Huan Chen[1], Yongquan Zhou[1,2], Peigang Guo[1], Xinxin Ouyang[1],Sucai He[1],Hongqing Zheng[1]

Guangxi University for Nationalities (1), Guangxi Key Laboratory of Hybrid Computation and IC Design Analysis (2)

A Hybrid Invasive Weed Optimization with Feasibility-Based Rule for Constrained Optimization Problem

Abstract. *During the past decade, hybrid algorithms combining evolutionary computation and constraint-handling techniques is one of the most popular method to solve constrained optimization problems. Usually, penalty functions are often used in constrained optimization. But it is difficult to strike the right balance between objective and penalty functions. As a novel population-based algorithm, invasive weed optimization (IWO) algorithm has gained wide applications in a variety of fields, especially for unconstrained optimization problems. In this paper, a hybrid IWO (HIWO) with a feasibility-based rule is proposed to solve constrained optimization problems. The feasibility-based rule does not need additional parameters, which is different from penalty functions. In addition, the complex method is used to provide direction for weed evolution, which can accelerate the convergence speed. Simulation and comparisons based on several well-studied benchmarks demonstrate the effectiveness, efficiency and robustness of the proposed HIWO.*

Streszczenie. *W artykule przedstawiono opracowaną metodę optymalizacji z funkcją kosztu, bazującą na hybrydowej metodzie IWO (ang. Hybrid Invasive Weed Optimizastion) oraz regułach związanych z wykonalnością. Zasady wykonalności, w przeciwieństwie do funkcji kar, nie wymagają dodatkowych parametrów. Dodatkowo zastosowano kompleksową metodę określania kierunki ewolucji trawy w algorytmie IWO, co pozwala na przyspieszenie konwergencji. Przeprowadzone badania symulacyjne i porównawcze dowodzą skuteczności i sprawności proponowanej metody HIWO. (**Optymalizacja hybrydowa IWO z wykorzystaniem reguł wykonalności w optymalizacji z funkcją kosztu**).*

Keywords: Invasive weed optimization; Feasibility-based rule; Constrained optimization; Complex method
Słowa kluczowe: optymalizacja IWO, reguły wykonalności, optymalizacja z funkcją kosztu, metoda kompleksowa.

Introduction

Constrained optimization problems are very important and frequently appear in many science and engineering disciplines, such as pressure vessel design problem [1], welded beam problem [2], reliability optimization problems [3] and so on. The aim of constrained optimization problems is to search for better objective values decided by feasible solutions that need to satisfy. Generally, a constrained optimization problem can be described as equation (1).

$$\min(f(x)), x = (x_1, x_2, \cdots, x_N)$$

$$(1)\qquad s.t.\begin{cases} g_p(x) \le 0, & p = 1,2,\cdots,N_g \\ h_q(x) = 0, & q = 1,2,\cdots,N_h \\ x_i \in [x_{iL}, x_{iU}], & i = 1,2,\cdots,N \end{cases}.$$

where x_{iL} and x_{iU} are the lower bound and upper bound of the decision vector x , N_g is the number of inequality constraints and N_h is the number of equality constraints. In a common practice, an equality constraint $h_q(x) = 0$ can be replaced by $|h_q(x)| - \delta \le 0$, δ is a small tolerant amount. Therefore all constraints can be transformed to $N_c = N_g + N_h$ inequality constraints. x is a feasible solution if it satisfies all the constraints.

There exist many studies on solving constrained optimization problems, and the traditional mathematical methods are based on the derivative information of the objective function and constraints, such as the Gradient Projection methods and the Lagrange multiplier methods [4]. The main problems are that these methods are sensitive to initial values and need derivative information of the objective function, while some reality problems have no explicit mathematical expression and the feasible region is not connected, so these methods are powerless to them. In recent years, evolutionary algorithms have attracted much attention for constrained optimization problems due to their well balance between the exploration and exploitation of the whole search space. Besides, evolutionary algorithms are not need the objective function to be derivable or continuous search space.

So far, a number of constraint handling techniques have been proposed to incorporate with evolutionary algorithms to solve constrained optimization problems. One of the most popular constraint handling techniques is the penalty function method due to its simple principle and ease to implementation. The violations of constraints of the solutions are incorporated into the objective function so that the original constrained problems are transformed into unconstrained ones. Homaifar A et al. [5] proposed an approach where penalty factors are set to different values for each level corresponding to the violations of constraints and are not depend on the evolution iterations. In [6], Joines J, et al put forward a penalty factors changed with evolution iterations scheme, which aimed at performing wide exploration of the search space at the early stage and gradually guided the search to focus on the feasible region. Although penalty function method is ease to implementation, better penalty factors are hard to be selected. So some self-adaptive techniques are proposed to avoid the trial and error process of tuning factors. Huang F et al [7] proposed a co-evolution particle swarm optimization, where one swarm evolution decision solutions and another evolution penalty factors. In generally, self-adaptive techniques can obtain better performance due to penalty factors adjusted dynamic depend on the feedback information in evolution process.

