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Comprehensive Evaluation on Design Scheme of Cartesian Robot

***Abstract**. indexes and factors that effect on the performance of Cartesian robot are analyzed. A comprehensive evaluation index system of Cartesian robot is constructed. Basic concepts and methods about grey theory and fuzzy mathematics theory are used to establish the grey fuzzy comprehensive evaluation model on the design scheme of Cartesian robot. The decision can be more scientific by weighting different factors through entropy weight method. A design scheme case is evaluated by using the presented model, results show that the model can reflect whole performance of different design scheme.*

***Streszczenie.** W artykule przedstawiono analizę parametrów wpływających na pracę robota liniowego (ang. Cartesian Robot). W badaniach wykorzystano logikę rozmytą oraz teorię odcieni szarości w celu opracowania modelu w logice rozmytej, służący do ogólnej oceny projektu robota liniowego. Przy użyciu proponowanego modelu, analizie poddano przykładowy projekt robota, dzięki czemu uzyskano pełne odzwierciedlenie jego możliwości działania. (**Model oceny ogólnej projektu robota Kartezjańskiego (liniowego)**).*

Keywords: Cartesian robot, design scheme, grey fuzzy theory, comprehensive evaluation, entropy weight method
Słowa kluczowe: robot kartezjański, schemat projektowy, rozmyta teoria szarości, ocena ogólna, metoda wag entropii.

1. Introduction

Generally, there are many alternative design schemes in designing mechanical product. And one of the crucial part in designing a product is how to analysis and evaluate the design scheme with feasible method so as to make sure the reliability of the design. However, it is rather complicated to make an evaluation on the scheme of cartesian-coordinate robot, because many evaluating indexes are required to be taken into consideration and there exist interaction and inter-restriction among each of them, which makes the evaluation more difficult.

Currently, there are some commonly used comprehensive evaluation methods such as fuzzy comprehensive evaluation method, grey comprehensive evaluation, analytic hierarchy process, data development analysis, artificial neuronal network evaluation method[1] and entropy method, etc., which all have advantages and disadvantages. Given that both grey and fuzziness characters are contained in the evaluating issues on the design scheme of Cartesian-coordinate robot, this paper will make an evaluation on the design scheme of Cartesian-coordinate robot with a method integrated fuzzy evaluation method with grey evaluation method and make a confirmation on the weight of evaluation indexes with entropy method, so as to optimize the design scheme.

2. Establishment of grey and fuzzy evaluation model

In the evaluation indexes of mechanical product, there are “connotation” and “extension” properties. Here “connotation” simply refers to the meaning of index and “extension” the value of index[2]. Generally, the difference between “grey system” and “fuzzy set” mainly lies in their attitudes toward the connotation and extension of system and difference in properties of subject investigated. Grey system gives emphasis to issues with definite extension and indefinite connotation; while fuzzy mathematics puts stress on issues with indefinite extension and definite connotation[3]. Fuzzy evaluation method overcomes the weakness of single result in traditional mathematics and well solves the issue on fuzzy and indefinite estimation; while grey evaluation method is a comprehensive method, combined qualitative analysis with quantitative analysis, able to better solve issues that are hard to be accurately quantized and counted and eliminate the influence caused by human factors, which leads to a more objective and accurate evaluation result[1]. Based on the advantages of those two methods, it will be an effective way to solve issues on information loss and analytic bias caused by taking single evaluation method. Entropy method is an objective comprehensive evaluation method, determining the weight of each index according to the quantity of information provided to decision maker. In that way, the influence of human factor brought by traditional subjective weighting method can be eliminated. The evaluating procedures of this evaluation model are as follows.

2.1 Establishment of scheme set and index set

For a set provided with n alternative schemes in designing, it is given by

$$X = \{x_1, x_2, \ldots x_n\} \tag{1}$$

For a set offered m technical indexes for the system, it is given by

$$Y = \{y_1, y_2, \ldots y_m\} \tag{2}$$

2.2 Establishment of decision index matrix

For a system of n alternative schemes and m evaluation indexes, the decision index matrix is given by

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{1m} \\ \vdots & \vdots & \vdots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nm} \end{bmatrix} \tag{3}$$

2.3 Normalized treatment on evaluation series

2.3.1 Normalized treatment on quantitative index data

For data series of different unit or different initial value, it generally shall be treated to be dimensionless and unified in analyzing their correlation degree [4]. Then transform the data series into the same order of magnitude according to membership function theory of fuzzy mathematics. The treated data shall be limited within the range of [0, 1] and denoted by preference membership grade[5].

