Assembly Operation Optimization Based on A Hybrid Particle Swarm Optimization and Genetic Algorithm

Abstract. The assembly dimensional quality can be improved by optimizing the assembly operations between parts. Firstly, as there is large quantity of geometric feasible assembly sequences for the auto-body, the multi-attribute directed liaison graph is applied to describe the precedence relationships and assembly control characteristics between parts. Secondly, with assembly deviation propagation as the fitness function, the hybrid particle swarm optimization and genetic algorithm is proposed to optimize the assembly operations between parts. Finally, the optimal assembly sequence is selected through assembly variation propagating based on linear assembly variation analysis model. The optimization of the key control characteristics is illustrated by the auto-body side assembly.

Streszczenie. W artykule zaproponowano wykorzystanie hybrydowego połączenia optymalizacji rojem cząstek i algorytmu genetycznego w optymalizacji procesu montażu/łączenia części. Metoda bazuje na funkcji użyteczności wynikającej z propagacji błędu złożenia. Efekty działania zaprezentowano na przykładzie składania boku karoserii samochodu. (Optymalizacja składania części, bazująca na optymalizacji rojem cząstek i algorytmie genetycznym).

Keywords: Assembly Operation, Hybrid Algorithm, Optimization
Słowa kluczowe: operacja składania, algorytm hybrydowy, optymalizacja.

Introduction
The quality of auto-body dimension relates to the whole external appearance and wind noise, the effect of closing the door and even the steady of driving. The quality of auto-body dimension is influenced by automobile parts design, assembly process and manufacturing variations [1]. As the current level of manufacturing precision control is close to the limit, deviation reduced in manufacturing technology is need for a qualitative breakthrough. There is still much room to improve the dimension quality through selecting assembly sequence, fixture locators and assembly control features [2~4]. The work apply a linear variation method to select optimal assembly operations and propagating assembly deviation.

Assembly model
The assembly modeling is usually used a directed graph \( G = \{P, C\} \) for generating geometric feasible sequences, where \( P \) is the set of parts, \( C \) is the set of precedence relationships between parts. In the study, the assembly modeling of engineering feasible sequences can be represented by the multi-attribute directed graph \( G = \{P, C, J_c\} \), where \( J_c \) stands for the set of control features between two parts. In this work, a case of auto-body side assembly is used to describe the above assembly modeling, in which the numbers with underlined stands for hole or pin features (Fig. 1).

Fig. 1. (1) An auto-body side assembly (2) Directed liaison graph

Based on the graph theory, an adjacency matrix is mapped from the liaison graph because the matrix is easier to be handled, which is represented by \( A = [a_{ij}]_{nxn} \). \( n \) represents the number of the parts, \( k \) represents the number of control features between parts. A few of rules established for different relationships are shown as follows:

1) If part \( i \) and part \( j \) have no precedence relationship, \( a_{i,j} \) and \( a_{j,i} \) are equal to \( k \).

2) If part \( i \) and part \( j \) have precedence relationship, the graph is a directed graph. If part \( i \) is assembled before part \( j \), \( a_{i,j} \) is equal to \( k \) and \( a_{j,i} \) should be equal to \(-k\).

3) If part \( i \) and part \( j \) have no physical connection and non-precedence relation, \( a_{i,j} \) is equal to 0.

Based on the above rules, the adjacency matrix of the assembly is shown as follows:

\[
A = \begin{bmatrix}
0 & +18 & +4 & +9 \\
-18 & 0 & 7 & 0 \\
-4 & 7 & 0 & 9 \\
-9 & 0 & 9 & 0 \\
\end{bmatrix}
\]

Assembly operation optimization
The translation and orientation variations at any critical characteristic point (CCP) can be expressed as a vector:

\[
\delta q = -J^T \Phi \delta R
\]

The fitness function should be accord with the principle of the minimum deviation propagation due to optimizing the match features between two parts, which is as follow:

\[
F(X) = \|J^T X\|
\]

where \( X \) represents the choice of match features, \( J^T \) stands for the Euclidean norm of the inverse Jacobian matrix. The set of assembly sequences is an important constraint condition because the sequence affects the optimal range of assembly features. The engineering feasible sequences are \( \{1,2,3,4\} \), \( \{1,2,4,3\} \) and \( \{1,4,3,2\} \).