Apart form the penalty function method, there are several novel techniques have been incorporated into evolution algorithms to handle constraints. Deb [8] proposed a select operator that no penalty factor is needed and three rules are adapted to compare decision solution. Motivated by [8], Qie He and Ling Wang [9] proposed a feasibility-based rule to solve constrained optimization problems and simulated annealing is adapted to incorporate with PSO. Runarsson and Yao [10] proposed a stochastic ranking method. In [11], constrained optimization problems are transformed to multi-objective optimization problems by Coello.

In 2006, a novel stochastic optimization model, invasive weed optimization (IWO) algorithm [12], was proposed by Mehrabian and Lucas, which is inspired from a common phenomenon in agriculture: colonization of invasive weeds. Not only it has the robustness, but also it is easy to understand and program. So far, it has been applied in many fields mainly for unconstrained continuous optimization problems [13-23][30-31]. As for constrained optimization problems, relatively less work based on IWO can be found than those based on other kinds of evolution algorithms. Zhang Qing et al [24] used IWO to optimize Speed Reducers with the same constraint handling techniques described in [8]. Su Shou-bao [25] proposed an invasive weed optimization algorithm with penalty function strategy to deal with constrained engineering design problems.

In this paper, a hybrid IWO with a feasibility-based rule (HIWO) is proposed to solve constrained optimization problems. The feasibility-based rule does not need additional parameters, which is different from penalty functions. In addition, the complex method is used to provide direction for weed evolution, which can accelerate the convergence speed.

The rest of the paper is organized as follows. In Section 2, IWO, the feasibility-based rule and Complex method are simply introduced. In Section 3, the HIWO is proposed. Simulation and comparisons are presented in Section 4, and the conclusions and future work are provided in Section 5.

IWO, Feasibility-Based Rule and Complex Method

IWO

In the basic IWO, weeds represent the feasible solutions of problems and population is the set of all weeds. A finite number of weeds are being dispread over the search area. Every weed produces new weeds depending on its fitness. The generated weeds are randomly distributed over the search space by normally distributed random numbers with a mean equal to zero. This process continues until maximum number of weeds is reached. Only the weeds with better fitness can survive and produce seed, others are being eliminated. The process continues until maximum iterations are reached or hopefully the weed with best fitness is closest to optimal solution.

The process is addressed in details as follows:

Step 1 Initialize a population

A population of initial solutions is being dispread over the D dimensional search space with random positions.

Step 2 Reproduction

The higher the weed's fitness is, the more seeds it produces. The formula of weeds producing seeds is

(2) $$weed_n = \frac{f - f_{\min}}{f_{\max} - f_{\min}}(s_{\max} - s_{\min}) + s_{\min}$$

where f is the current weed's fitness $f_{\max}$ and $f_{\min}$ respectively represent the maximum and the least fitness of the current population. $s_{\max}$ and $s_{\min}$ respectively represent the maximum and the minimum number of seeds that the current population can produce.

Step 3 Spatial dispersal

The generated seeds are randomly distributed over the D dimensional search space by normally distributed random numbers with a mean equal to zero, but with a varying variance. This ensures that seeds will be randomly distributed so that they abide near to the parent plant. However, standard deviation (σ) of the random function will be reduced from a previously defined initial value (σ_{init}) to a final value (σ_{final}) in every generation. In simulations, a nonlinear alteration has shown satisfactory performance, given as follows

(3) $$\sigma_{cur} = \frac{(iter_{\max} - iter)^n}{(iter_{\max})^n}(\sigma_{init} - \sigma_{final}) + \sigma_{final}$$

where, $iter_{max}$ is the maximum number of iterations, σ_{cur} is the standard deviation at the present time step and n is the nonlinear modulation index. Generally, n is 3.

Step 4 Competitive exclusion

After passing some iteration, the number of weeds in a colony will reach its maximum (P_MAX) by fast reproduction. At this time, each weed is allowed to produce seeds. The produced seeds are then allowed to spread over the search area. When all seeds have found their position in the search area, they are ranked together with their parents (as a colony of weeds). Next, weeds with lower fitness are eliminated to reach the maximum allowable population in a colony. In this way, weeds and seeds are ranked together and the ones with better fitness survive and are allowed to replicate. The population control mechanism also is applied to their offspring to the end of a given run, realizing competitive exclusion. Figure 1 is the flowchart of IWO.