If the jth index is a positive index, then

$$x_i(j) = \frac{y_{ij} - \min y_{ij}}{\max y_{ij} - \min y_{ij}} \tag{4}$$

If the jth index is a negative index, then

$$x_i(j) = \frac{\max y_{ij} - y_{ij}}{\max y_{ij} - \min y_{ij}} \tag{5}$$

where i=1,2， …， n； j=1,2， …， m.

2.3.2 Normalized treatment on qualitative index

In this paper, the qualitative indexes specified in the scheme are fuzzily quantized with fuzzy numbers [6]. The commonly used fuzzy numbers are trapezoid fuzzy numbers, denoted by (α, m; n, β); for the convenience of calculation, it can also be written as L-R typed fuzzy number (m, n; γ, δ), where $\gamma=m-\alpha$ and $\delta=\beta-n$. The universally used linguistic scales on evaluation of qualitative index are: very good, good, relatively good, ordinary, relatively bad, bad, and very bad; and it can be continuously defined with fuzzy number, see table 1.

Table 1 linguistic scales defined with fuzzy number

No.	Satisfaction level	trapezoid fuzzy numbers
1	Very bad	（0,0,0,0.2）
2	bad	（0,0，0.1,0.3）
3	Relatively bad	（0,0.2,0.2,0.4）
4	ordinary	（0.3,0.5,0.5,0.7）
5	relatively good	（0.6,0.8,0.8,1.0）
6	good	（0.7,0.9,1.0,1.0）
7	Very good	（0.8,1.0,1.0,1.0）

Let the two L-R typed trapezoid fuzzy numbers $M=(a, b; \alpha, \beta)$ and $N=(c, d; \gamma, \delta)$, the fuzzy number can be calculated approximately with the following formula:

(6) $$M/N\approx[a/d, b/c; (a\delta+d\alpha)/d(d+\delta), (b\gamma+c\beta)/c(c-\gamma)]$$

After getting the approximate solution of this fuzzy number, evaluate the overall expected value for the calculated fuzzy number, which shall also be limited within the range of [0, 1] and be regarded as the normalized data of qualitative index.

2.3.3 Confirmation on the reference data series

Reference data is usually the optimal index value in each scheme, thus the preference membership grades of reference data series are all 1, that is:

(7) $$x_0(j)=\{x_0(1), x_0(2), \cdots, x_0(m)\}=\{1,1,\cdots 1\}$$

2.3.4 Calculation of grey correlation coefficient

According to grey theory, the grey correlation coefficient can be calculated with the given formula:

(8) $$\xi_i(k)=\frac{\min\limits_i\min\limits_j|x_0(j)-x_i(j)|+\rho\max\limits_i\max\limits_j|x_0(j)-x_i(j)|}{|x_0(j)-x_i(j)|+\rho\max\limits_i\max\limits_j|x_0(j)-x_i(j)|}$$

Where in formula (8), ρ refers to recognition differential and generally $\rho=0.5$. In application, we can first evaluate the absolute difference matrix between $x_0(j)$ and $x_i(j)$, denoting by:

(9) $$\Delta=\begin{bmatrix}\Delta_1(1) & \Delta_1(2) & \cdots & \Delta_1(m)\\ \Delta_2(1) & \Delta_2(2) & \cdots & \Delta_2(m)\\ \vdots & \vdots & \vdots & \vdots\\ \Delta_n(1) & \Delta_n(2) & \cdots & \Delta_n(m)\end{bmatrix}$$

Where in formula (9), $\Delta_i(j)=|1-x_i(j)|$.

Then substitute the data in matrix Δ into formula (7) to get the matrix of grey correlation coefficient as shown in formula (10).

