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## Separating Storm surge from Tidal Level Based on Wavelet Packet Transform

**Abstract**. A method of wavelet packet transformation (WPT) is presented in this paper, which regards the tidal level as a signal with containing different frequency of astronomical tidal components and storm surge components. The tidal level signal can be decomposed serials of frequency bands signals by using WPT to carry out time-frequency analysis for procuring the representative frequency bands of storm surge. Then the storm surge process gained by reconstructing the exclusive decomposed wavelet packet coefficients. Taken the typhoon NO.9608 measured tidal level of Xiamen tide station as an example, the storm surge calculates by using the presented method, which compared the value calculated by transformation linear method with the measured value. The result shows that the suggested method can preferably separate storm surge and provide a new way to separate storm surge from short-term tidal level material.

**Streszczenie.** W artykule przedstawiono algorytm realizacji transformaty Falkowej, analizujący dane o astronomicznych parametrach pływów oraz podniesieniu się poziomu morza w czasie sztormu (nagon). Opracowana metoda pomaga w efektywnym odróżnieniu tych dwóch zjawisk, co ogranicza ryzyko fałszywego alarmu. Na podstawie rzeczywistych danych, dokonano jej porównania z metodami liniowymi, co wykazało sprawniejsze działanie. (Odróżnianie nagonu szturmowego od wysokości pływu w oparciu o transformatę Falkową WPT).

Keywords: Storm Surge Separation, Wavelet Packet Transform, Frequency Bands, Astronomical Tidal Constituent. Stowa kluczowe: nagon szturmowy, transformata falkowa WPT, pasma częstotliwościowe, astronomiczna składowa pływów.

## Introduction

In the research of storm surge physical mechanism and forecast method, the most basic question is separation of storm surge1, that's to say, how to objectively separate the pure level caused by meteorological factors including by the hydrology factors derived from the meteorological factors. Currently, the commonly solved is difference method at home and abroad2, namely linearly minus the forecasted astronomical tide process from the measured tidal level. But the separated storm surge has distinct tide cycle3. For this physical phenomenon, domestic and foreign researchers believe that there are two main reasons4; the one is from the perspective of dynamics5, which holds that the water movement is a nonlinear phenomenon under the joint action of astronomical tide-generating force and meteorological forcing force, which also be called nonlinear interaction effects between storm surges and astronomical tides, that's the main reason lead to the storm surge process curve has tide cycle6. Another is from the perspective of kinematics7, which considers that storm surge is a long-wave motion, so that great changes for the tidal level of tide station have taken place when typhoon was away from the coast, and the details of the changes are the ahead of appearance time for high and low tide and the risen of height for tidal level actually. But at this time, the forecast astronomical tide level and hour which have ignored the changed weather factors would induce errors, so that the storm surge calculated by measured level minus forecast astronomical tide would show up tide cycle8. In this paper, a new method called WPT is found to solve this problem. Wavelet packet transform (WPT) can accurately describe the transient partial feature of the signal and decompose signals to various frequency bands.

# Wavelet packet Decomposition and reconstruction algorithm for the measured tidal level signal

2.1 The Optimal Parameters Selection for Wavelet Packet Decomposition

(1) The selection of orthogonal mother wavelet. WPT for the measured tidal level signal is essentially crosscorrelation between different scales of wavelet function and the signal. According to the tidal theory, the astronomical tide from the recorded water level is superposition of sine wave with different frequency. Generally speaking, the closer wavelet generating function to the analyzed signal, the more concentrated energy distribution for the signal in the time-frequency domain is. So that, it can consider choosing the wavelet function whose shape is similar to astronomical tidal constituent as the mother wavelet when we process WPT for the tidal level. Daubechies wavelets have excellent local characteristics, and it can well extract useful signal components in all kinds of transient waveform. So db4 tight orthogonal wavelet generating function is selected as a wavelet packet for decomposition the tidal level signal eventually.