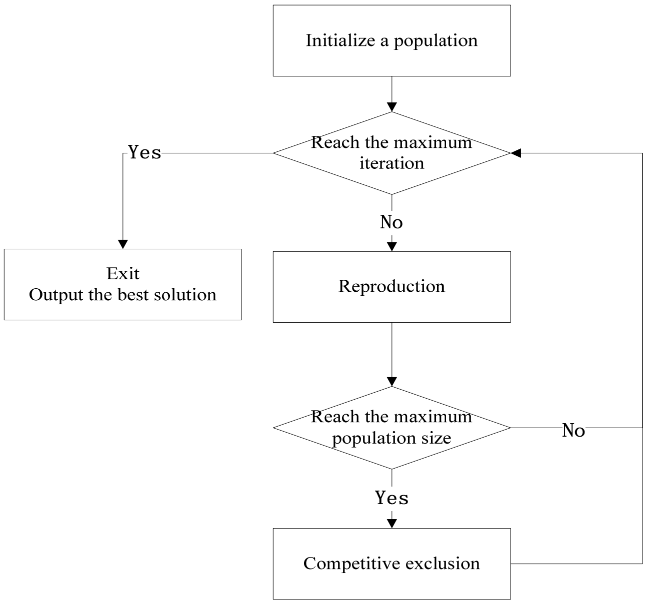

Fig.1. Flowchart of IWO

The feasibility-based rule

Although penalty function method is easy to implement, suitable penalty factors that affected its performance are difficult to determine and problem-dependent.

Motivated by [9], a feasibility-based rule is employed in this paper to handle constraints, which is described as follows:

(1) Any feasible solution is preferred to any infeasible solution.
(2) Between two feasible solutions, the one having better objective function value is preferred.
(3) Between two infeasible solutions, the one having smaller constraint violation is preferred.

Based on the three criteria, there is no need to use penalty factors at all. It can be see that objective function and constraint violation information are considered separately. Moreover, the search tends to the feasible region rather than infeasible region in the first and the third cases, and it tends to the feasible region with good solution in the second case. So, additional fitness function is not need to design, and the objective function value is the fitness of IWO.

Complex method

Simplex Method which is transformed for solving constrained optimization by M. J. Box in 1965 is called Complex Method. It was demonstrated to be easily applied and effective in obtaining optimum solutions as compared with other methods.Suppose arrange $(x_1, x_2, \cdots, x_n)$ sorted by ascending order is set of vertex of Complex,. The key step of Complex Method is as follow:

Step 1 Calculate the centroid x_c

(4) $$x_c = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i - x_n\right)$$

Step 2 Calculate the reflecting point x_r

(5) $$x_r = x_c + \rho(x_c - x_n)$$

where ρ usually set to 1.3 is reflectance. If objective function value $f(x_r) > f(x_n)$, then x_n will be replaced by x_r and go to Step 3, otherwise go to Step 4.

Step 3 Extend operation

(6) $$x_e = x_r + \gamma(x_r - x_c)$$

where γ is lengthening coefficient $\gamma \in [0.5, 0.8]$. If objective function value $f(x_e) > f(x_n)$, then x_n will be replaced by x_e and go to Step 1, otherwise go to Step 4.

Step 4 Compress operation

(7) $$x_s = x_n + \beta(x_c - x_n)$$

where β usually set to 0.7 is contraction coefficient. Check whether the objective function value $f(x_s) > f(x_n)$, if true, x_n will be replaced by x_s and go to Step 1, otherwise constructed new complex again.

Step 5 Repeated more than implementation process, complex then convergence, so every time form the new complex, each times should be discriminant, the discriminant as

(8) $$\left\{\frac{1}{n}\sum_{i=1}^{k}[f(x_c) - f(x_i)]^2\right\}^{\frac{1}{2}} \le \varepsilon$$

where $\varepsilon > 0$. When above inequality was founded, we can stop calculate, output the optimal solution.

HIWO Hybrid Strategy

Updating strategy

In this paper, the constraint violation value of a decision solution is calculated as follows:

(9) $$viol(x) = \sum_{j=1}^{N_c}[\max(g_j(x), 0)]$$

If $viol(x) = 0$, x is feasible solution, otherwise it is infeasible solution. In reproduction case, population is divided into two parts. One part is feasible solution set, another is infeasible solution set. Then the two parts reproduce seeds respectively, and the feasible solution produce more seed, while the infeasible solution produce less seed. In detail, $s_{\max}$ and $s_{\min}$ respectively represent the maximum and the minimum number of seeds that the current population can produce, then $s_{\max}$ and $(s_{\max} + s_{\min})/2$ will be the maximum and the minimum number of seeds that the feasible set can produce, $(s_{\max} + s_{\min})/2$ and $s_{\min}$ will be the maximum and the minimum number of seeds that the infeasible set can produce.

