Optimal Sizing and Placement of Distributed Generation in Distribution System Considering Losses and THDv using Gravitational Search Algorithm

Abstract. This paper presents a new method for determining optimal sizing and suitable placement for distributed generation (DG) in distribution system. A multi-objective function is created to minimise the total losses and average voltage total harmonic distortion (THDv) of the distribution system. The proposed method utilizes gravitational search algorithm (GSA) in the optimization process and its performance is compared with other optimization techniques such as particle swarm optimisation (PSO) and evolutionary programming (EP). The results show that the GSA performs better than PSO and EP by giving the best fitness value and convergence rate.

Streszczenie. W artykule opisano metodę optymalizacji rozmięsczenia i mocy generatorów energii w systemie rozproszonym. Opracowana została funkcja wielokryterialna, służąca do minimalizacji całkowitych strat i wskaźnik THD napięcia. Optymalizacja dokonywana jest z wykorzystaniem algorytmu GSA. Jego działanie zostało porównane z działaniem innych metod, jak PSO i EP. Przedstawiono wyniki porównania. (Optymalizacja rozmięsczenia i mocy generatorów energii w systemie rozproszonym z wykorzystaniem algorytmu GSA – zagadnienia całkowitych strat i THD napięcia).

Keywords: Gravitational search algorithm, optimization, harmonic distortion, distributed generation.

Introduction

The presence of DGs in the distribution system may lead to several advantages such as voltage support, improved power quality, loss reduction, deferment of new or upgraded transmission and distribution infrastructure and improved utility system reliability [1]. However, the insertion of DG into the distribution system could either have a positive or negative impact depending on the operating characteristics of the DG and the distribution network. DG can be beneficial if it meets the basic requirements of the system’s operating philosophy and feeder design [2].

However, when DG is connected to a distribution system, it may contribute to harmonic distortion in the system, depending on the type of DG unit and the power converter technology. In terms of the DG interfacing, DGs can be classified into two types, namely, inverter based DG and non-inverter based DG [3-4]. Examples of inverter based DG are photovoltaic systems, wind turbine generators, fuel cells, and micro turbines, which use power converters as interfacing devices to the grid. On the other hand, small hydro synchronous generators and induction generators are considered to be non-inverter based DG units.

It is well known that DGs need to be installed at the distribution system level of the electric grid and should be located close to the load centre. The impact of DG on power losses, voltage profile, short circuit current, harmonic distortion and power system reliability are usually tested separately before connecting it to the distribution system. The achievement of the benefits from DGs depends greatly on how optimally they are installed. Studies have indicated that approximately 13% of the generated power is consumed as losses at the distribution level [5]. Another problem in the distribution system is the voltage profile, which tends to drop below acceptable operating limits along distribution feeders with increased loads. This arises due to the increasing electricity demand, which will require the upgrading of the distribution system infrastructure [6]. Therefore, to reduce the power losses and to improve both the voltage profile and the THDv, appropriate planning must be carried out for incorporating DG into power systems. In this process, several factors need to be considered, such as the technology to be used, the number and the capacity of the units, the optimal location, and the type of network connection.

Currently, the problem of DG placement and sizing is of importance. The installation of DG units at non-optimal places with non-optimal sizing can cause higher power losses, power quality problems, instability of the system, and escalating operational costs [7-8]. Optimisation approaches are capable of indicating the best solution for a given distribution network. There are several methods to allocate and size the DG in the distribution power system. The power flow algorithm [7, 9] can be used to find the optimum DG size at each load bus by assuming each load bus is able to have a DG unit. This method is inefficient due to the requirement of a large number of load flow computations. Another method for determining the location and size of DG is to use optimization algorithms such as the genetic algorithm (GA) [10-11]. GA is suitable for multiobjective problems such as DG allocation and gives very satisfactory solutions. However, the computational time for GA is very long, with an extremely lengthy convergence time. Analytical methods can also be used to allocate the DG in radial or meshed systems [12]. In this technique, separate expressions for radial and meshed network systems are required. Furthermore, complex procedures based on phasor current are used to solve the location problem. However, this technique only optimises the location by considering a fixed size of DG. For the same purpose, a combination of sensitivity analysis (SA) technique [5] and other heuristic algorithm techniques are commonly used [5, 13-18]. In these techniques, the location of the DG is determined through SA and the sizing of the DG is determined through a heuristic algorithm technique. The advantage of this technique is the reduction of the search space, which eventually increases the overall speed of optimisation processes.