(10) $$\xi=\begin{bmatrix}\xi_1(1) & \xi_1(2) & \cdots & \xi_1(m)\\ \xi_2(1) & \xi_2(2) & \cdots & \xi_2(m)\\ \vdots & \vdots & \vdots & \vdots\\ \xi_n(1) & \xi_n(2) & \cdots & \xi_n(m)\end{bmatrix}$$

2.3.5 Confirmation on the weight of index

In this paper, the weight of index is confirmed with entropy method [7]. The following shows the detailed calculation procedures:

1) Calculate p_{ij}, namely the specific weight of index value of the ith evaluated object under the jth index:

(11) $$p_{ij}=x_i(j)/\sum_{k=1}^{n}x_k(j)$$

2) Calculate entropy value E of the jth index:

(12) $$E_j=-\frac{1}{\ln n}\sum_{k=1}^{n}p_{ij}\ln p_{ij}, j=1,2,\cdots,m$$

3) Calculate difference coefficient of the jth index. For the jth index, the smaller the value of E_j is, the larger the variation degree of index value is; and in reverse, the larger the value of E_j is, the smaller the variation degree of index value is. Coefficient of variation is:

(13) $$g_j=1-E_j$$

4) Calculate weight w_j of the jth index.

(14) $$w_j=g_j/\sum_{j=1}^{m}g_j$$

And then, the weight set can be given by:

(15) $$W=\{w_1, w_2\cdots w_m\}$$

2.3.6 Making first-level comprehensive evaluation

Based on the grey correlation coefficient matrix ζ and weight set W evaluated in accordance with the above steps, we can get a first-level comprehensive evaluation model:

(16) $$R=W\xi^T$$

2.3.7 Making multilevel comprehensive evaluation

As the evaluation index system of cartesian-coordinate robot is complex and needs to consider the indexes of different grade and type, it is necessary to make a gradation treatment on the index system. In making multilevel comprehensive evaluation, we shall firstly make comprehensive evaluation on index of the lowest grade and regard the evaluated value as the evaluation index series data of the higher grade, then reevaluate that of the higher grade, and go on until that of the highest grade. In the final comprehensive evaluation value series, principle of maximum membership grade, the optimal design scheme shall be given to that which owes the maximum membership grade value.

3. Grey fuzzy comprehensive evaluation on the design scheme of Cartesian-coordinate robot

Cartesian-coordinate robot is mainly composed of control system, driving system, linear movement shaft and hand grasp system. To demonstrate the application of the above-mentioned evaluation model, here a PLC controlled three-dimensional cartesian-coordinate robot is taken as a subject investigated to enter into detail application of fuzzy grey evaluation method in evaluating the design scheme of cartesian-coordinate robot. Fig.1 is the index system of the overall design scheme for this robot.