After reproduction, the feasible solution and infeasible solution set will be sort again with their offspring, because the offspring of a feasible solution may be infeasible solution, an infeasible solution's offspring may be feasible solution. In competitive exclusion, if the number of the feasible solution reaches the maximum allowable population in a colony, then better feasible solution will be selected as next generation, otherwise, if there are some feasible solutions but the number is not reach the maximum, the feasible solutions and some infeasible solutions with smaller constraint violation values are selected as next generation. Obviously, if there is no feasible solution in current population, infeasible solutions with smaller constraint violation values are selected as next generation. In brief, the better the object function value of a feasible solution, the greater the opportunity of being selected, the smaller the constraint violation values of an infeasible solution, the greater the chance of being selected.

Complex method-based local search

The complex method is a multivariable, direct-search technique that is efficient and convenient in optimizing problems with nonlinear objective functions subject to inequality constraints on explicit or implicit variables. This method has been applied successfully too many problems in the chemical industry.

In HIWO, after competitive exclusion of IWO, we use the complex method to optimize population that will go to next generation, because the complex method could provide direction for weed evolution. Let $feasible\_num$ be the number of feasible solution after competitive exclusion, and λ the threshold value. If $feasible\_num > \lambda$, then feasible solutions are selected and evolution with complex method, else infeasible solutions are selected and evolution with complex method. The step of complex method is according to section 2.3.

When use the complex method guide feasible, the object function value are used to compare between two seeds, and when use the complex method guide infeasible, the constraint violation value are used to compare between two seeds. In this way, both the results are better than before without using complex method.

Simulation experiments and results analysis

In this section, numerical simulations are carried out to investigate the performances of the proposed HIWO, where 12 constrained benchmark functions are used for testing [10]. The experimental program testing platform as: Processor: CPU Intel Core i3-370, Frequency: 2.40GHz, Memory: 4GB, Operating system: Windows 7, Run software: Matlab7.6. Parameters settings are given by Table1.

Table 1. Parameters settings

Parameter meaning	Variable	Value	Parameter meaning	Variable	Value
The initial number of population	G_SIZE	20	The maximum number of seed generated	$seed\_\max$	10
The maximum number of population	P_MAX	30	The minimum number of seed generated	$seed\_\min$	1
The maximum iteration number	$iter\_\max$	1000	The nonlinear modulation index	n	4

The influence of complex method

To test the influence of complex method, function g02 are selected to investigate the performance. Let f_n be the times of feasible solutions optimized by complex method and $\inf_n$ be the times of infeasible solutions optimized by complex method. Both f_n and $\inf_n$ are integer and ranked in $[0,7]$. For each pair f_n and $\inf_n$, g02 function runs 20 times independently.In order to facilitate observation, Figures from 2 to 5 give the comparison histogram about results of maximum, average, minimum and standard deviation obtained by each pair f_n and $\inf_n$.