The optimal placement and sizing of DG using particle swarm optimisation (PSO) and sensitivity analysis was also studied in [5]. Here, the aim was to minimise the total cost of the system by reducing losses and THD and by improving the voltage profile. Another similar study was reported in [15] in which harmony search algorithm (HSA) was used for solving the optimal placement and sizing of DG. The objective function of this study was to improve the voltage profile and to minimise loss and THDv. Most of the
aforementioned DG sizing and placement problem requires harmonic analysis methods to study the impact of DGs on harmonic propagations in the distribution system. A fast harmonic load flow method was introduced in [19-20] for a three-phase radial distribution system in order to implement harmonic analysis. The results indicates that it is more efficient and accurate as compared to other conventional harmonic loadflow algorithms.

Similar to the work in [5, 15 - 16], this paper proposes sensitivity analysis for determining the suitable location of DG in a radial distribution system. In order to determine the optimal size of the DG, a gravitational search algorithm (GSA) technique integrated with harmonic distribution loadflow (HDLF) is proposed. The proposed methodology is then tested in a 69-bus radial distribution system. The proposed technique has been compared with other optimisation techniques such as PSO and differential evolution (DE) algorithms. The results indicates that the proposed technique indicates the highest performance in getting the best fitness and the highest convergence rate. The results also shows the efficiency of the proposed technique for minimising the total losses and average THDv.

**Problem Formulation**

A multi-objective optimisation technique, formulated as a constrained non-linear integer optimisation problem, is proposed for DG placement and sizing in a distribution system. The objective is to minimise the total power loss and the THDv. The fitness function is given by Eq. (1):

\[
F_{\text{min}} = \alpha (P_{\text{loss}}) + \beta (THD_v)
\]

where \( F \) is the fitness function, \( P_{\text{loss}} \) is the total power loss, \( \alpha \) is the weighted factor for total power loss, \( THD_v \) is the average THDv, at all system busbars, and \( \beta \) is the weighted factor for THDv. The total real power loss is defined by

\[
P_{\text{loss}} = \sum_{i=1}^{n} P_{\text{loss} i}
\]

where \( n \) is the number of lines. The average THDv is defined by

\[
THD_v = \frac{\sum_{i=1}^{m} THD_{v i}}{m}
\]

where \( m \) is the number of buses.

The total power loss and the average THDv must be minimised according to the network power flow equations at fundamental and harmonic frequencies. Generally, multi-objective methods provide a set of optimal solutions. For this paper, the sum of the weighted methods is used to decide the relative importance of the objectives in order to obtain the best optimisation solution. The weighted factor for total power loss is 0.7 while the average THDv is 0.3. The factor for power loss is considered greater than that for THDv because the reduction of power loss in distribution networks has a significant impact on economic and technical prospects.

The inequality constraints involve those associated with the bus voltages and the DG to be installed. The bus voltage magnitudes are to be kept within acceptable operating limits throughout the optimisation process, as follows:

\[
V_{\text{min}} \leq |V_i| \leq V_{\text{max}}
\]

where \( V_{\text{min}} \) is the lower bound of bus voltage limits, \( V_{\text{max}} \) is the upper bound of the voltage limits, and \( |V_i| \) is the root mean square (RMS) value of the \( i \)th bus voltage.

The total harmonic level at each bus is to be less than or equal to the maximum allowable harmonic level, as expressed as follows:

\[
THDv_i (\%) \leq THD_{v \text{max}}
\]

where \( THD_{v \text{max}} \) is the maximum allowable level at each bus (5%).

**Proposed Algorithm**

With the growing use of DGs in distribution systems, several methods have been used to achieve various objectives in power system optimisation problems. In this paper, sensitivity analysis and gravitational search algorithm are used to determine the suitable placement and optimal sizing of DG in a distribution system. Harmonic loadflow analysis was integrated with this optimisation technique in order to obtain the fitness functions for the total power loss and average THDv.

**Sensitivity Analysis**

The sensitivity analysis method was used to find the most sensitive candidate for allocating the DG based on loss reduction. The advantage of this method is that it reduces the research space and increases the speed of the optimisation algorithm convergence. The theory behind this method is illustrated in Figure 1 [5].