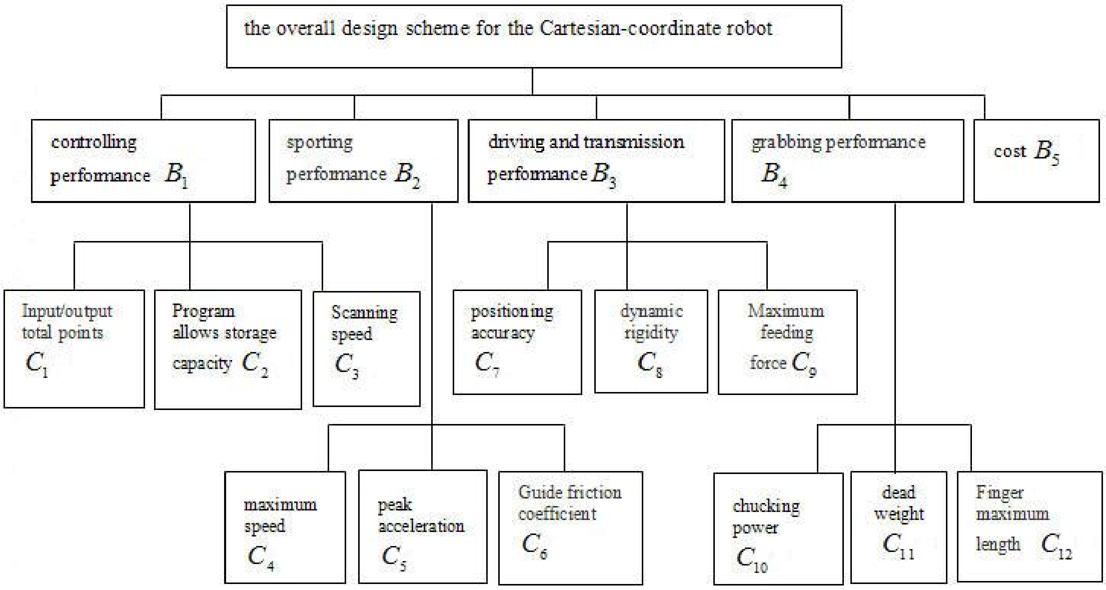

Fig.1. Index system of the overall design scheme for the Cartesian-coordinate robot

3.1 First-level comprehensive evaluation

Table 2 shows the secondary-grade index values of three design schemes.

Table.2. The secondary-grade index values of three design schemes

index	Scheme No.1	Scheme No.2	Scheme No.3
C_1	24	20	48
C_2(B)	8	2.8	8
C_3(μs/step)	0.37	0.75	0.74
C_4(m/min)	60	120	150
C_5(g)	1.5	3	10
C_6	0.002	0.0005	0.003
C_7(μm/300mm)	10	10	0.5
C_8(N/μm)	90	350	120
C_9(N)	240000	300000	14500
C_{10}(N)	540	460	540
C_{11}(kg)	4.2	1.1	0.43
C_{12}(mm)	160	50	80

3.1.1 Evaluation on control performance B_1

1) The index set of control performance B_1 is given by

$$C^1 = \{C_1, C_2, C_3\}$$

2) The decision index matrix of scheme set for index set gotten from table 2 is:

$$Y_1 = \begin{bmatrix} 24 & 8 & 0.37 \\ 20 & 2.8 & 0.75 \\ 48 & 8 & 0.74 \end{bmatrix}$$

3) Through formula (4) and (5), decision matrix can be normalized as:

$$X_1 = \begin{bmatrix} 0.1667 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0.0263 \end{bmatrix}$$

4) Through formula (8), difference matrix can be calculated as:

$$\Delta_1 = \begin{bmatrix} 0.8333 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0.9737 \end{bmatrix}$$

5) Through formula (7), grey correlation coefficient matrix can be calculated as:

$$\xi_1 = \begin{bmatrix} 0.3750 & 1 & 1 \\ 0.3333 & 0.3333 & 0.3333 \\ 1 & 1 & 0.3393 \end{bmatrix}$$

6) Evaluate the weight of index that has effect on control performance.

Calculate the entropy value E_{1j}, technical index weight w_{1j} and weight set W_1 of each technical index that has effect on control performance in accordance with procedure 1to 6, as shown in table 3.

Table 3. Entropy value E_{1j} and weight w_{1j} of technical indexes

Technical index	C_1	C_2	C_3
E_{1j}	0.3734	0.6309	0.1085
w_{1j}	0.3320	0.1956	0.4724

Thereby the weight set is

$$W_1 = (0.3320 \quad 0.1956 \quad 0.4724)$$

7) Make comprehensive evaluation

$$R_1 = W_1 \xi^T{}_1 =$$

$$(0.3320 \quad 0.1956 \quad 0.4724) \begin{bmatrix} 0.3750 & 1 & 1 \\ 0.3333 & 0.3333 & 0.3333 \\ 1 & 1 & 0.3393 \end{bmatrix}^T =$$

（0.7925 0.3333 0.6879）

Thus get comprehensive evaluation value of control performance B_1 in each scheme.