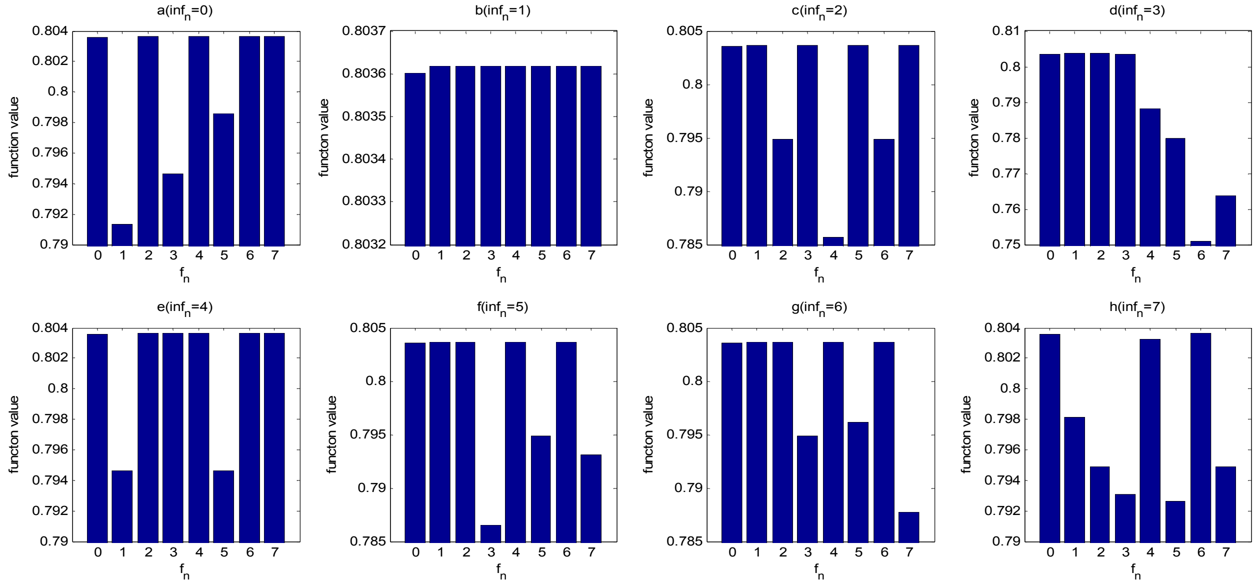

Fig.2. The histogram of maximum

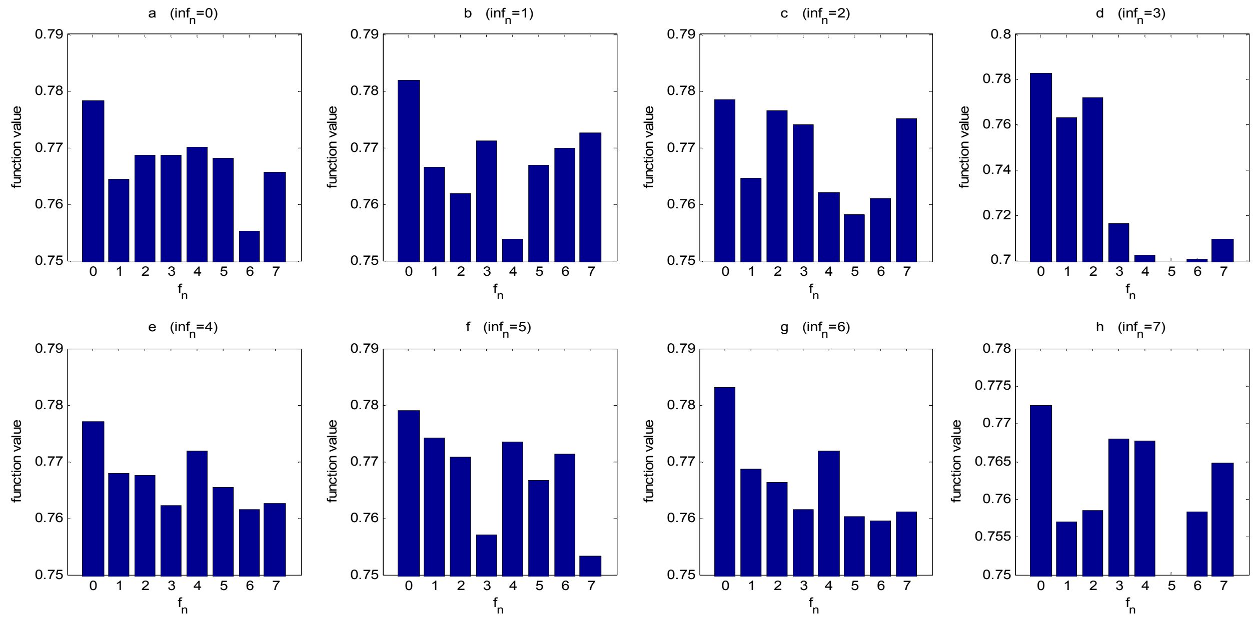

Fig.3. The histogram of average

For Figure 2 (a), the first column is both f_n and $\inf_n$ equal 0, that is the basic IWO, when f_n increase steadily, not all results can reach to the optimal. The same situation emerge other histograms, for example, in Figure 2 (h), $\inf_n$ equal 7 and when f_n increase steadily, not all results can reach to the optimal also. Then we survey the first column of 8 histograms in Figure 2, when f_n is 0, no matter what inf_n equal, the results can reach to the optimal all. That is to say, the effect of f_n is bigger than $\inf_n$ for g02.

In the same way, for each histograms of Figure 3, the average value do not improve with the increase of f_n, while the result is stable when f_n equal 0 and $\inf_n$ increase steadily, which also point out that the effect of f_n is bigger than $\inf_n$ for g02.

From Figure 4, we can see from each histogram that the value with f_n equal 0 is better than f_n equal other values. The same situation emerges in Figure 5. In addition, From Figure 2,3,4,5, when f_n equal 0 and $\inf_n$ equal 3, the maximum, average, minimum and standard deviation are all better than others.