![Fig. 1. Connected line between bus i and j](image)

Figure 1 shows a line impedance of \( R + jX \) between bus \( i \) and \( j \) connected to the load \( P + jQ \). The active power loss in the \( k \)th line is indicated by Eq. (6):

\[
P_{\text{loss}} = |I_k|^2 \times R[k]
\]

where \( I_k \) is the branch current and \( R \) is the resistance of the line. In addition,

\[
I_k = \left( \frac{P[j] + jQ[j]}{V[j]} \right)^* = \frac{P[j] + jQ[j]}{V[j]^*}
\]

where \( P \) is the real power load at the receiving bus, \( Q \) is the reactive power load at the receiving bus, and \( V \) is the voltage at the receiving bus. By substituting Eq. (7) into Eq. (6), we obtain:

\[
P_{\text{loss}} = \frac{\left( P[j] + Q[j] \right) R[k]}{\left( V[j]^* \right)^2}
\]

Thus, the sensitivity analysis factor is a derivative of the power loss with real power, \( P \), as indicated in Eq. (9):

\[
\frac{\partial P_{\text{loss}}}{\partial P} = \frac{\left( 2 \times P[j] \times R[k] \right)}{\left( V[j]^* \right)^2}
\]

Hence, the buses can be ranked based on Eq. (9) accordingly and some buses can be nominated as the most sensitive to DG placement in order to have the best effect on loss reduction.

**Gravitational search algorithm**

Gravitational search algorithm (GSA) was developed by Rashedi et al. in 2009 based on metaphor of gravitational kinematics. This algorithm is based on the Newtonian gravity: “Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them” [21]. The advantages and differences of GSA over other conventional optimisation techniques are as follows [21]:
I. The agent direction is calculated based on the overall force obtained by all other agents.
II. The force is proportional to the fitness value and so the agents see the search space around themselves in the influence of force.
III. GSA is memory-less and only the current position of the agents plays a role in the updating procedure.
IV. Balance between exploitation (heavier mass have higher attractions and move slowly) & exploration (lighter mass for fast moving and avoid trapping in a local optimum).
V. The force is reversely proportional to the distance between solutions.

The computational procedures of the GSA technique are described as follows [21]:

i. The position of the $i^{th}$ agent is given in Eq. (10):
$$ x_i^d = (x_i^1, x_i^2, ..., x_i^n), \text{for } i = 1, 2, ..., N $$
where $x_i^d$ presents the position of $i^{th}$ agent in the $d^{th}$ dimension.

ii. Update gravitational constant ($G$) is given in Eq. (11):
$$ G(t) = G_0 \times \frac{T - t}{T} $$
where $G(t)$ is the value of the gravitational constant at time $t$. $G_0$ is the value of the gravitational constant at the first cosmic quantum-interval of time.

iii. Update mass ($M$). Give weighting in range [0,1] correspond to their fitness are given in Eq. (12)-(13):
$$ m_i(t) = \frac{\text{fitness}(t) - \text{worst}(t)}{\text{best}(t) - \text{worst}(t)} $$
$$ M_i(t) = \sum_{j=1}^{N} m_j(t) $$
where fitness($t$) represent the fitness value of the agent $i$ at time $t$, worst($t$) and best($t$) are defined as maximum and minimum fitness, respectively.

iv. Update $k_{best}$ is given in Eq. (14):
$$ k_{best} = K_{best_{final}} + [\frac{T - t}{T} \times (100 - K_{best_{final}})] $$

v. Calculate total force ($F$) are given in Eq. (15)-(17):
$$ F_{ij} = G \times M_i \times M_j \times (x_i^d - x_j^d) $$
$$ R_{ij} = \|x_i - x_j\|_2 = \sqrt{\sum_{d=1}^{N} (x_i^d - x_j^d)^2} $$
$$ \varepsilon = \text{small coefficient}, 2^{-52} $$

vi. Calculate acceleration ($a$) is given in Eq. (18):
$$ a^d = \frac{F^d}{M_i} $$

vii. Update velocity ($v$) is given in Eq. (19):
$$ v_i^d (t + 1) = \text{rand} \times v_i^d (t) + a_i^d (t) $$
where rand is the random variable in the interval [0,1]. This random number will gives a randomized characteristic to the search.

viii. Update position ($x$) is given in Eq. (20):
$$ x_i^d (t + 1) = x_i^d (t) + v_i^d (t + 1) $$

Harmonic Distribution Loadflow
The growing number of DG units may contribute to harmonic pollution in power system networks. Therefore, the harmonic analysis tool is very important to distribution system analysis and design. It can be used to assess the harmonic distortion in the voltage and current at various buses and can also determine the existence of unsafe resonance phenomena in the power system. Generally, harmonic analysis algorithms can be divided into two categories. The first category is based on transient-state analysis techniques, such as time domain analysis and wavelet analysis [19-20]. The second category is steady-state analysis, which is based on load flow programs and the use of frequency-based component models [22-23]. Steady-state based algorithms are more efficient compared to transient state based algorithms due to their large-scale power system application and less computational time [24].