(2) The determination of optimal level for wavelet packet decomposition. Set a measured tidal level signal S with the frequency range [ $f_0$ ,  $f_1$ ], and conduct N layers wavelet packet decomposition, then get corresponding frequency range of each node for NO.N layer leaf of the wavelet tree, which can be described as below table 1.

Table 1 The corresponding frequency range of each node for NO.N layer leaf of the wavelet tree

NODE NUMBE RS	FREQUENCEY RANGE				
[N,0]	$[f_0, \frac{(2^N - 1)f_0 + f_1}{2^N}]$				
[N,1]	$[\frac{(2^{N}-1)f_{0}+f_{1}}{2^{N}},\frac{(2^{N}-2)f_{0}+2f_{1}}{2^{N}}]$				
[N,2]	$[\frac{(2^{N}-2)f_{0}+2f_{1}}{2^{N}},\frac{(2^{N}-3)f_{0}+3f_{1}}{2^{N}}]$				
:	:				
[N,N- 1]	$\left[\frac{[2^{N}-(2^{N}-2)]f_{0}+(2^{N}-2)f_{1}}{2^{N}},\frac{f_{0}+(2^{N}-1)f_{1}}{2^{N}}\right]$				
[N,N]	$\left[\frac{f_0 + (2^N - 1)f_1}{2^N}, f_1\right]$				

Leaf nodes [N, 0] is the lowest frequency interval, if we assume that the lowest frequency of the astronomical tidal constituent was separated, the upper of frequency interval for leaf nodes [N, 0] is less than the frequency of second-lowest astronomical tidal constituent, so that the optimal wavelet packet decomposition layers can be determined finally.

(3)Obtaining the best wavelet packet basis. The optimum wavelet packet basis selection principle is that,

according to the prior knowledge of the frequency for each astronomical tidal constituent and the corresponding starting and ending frequency for each leaf node of wavelet tree, the located frequency bands of each astronomical tidal constituent can calculated, and then through pruning or merge method, wavelet tree leaf nodes which are not belong to any astronomical tidal constituent would be processed to obtain the optimal wavelet packet basis of the tidal level signal and then to automatically construct the optimal wavelet tree.

2.2 Wavelet Packet Decomposition and Reconstruction for the Measured Tidal Level Signal

Taken a measured tidal level signal to wavelet packet decomposition step by step,  $2^{L-1}$  group sequences with different frequency bands would be gotten in the L layer, and each group sequence is respectively made up by the resulting for low pass filter  $W_{2n}^{L}$  and high pass filter  $W_{2n+1}^{L}$ . It supposed that the length of original tidal level f(i) is  $2^{N}$  and sampling frequency is  $f_{s}$ , so that the length of each  $W^{L}$  is  $N/2^{L}$  and sampling frequency of each  $W^{L}$  is  $f_{s}/2^{L}$ 

Set  $W^{0}(i) = f(i)(i = 0, 1, \dots, 2^{N} - 1)$ , there is the following recursive decomposition formula:

$$\begin{cases} W_{2n}^{L}(i) = \sum_{k} h(k-2i) W_{n}^{L-1}(k) \\ W_{2n+1}^{L}(i) = \sum_{k} g(k-2i) W_{n}^{L-1}(k) \end{cases}$$

where,  $i = 0, 1, \dots, N/2^{L-1}$ ;  $n = 0, 1, \dots, 2^{L-1} - 1$ ;  $L = 0, 1, \dots, N$ 

 $h_k$  denotes conjugate orthogonal filter and  $g_k = (-1)^{k-1} h_{1-k}$ 

The reconstruction formula for Wavelet decomposition coefficients is as follows.

$$NW = {}_{n}^{L}(i) = 2 \sum_{k} h(i - 2k) NW = {}_{2n}^{L+1}(k) + 2 \sum_{k} g(i - 2k) NW = {}_{2n+1}^{L+1}(k)$$

Where,  $i = 0, 1, \dots, N/2^{L-1}; n = 0, 1, \dots, 2^{L-1} - 1; L = 0, 1, \dots, N$ 