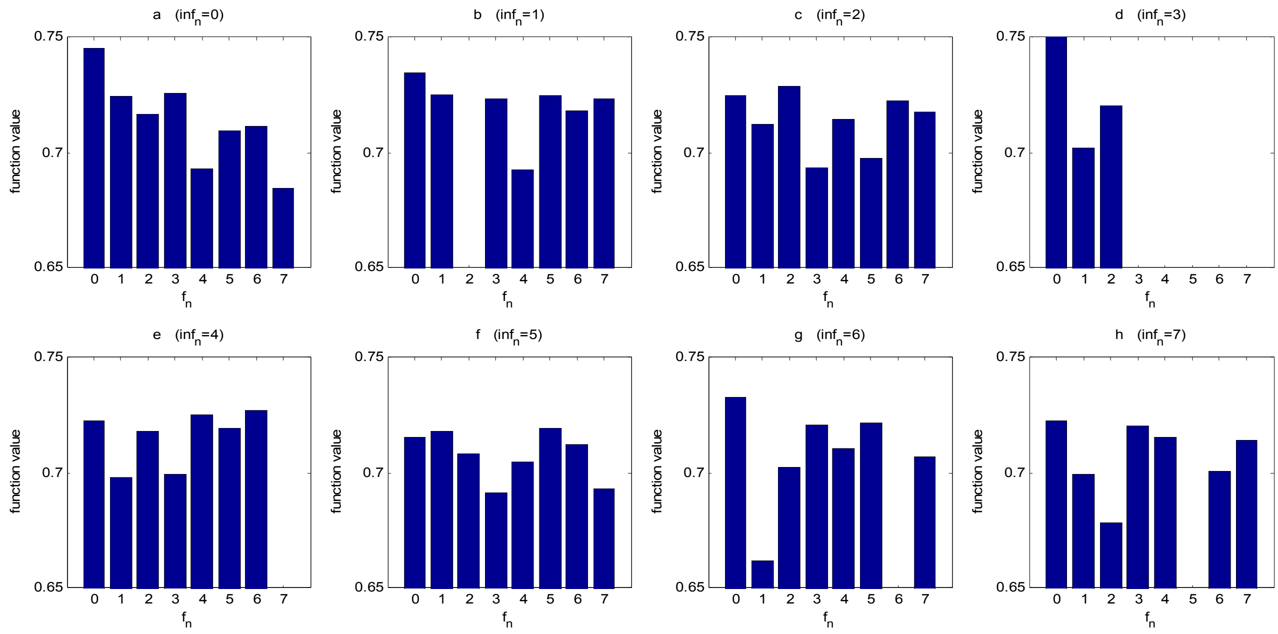

Fig.4. The histogram of minimum

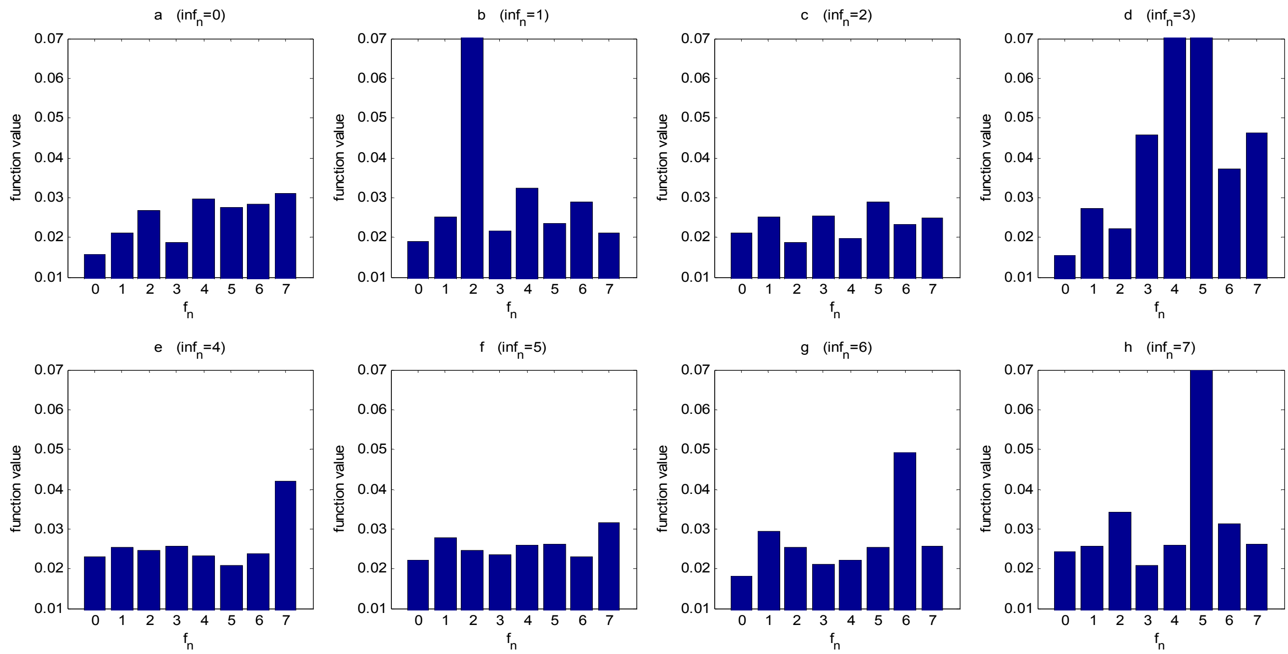

Fig.5. The histogram of standard deviation

Figure 6 gives the box plot about errors with f_n equal 0 and $\inf_n$ equal different values. The first column is the basic IWO. We can see that, the result with $\inf_n$ equal 3 is better than others. Figure7 to Figure12 give some test functions' curve evolution diagram about IWO and HIWO. From the figures, we can see that the convergence speed of HIWO is faster than IWO. So, the complex method is effective.