Some particles cannot satisfy the principle of six locating points because the particles are randomly generated. Therefore, the work proposes a constraint function, which can evaluate unfeasible particles. The constraint function is shown as follow:
where $J_{R_j}(X)$ is the sub-matrix of $J(X)$ and $R_j$ is the rank of $J(X)$. The major fitness value is 63215 and the minor is 7.61. In order to enhance the effect the constraint function during evaluation process, the evaluation can be written as follow:

$$E(X) = Lg(\alpha_p F(X) + \alpha_h H(X))$$

where $\alpha_p$ and $\alpha_h$ represent the coefficients of the fitness and constraint functions respectively.

At each iteration $k$, the particles are represented by a vector $x$ in multi-dimensional space to characterize its position, which is shown as follow:

$$x = \{x_1, x_2, \ldots, x_d\}$$

The vector $v$ is used to characterize its velocity, which is shown as follow:

$$v = \{v_1, v_2, \ldots, v_d\}$$

Each particle changes its position $x_{ik}$ in per iteration. The new position $x_{i(k+1)}$ of the $i^{th}$ particle is biased towards its best position $p_{ik}$ with best function value, referred to as personal best or $p_{Best}$, and the global best position $g_{Best}$, referred to as the global best or $g_{Best}$. The $g_{Best}$ is the best position in the set:

$$P = \{p_1, p_2, \ldots, p_N\}$$

where $p_i = x_i^0$, $i = 1, 2, \ldots, N$.

At each iteration $k$, the position $x_{ik}$ is updated by a velocity $v_{ik}$ which depends on three factors: (1) its current velocity $v_{ik}$, (2) the weight difference vector $(p_{ik} - x_{ik})$ and (3) the weight difference vector $(p_{gk} - x_{ik})$. The set $X$ is updated for the next iteration using the following equation:

$$x_{ik(k+1)} = x_{ik} + v_{ik(k+1)}$$

The velocity $v_{ik}$ can be calculated as bellow:

$$v_{ikk+1} = \omega v_{ik} + r_1 c_1 (p_{ik} - x_{ik}) + r_2 c_2 (p_{gk} - x_{ik})$$

The parameter $\omega$ is commonly equal to 0.9. The parameters $r_1$ and $r_2$ are uniformly distributed random numbers in $[0, 1]$. $c_1$ and $c_2$ are popularly equal to 2. The set $P$ is updated when the new positions are calculated through the follow equation:

$$p_{ik} = \begin{cases} x_{ik} & f(x_{ik}) < f(p_i^0) \\ p_i^0 & f(x_{ik}) \geq f(p_i^0) \end{cases}$$

In this study, partially matched crossover is used to increase the diversity of particles. The positions of particles are changed according to the mutation rate. The particle swarm optimization starts with a random population of particles, generates new populations of particles through updating the positions, and terminates when the stopping condition is met. In each iteration, every particle is updated through tracing two extremes: individual extreme point $p_{Best}$ and global extreme point $g_{Best}$. Assuming the D-dimensional space, there are $N$ particles, the flowchart of the hybrid algorithm is as follows:

Step 1. Initializing parameters

Step 1-a: Initializing iteration counter $k=0$; Step 1-b: Initializing $N$ random positions of the particles; Step 1-c: Initializing $N$ random velocities; Step 1-d: Initializing individual extreme point; Step 1-e: Setting the crossover rate and mutate rate.

Step 2. Evaluating particles

Step 2-a: Checking the feasibility of particles through the Jacobi matrix; Step 2-b: Calculating the evaluation function value of each particle; Step 2-c: Recording the set $FP$ of the feasible particles.