3.1.2 Make evaluation on other first-grade indexes such as B_2, B_3 and B_4, with the same procedures

The comprehensive evaluation value of motion performance B_2 in each scheme is

R_2=（0.3712 0.6312 0.7917）

The comprehensive evaluation value of driving performance B_3 in each scheme is

R_3=（0.3951 0.7038 0.6403）

The comprehensive evaluation value of grabbing performance B_4 in each scheme is

R_4=（0.8384 0.4314 0.6932）

3.2 Second-level comprehensive evaluation

1) According to Fig.1, the first-grade index set can be given by

$$B = \{B_1, B_2, B_3, B_4, B_5\}$$

Where, B_1 is control performance, B_2 is motion performance, B_3 is driving performance, B_4 is grabbing performance and B_5 is the cost. On the basis of the first-grade comprehensive evaluation result, the first-grade decision index value is as shown in table 4.

Table 4 the first-grade decision index value

Alternative scheme	B_1	B_2	B_3	B_4	B_5
Scheme No.1	0.7925	0.3712	0.3951	0.8384	relatively bad
Scheme No.2	0.3333	0.6312	0.7038	0.4314	ordinary
Scheme No.3	0.6879	0.7917	0.6403	0.6932	good

2) Normalize the decision index series of qualitative index cost B_5

In three design schemes, decision index series of cost are low, ordinary and high, where low cost indicates a relatively good design and high cost means bad design. According to formula (6), the cost of L-R type is evaluated as: ordinary/relatively good (0.625, 0.625; 0.325, 0.5417) and bad/relatively good = (0, 0; 0.125, 0.5).

And the corresponding overall expected values are 0.6792 and 0.1563. Thus we can get the normalized decision series as:

$$R_5 = (1 \quad 0.6792 \quad 0.1563)$$

3) Then the decision index matrix composed by decision index series of five indexes B_1, B_2, B_3, B_4 and B_5 after normalization of first grade can be got from the following matrix.

$$Y = \begin{pmatrix} R_1^T & R_2^T & R_3^T & R_4^T & R_5^T \end{pmatrix} = \begin{bmatrix} 0.7925 & 0.3712 & 0.3951 & 0.8384 & 1 \\ 0.3333 & 0.6312 & 0.7038 & 0.4314 & 0.6792 \\ 0.6879 & 0.7917 & 0.6403 & 0.6932 & 0.1563 \end{bmatrix}$$

4) According to procedure1.3, 1.4 and 1.5, the first-grade grey correlation coefficient matrix is given by

$$\xi = \begin{bmatrix} 1 & 0.3333 & 0.3333 & 1 & 1 \\ 0.3333 & 0.5671 & 1 & 0.3333 & 0.6092 \\ 0.687 & 1 & 0.7085 & 0.5836 & 0.3721 \end{bmatrix}$$

5) Evaluate the weight of index that has effect on overall performance

Calculate the entropy value E_j, technical index weight w_j and weight set W that have effect on the each technical index of the first-grade in accordance with procedure 1 to 6, as shown in table 5.

Table 5 the entropy value E_j and technical index weight w_j that have effect on the each technical index of the first-grade

Technical index	C_1	C_2	C_3	C_4	C_5
E_{1j}	0.6232	0.6057	0.6249	0.6089	0.8299
w_{1j}	0.2207	0.2309	0.2197	0.2291	0.0996

Then the weight set W is

$$W = (0.2207 \quad 0.2309 \quad 0.2197 \quad 0.2291 \quad 0.0996)$$

6) Making overall comprehensive evaluation on design scheme

$$R = W\xi^T = (r_1, r_2, r_3) = (0.6996, 0.5612, 0.7089)$$

7) The evaluation result of $r_3 > r_1 > r_2$ shows that the third scheme is the optimal one in the three design schemes for cartesian-coordinate robot, followed by the first scheme and the second scheme.

4. Conclusion

Through making research on fuzzy comprehensive evaluation method and grey evaluation method and in combination with the two advantages, a grey fuzzy evaluation model on design scheme for cartesian-coordinate robot was established in this work. The application of membership grade and grey correlation and full consideration of many indexes influencing on the whole system made the evaluating process more scientific and comprehensive. Moreover, the application of entropy method to calculate weight value fully avoided the influence of human factors and weakened the randomness in making weight decision; therefore a more reliable result was evaluated. In conclusion, this method is of significant and able to be effectively used in the evaluation of design scheme for cartesian-coordinate robot.

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