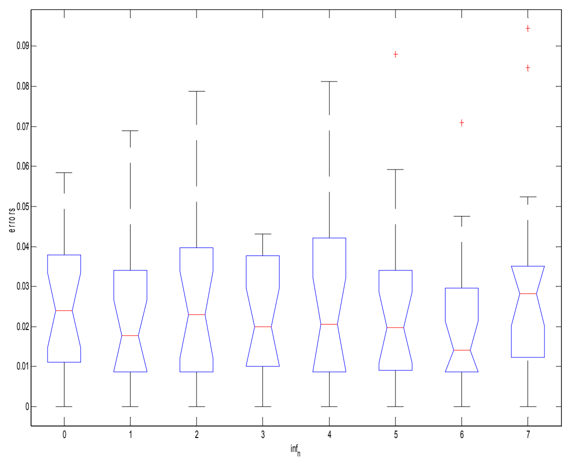

Fig.6. The box plot about errors with f_n equal 0

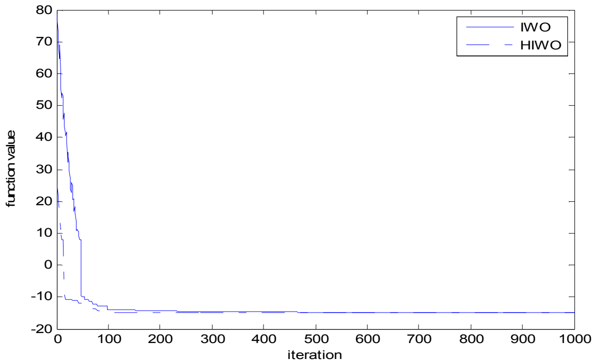

Fig.7. Curve evolution diagram of g01

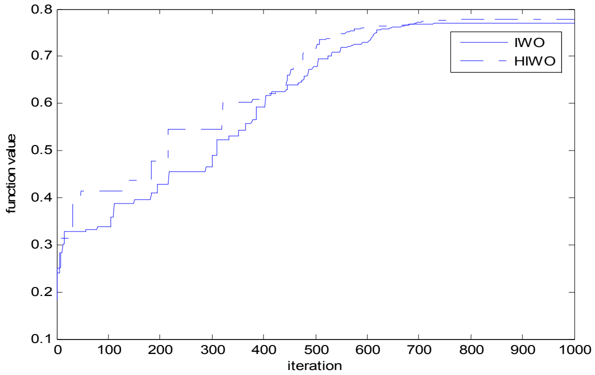

Fig.8. Curve evolution diagram of g02

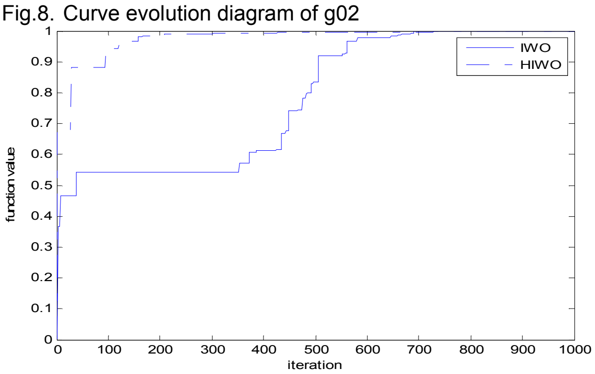

Fig.9. Curve evolution diagram of g03

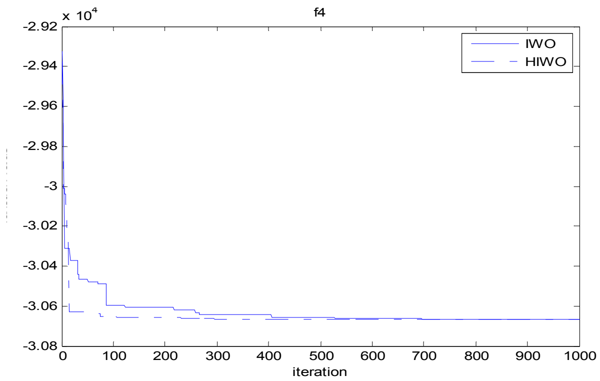

Fig.10. Curve evolution diagram of g04

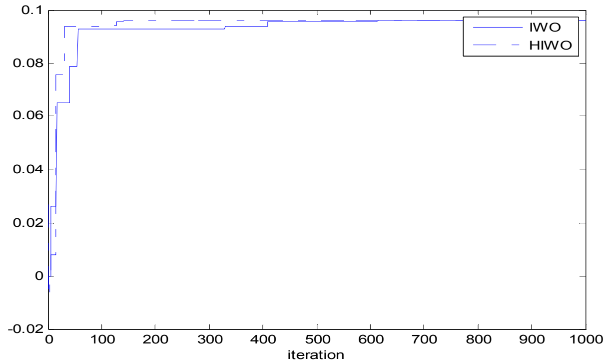

Fig.11. Curve evolution diagram of g08

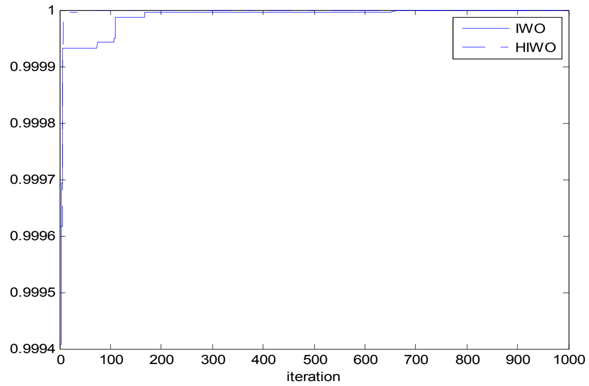

Fig.12. Curve evolution diagram of g12

Comparison with other algorithm

To investigate the performances of the proposed HIWO, RY proposed in [10], KM proposed in [26], SAFF proposed in [27], SMES proposed in [28], ICMOA proposed in [29] are selected to compare with HIWO. In this paper, all equality constraint $h_q(x)=0$ has been replaced by $|h_q(x)|-\delta\le 0$, using the degree of violation $\delta=1e-4$.

Table 2 is the settings of f_n and $\inf_n$. Table 3 gives the results of 12 functions obtained by these 6 algorithms.

Table 3 gives the conclusion that the best results obtained by RY, SAFF, SMES, ICMOA and HIWO are better than KM. For g01, g03, g04, g08, g11, g12, RY, SAFF, SMES, ICMOA and HIWO can get optimal in every run. For g02, ICMOA and HIWO can obtain the better result than other 4 algorithms; what's more, the standard deviation of HIWO is 1.5E-02, which is better than 5 other algorithms. In other words, the robustness of HIWO is better. For g05, result obtained by ICMOA is same with HWIO. For g06, g07 and g09, the standard deviation of HIWO is better than other 5 algorithms. Although the