Step 3. If current iteration is larger than the maximum iteration, the algorithm terminates; otherwise the algorithm goes to step 4.

Step 4. Crossover is an important operation for generating the offspring population. Many kinds of crossover are presented to deal with different problems. This study adopts a partially matched crossover for the particles.

Step 5. Mutation is applied to alter the genetic material of a very small number of individuals in a random number, to enhance the diversity of the population and expand the volume of the current search space. Mutation operation is executed on the same constraints as crossover. When one position of the particle is changed, another position is changed for keeping the meaning of the assembly operations at the same time.

Step 6. Generating a new population of PSO

Step 6-a: Updating particle velocities; Step 6-b: Updating particle positions; Step 6-c: Updating $g_{Best}$; Step 6-d: Removing the duplicate particles and creating new particles; Step 6-e: Setting $k = k + 1$.

Step 7. Repeating from step 2 to step 6.

Case of the body side assembly

The body side assembly is illustrated to the flowchart of assembly operation optimization (Fig.1). The given assembly features are 47 between parts, in which 18 features will be selected to assemble four parts together. The pin (/hole) feature should be as one feature while coding the particles; however, the pin feature needs to be divided into two points for calculating the Jacobian matrix. The curves of the figure 3 represent the values of the evaluation function after ten cycles between the rear door and the side frame. The figure 3 shows that different cycles are different; moreover, the best value of the initial population is small, but many iterations is required to generate the best value. The table 1 shows the results of the best value, the iteration and the initial best value after 20 cycles.

From the table 1, the particle swarm optimization can generate the local optimal solution firstly and the genetic algorithm generates the local optimal solution finally. The
A hybrid algorithm is best for optimizing assembly operation according to the best value. Measurement operations are an important means to evaluate the assembly deviation propagation. In this work, five measurement operations are selected to evaluate assembly sequences shown as Fig. 4.

![Fig. 4 Measurement operations of the body side](image)

**Table. 1. Comparison of different optimization algorithms**

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Iterations</th>
<th>Best value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>15</td>
<td>12.04</td>
</tr>
<tr>
<td>GA</td>
<td>40</td>
<td>9.84</td>
</tr>
<tr>
<td>Hybrid algorithm</td>
<td>24</td>
<td>7.38</td>
</tr>
</tbody>
</table>

All assembly operations can be optimized according to repeat the flowchart of the hybrid algorithm. The assembly operations of the sequence \{1, 2, 3, 4\} are shown as figure 5. The assembly variation can be calculated according to Eq. (1). For an assembly, some key product characteristics (KPCs) can be used to evaluate assembly quality; therefore, the objective function can be written as follows:

\[
F_{seq} = \sum_{i=1}^{m} T_{MO}
\]

where \(F_{seq}\) represents the fitness function, \(T_{MO}\) stands for the assembly tolerance of the \(i^{th}\) KPC, and \(m\) is the number of KPCs.

![Fig. 5 All assembly operations of the best sequence](image)

The objective function value of the assembly sequence \{1, 2, 3, 4\} is 1.75. The objective function values are 5.37 and 8.72 for the sequences \{1, 2, 4, 3\} and \{1, 4, 3, 2\}. Therefore, the assembly sequence \{1, 2, 3, 4\} is the optimal sequence.

**Conclusions**

The multi-attribute directed liaison graph is used to describe the assembly, which can initially remove unfeasible engineering sequences through the number of assembly control features between parts. The study proposes the fitness function and the constraint function to evaluate the feasible and unfeasible sequences. The hybrid algorithm can improve the optimization effect through comparing the particle swarm optimization, the genetic algorithm and the enumeration method. The auto-body side assembly is used to illustrate the flowchart of the assembly sequences. The results show that the difference of assembly tolerance is due to different selection of assembly control features.

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**REFERENCES**


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