Conventional harmonic analysis methods utilise Newton-Raphson and Gauss-Seidel methods, which need an admittance matrix to obtain the harmonic penetration in distribution systems. These methods do not consider the particular topology characteristics of the distribution systems, such as radial and weakly meshed configurations. Therefore, they take more computational time to calculate the solution for each harmonic order. In order to save computational time and to apply in large-scale distribution systems, a harmonic distribution loadflow (HDLF) method is used in this paper, as proposed in [25].

This study aims to determine optimal sizing for DGs when they are installed in a distribution system. This technique is based on population-based search techniques that apply both random variation and selection. The technique estimates the value of multiple DG units and then the values are used as inputs for the harmonic distribution load flow program. Again, the goal is to minimise the power loss and THD$_v$. The proposed GSA technique is used to find the best solution of the formulated problem. The flow chart of SA and GSA-HDLF algorithm is shown in Figure 2. The assumption are made for DG placement by installed 2 DG units in the distribution system.

Results and Discussion
The proposed method for DG placement and sizing is tested on a 69-bus radial distribution system as shown in Figure 3. The load and bus data of the 69-bus radial distribution system are indicated in [13]. The system loads are considered as spot loads, with the total being 3.8 MW and 2.69 MVAr. The minimum and maximum voltage limits are set at 0.9 p.u and 1.05 p.u. The maximum iteration for the GSA, PSO and EP algorithm is chosen as 100. The only
supply source in the system is the substation at bus 1, which is a slack bus with constant voltage.

The occurrence of the harmonics in the system can be incorporated with the harmonic producing loads, such as adjustable speed drives (ASD). These non-linear loads are located at buses 19, 30, 38, and 57. Another harmonic producing device is added to the distribution system when inverter based DGs are installed in the system. The typical harmonic spectrum of these non-linear loads is provided in Table 1 [24].

Table 1. Harmonic spectrum of non-linear loads and inverter based DG

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>Non-linear loads at bus 19, 30, 38, and 57 (%)</th>
<th>Inverter based DG (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>68.3</td>
<td>38.4</td>
</tr>
<tr>
<td>7</td>
<td>47.8</td>
<td>11.41</td>
</tr>
<tr>
<td>11</td>
<td>0.2</td>
<td>10.8</td>
</tr>
<tr>
<td>13</td>
<td>6.1</td>
<td>7.3</td>
</tr>
<tr>
<td>17</td>
<td>4.2</td>
<td>5.4</td>
</tr>
</tbody>
</table>

The GSA technique is applied to determine the optimal sizing of DGs in the 69-bus radial distribution system, considering the harmonic propagation in the analysis. The total harmonic distortion levels of each DG unit are to be maintained within 5% according to the IEEE standard 51-1992. Two cases are considered with regards to the impact of DG installation on harmonic distortion and power loss, in the 69-bus radial distribution system, as indicated below:

i. No DG installed in the system

ii. Two DG units are installed in the system

In this paper, the number of DG units is assumed to be two and the range of the DG size is between 400 kW – 2000 kW. The application of the sensitivity analysis can reduce the exploration space of the optimisation technique and thus increase the speed of the simulation. Table 2 shows the five most sensitive buses in the system obtained from the sensitivity analysis. Therefore, in this study, only two DGs, namely buses 61 and 21 are considered for placement.

Table 2. Sensitivity analysis results

<table>
<thead>
<tr>
<th>Bus number</th>
<th>Sensitivity to loss reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>61</td>
<td>16.790</td>
</tr>
<tr>
<td>21</td>
<td>2.246</td>
</tr>
<tr>
<td>65</td>
<td>1.351</td>
</tr>
<tr>
<td>59</td>
<td>1.324</td>
</tr>
<tr>
<td>18</td>
<td>1.048</td>
</tr>
</tbody>
</table>

Before applying GSA, parameters are tuned to enhance the performance of the proposed algorithm. A population size of 40 were selected for the GSA algorithm. The same population size are used for PSO and EP algorithm. Then, the proposed technique also has been compare with PSO and EP techniques. Figure 4 shows the best result among 30 simulation runs for these three optimisation techniques. The statistical results for best fitness, mean and standard deviation obtained using SPSS statistic software are summarized in the Table 3. The results indicated that the GSA gives the best fitness compared to PSO and EP. GSA also shows the highest convergence rate compared to other methods.

Table 3. Statistical results of the optimal sizing of DG for three optimisation techniques

<table>
<thead>
<tr>
<th></th>
<th>GSA</th>
<th>PSO</th>
<th>EP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum fitness</td>
<td>4.988e-4</td>
<td>4.993e-4</td>
<td>5.007e-4</td>
</tr>
<tr>
<td>Mean</td>
<td>5.008e-4</td>
<td>5.013e-4</td>
<td>5.030e-4</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.198e-4</td>
<td>2.613e-4</td>
<td>3.777e-4</td>
</tr>
</tbody>
</table>

To validate each algorithm separately, the paired t-test method was used in this study. The basic idea is to test the hypothesis of no difference between two variables. The results are shown in Table 4.