## The Analysis of real events

Taken the event NO.9608 of hourly measured tidal level of Xiamen tide station (east longitude 118.083°, north latitude 24.45°) as an example (as the figure 2 shows), the storm surge process curve can be exacted by WPT. Eleven of greatly influenced astronomical tidal constituent Q1, O1, P1, K1, N2, M2, S2, K2, M4, MS4, M6 are selected as the primary vibration frequency components in the measured signal to decompose into different frequency bands by 11 layers WPT. And the optimal wavelet tree and the corresponding node number for the entire astronomical tidal constituent are as follows. Extracted the corresponding wavelet decomposition coefficients for the eleven astronomical tidal constituents from the wavelet tree nodes to reconstruct respectively, eleven astronomical tidal constituents' process curves can be obtained in the Fig.3. From the above chart and figure, we can see that the total energy percentage of eight astronomical tidal constituents is 91.5663%. Due the extreme storm surge valued 160cm account for 25.20 percent of maximum amplitude valued 635cm, we can believe the sum of eight astronomical tidal constituents represent the total value of astronomical tide. Then taken out wavelet decomposition coefficient for wavelet leaf node7, 11 (which frequency range is [0.0429688, 0.046875]) to reconstruction, the storm water process curve would be achieved, which compares with the measured level curve and the linear separation level curve, and the result effect is shown as figure 4.



Fig.1 The Process Curves For The Measured Tidal Level and Forcasted Astronomical Level



Fig.2 The Optimal Wavelet Tree for the Measured Tidal Level Signal.

Table 2. The Corresponding Node Number For The Eleven Astronomical tidal constituents

Astronomical tidal constituents							
Nam	Frequenc	Node	Frequency bands for	Energy			
е	У	Number	wavelet packet	percenta			
			decomposition	ge			
Q1	0.037218	(9,38)	[0.037109,0.038085]	9.3865%			
01	0.038730	(9,39)	[0.038085,0.039062]	8.1545%			
P1	0.041552	(11,170 )	[0.041503,0.04174]	1.3712%			
K1	0.041780	(11,171 )	[0.04174,0.041992]	1.2074%			
N2	0.078999	(8,40)	[0.07812,0.080078]	10.6188 %			
M2	0.080511	(8,41)	[0.080078,0.082031]	10.0752 %			
S2	0.083333	(10,170 )	[0.083007,0.083496]	15.0012 %			
K2	0.083561	(10,171 )	[0.083496,0.083984]	10.0079 %			
M4	0.161022 8	(8,82)	[0.160156,0.162109]	6.5976%			
MS4	0.163844 7	(8,83)	[0.162109,0.164063]	6.6024%			
M6	0.241534 2	(3,3)	[0.1875,0.25]	12.5436 %			

As seen in the graph, the storm surge process curve separated by traditional linear methods was bigger than the actual measured value, and the WPT method is closer to. The specific error statistics is as follows.

## Conclusion

This paper provides a new method to separate the storm surge, which adopts WPT to decompose the measured tidal level signal with different frequency of astronomical tide constituents and storm surge to extract the storm surge process curve finally. This separation method is directly use the observation tidal level and not need the forecasted astronomical tide value, compared with the traditional linear separation method, which avoid tide cycle phenomenon cased by the prediction error for astronomical tide or nonlinear interaction effect in shallow water. The results of event calculation show that it is a good method to solve the problem of separating storm surge from the measured tidal level and also bring convenience for obtain the storm surge.



Fig. 3 Eleven Astronomical Tidal Constituents Process Curves



Fig.4 The Storm Surge Contrast Among Measured Curve, Linear Separation Curve And WPT Separation Curve

Table 3 The Error Statistics F	For The Storm Surge	Of Measured,
Linear Separation And WPT	Separation	

Contrast Items	Measured	Linear	WPT
	Value	Separation	Separation
Error Items		Value	Value
Extreme Storm Surge	124cm	160cm	129.003cm
Mean Square Error	24.493cm	24.577cm	23.929cm
Absolute Average Error	0	15.471cm	1.733cm

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