capacitor in instantaneous values using all mentioned approaches are summarized in table 1.

Table 1. Results of the single-phase asynchronous motor with starting capacitor model construction

The procedure for model form selection	1. general cubic approxi-mation	2. form based on physics analysis	3. Proposed approach
The number of iterations of optimization algorithm	$\sim 4\cdot 10^6$	$\sim 4\cdot 10^4$	$\sim 3\cdot 10^5$
Resulting model precision	11%	10.3%	10%
Size of vector *x*	3	2	2
Max power used for nonlinearity approximation	3	4	3
Total number of non-zero model coefficients	64	7	19

As it can be seen, the model of the single-phase asynchronous motor with starting capacitor in instantaneous values created using proposed algorithm is simpler than the model created without evolutionary model form selection, but is more complex than the model constructed using analysis of physical processes in the motor. But in this case the simpler form obtained using physics analysis is not available for the proposed algorithm, because it includes the coefficients of the 4-th power.

Same results have been obtained for the model of the asynchronous motor A051A4 with cage rotor using RMS values. The model obtained using proposed approach is simpler in comparison to the model for the same object constructed previously [3] (new model has 18 coefficients instead of 69). Both models are of the same accuracy (3%).

The main problem of the proposed approach to model form selection is the large amount of calculations needed to estimate alternative model forms. But there can be exceptions. For example, during construction of the model for the single-phase asynchronous motor with starting capacitor in instantaneous values the total number of iterations of optimization algorithm was smaller for the proposed approach in comparison to the optimization of single classical form with cubic approximation of nonlinearities. This can be explained by rapid increase of computational resources needed to solve the optimization task when number of unknown coefficients increases. In case of the proposed approach the number of model coefficients has a tendency to be limited during the whole process of evolutionary selection of the optimal model form.

From the other side, the large amount of needed computations can be easily parallelized. The most computation resources in the proposed approach are needed to perform optimization of all model forms in a "new generation" (see step #2 in the description of the proposed approach). But this step can be executed independently for each tested model form. Such parallelization does not require communication between computers performing optimization of different model forms. Thus such parallelization can be done even using internet communication what significantly simplifies building of big computational clusters.

Implementation of such parallelization using internet communication has been done by authors but its discussion goes beyond the scope of this article.

Summary

The proposed evolutionary selection of the best model form is based on the mathematical model construction that utilizes optimization, and allows performing systematic search through all possible forms of model representation. Owing to this it is possible to find automatically the optimal form for practical purposes.

The main disadvantage of the proposed approach is a large amount of calculations caused by the need to perform evaluation of different model forms. But the most CPU consuming part is easily parallelizable and does not require quick communication between computers, so it can be done using communication via internet.

Examples of models constructed to test this approach show that it is practically useful, and produces relatively simple and at the same time precise models.

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