Table 4. Paired t-test analysis

<table>
<thead>
<tr>
<th>Pairs</th>
<th>Mean</th>
<th>Correlation</th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EP-GSA</td>
<td>-2.195e-6</td>
<td>0.174</td>
<td>-3.199</td>
<td>29</td>
<td>0.0003</td>
</tr>
<tr>
<td>EP-PSO</td>
<td>-1.687e-6</td>
<td>0.006</td>
<td>-2.018</td>
<td>29</td>
<td>0.0053</td>
</tr>
<tr>
<td>GSA-PSO</td>
<td>-5.077e-7</td>
<td>-0.018</td>
<td>-0.961</td>
<td>29</td>
<td>0.0445</td>
</tr>
</tbody>
</table>

Table 5. Results of power loss and average THDv for both cases

<table>
<thead>
<tr>
<th>DG availability</th>
<th>Optimization Techniques</th>
<th>Size DG1 (MW)</th>
<th>Size DG2 (MW)</th>
<th>THDv (100%)</th>
<th>Losses (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No DG in the system</td>
<td>-</td>
<td>-</td>
<td>18.09</td>
<td>243.9</td>
<td></td>
</tr>
<tr>
<td>With 2 DGs in the system</td>
<td>GSA</td>
<td>1.71</td>
<td>0.60</td>
<td>1.42</td>
<td>34.92</td>
</tr>
<tr>
<td></td>
<td>PSO</td>
<td>1.88</td>
<td>0.55</td>
<td>1.56</td>
<td>34.95</td>
</tr>
<tr>
<td></td>
<td>EP</td>
<td>1.78</td>
<td>0.60</td>
<td>1.83</td>
<td>35.05</td>
</tr>
</tbody>
</table>

The first column in the paired-samples t test displays the average mean difference between compared algorithms. It clearly shows that there is a clear difference between EP and other two algorithms. Furthermore, at -0.018, the correlation between the GSA and PSO is not statistically significant unlike for the case of EP-GSA and EP-PSO. The t statistic in Table 4 is obtained by dividing the mean difference by its standard error which again indicates GSA-PSO have small difference. The Sig. (2-tailed) column displays that the obtained t statistic is quite satisfactory. Thus this test indicates that GSA and PSO can be equally used to obtain an appropriate fitness level. In addition to
above results, the optimal sizing of the DGs, the power loss and average THD, for the two cases (with and without DGs) with the three optimisation techniques are summarised in Table 5.

From the results shown in Table 5, it can be noted that installing the DG with optimal placement and sizing has significant impacts on the reduction of total loss and average harmonic distortion in the distribution system. The losses are decreased dramatically when two DG with optimal sizes are installed in the system. Table 5 also clearly shows that the proposed technique gives the best solution in term of fitness and convergence rate for minimizing the losses and THDv, compared to the others methods.

Conclusion

This paper proposed a new method for determining suitable placement and optimal sizing of DG units. Firstly, suitable placement of DG was obtained through sensitivity analysis. Then, the optimal sizing of the DG was performed with the GSA technique. The multi-objective function was to minimise the total power loss and THDv. The results indicated that the proposed algorithm is effective in finding optimum sizes of DGs in distribution power systems. Also, the reduction of losses and THDv is clearly seen after optimal DG placement and sizing. The proposed method performs better compared to the other methods such as PSO and EP in minimizing the losses and THDv.

Acknowledgment

This work was supported by Universiti Kebangsaan Malaysia under research grant UKM-GUP-2011-038, Universiti Teknikal Malaysia Melaka and the Ministry of Higher Education of Malaysia.

REFERENCES

[23] Xia D., Heydt G.T., Harmonic power study, Part 2 – Implementation and practical application, IEEE Trans. on IAS, 30(1982), No. 6, 1266-1270

Authors: Aida Fazliana Abdul Kadir, Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia, 43600, Bangi, Selangor, Malaysia. E-mail: fazliana@eng.ukm.my. Prof. Dr. Azah Mohamed, E-mail: azah@eng.ukm.my. Dr. Hussain Shareef. E-mail: shareef@eng.ukm.my. Dr. Mohd Zamri Che Wanik. E-mail: mzamri@eng.ukm.my. Ahmad Asrul Ibrahim. E-mail: asru@eng.ukm.my.