result of g10 obtained by HIWO is worse than ICMOA, it is better than other 4 algorithms, in addition, the best result of g10 obtained by HIWO is close to that obtained by ICMOA. In a word, the algorithm proposed in this paper is effective and robustness.

Table 2. The settings of f_n and $\inf_n$ for 12 test functions

	g01	g02	g03	g04	g05	g06	g07	g08	g09	g10	g11	g12
f_n	7	0	1	2	4	2	0	4	1	3	2	1
$\inf_n$	1	3	5	0	0	5	2	1	1	6	0	0

Table 3. Results obtained by RY, SAFF, SMES, ICMOA and HIWO

Function name	result	KM	RY	SAFF	SMES	ICMOA	HIWO
g01	Best	-14.7864	-15.000	-15.000	-15.000	-15.000	-15.000
	Average	-14.7082	-15.000	-15.000	-15.000	-15.000	-15.000
	Worst	-14.6154	-15.000	-15.000	-15.000	-15.000	-15.000
	Std	3.1E-02	0	0	0	0	0
g02	Best	0.79953	0.803515	0.80297	0.803601	0.803619	0.803618
	Average	0.79671	0.782134	0.79121	0.784257	0.789512	0.782741
	Worst	0.79119	0.721254	0.72157	0.750984	0.697845	0.760555
	Std	7.0E-03	3.7E-02	1.9E-02	3.8E-02	6.5E-02	1.5E-02
g03	Best	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000
	Average	0.9989	1.0000	1.0000	1.0000	1.0000	1.0000
	Worst	0.9978	1.0000	1.0000	0.999	1.0000	1.0000
	Std	9.5E-04	0	0	3.7E-05	0	0
g04	Best	-30664.5	-30665.539	-30665.50	-30665.539	-30665.539	-30665.539
	Average	-30665.3	-30665.539	-30664.20	-30665.539	-30665.539	-30665.539
	Worst	-30645.9	-30665.539	-30663.30	-30665.539	-30665.539	-30665.539
	Std	1.4E+00	0	2.3E-01	0	0	0
g05	Best	--	5126.497	5126.989	5126.610	5126.4981	5126.4981
	Average		5129.126	5431.884	5237.693	5126.4981	5126.4981
	Worst		5146.254	6081.547	5302.656	5126.4981	5126.4981
	Std		5.3E+00	4.0E+03	4.9E+01	0	0
g06	Best	-6952.141	-6961.814	-6961.800	-6961.814	-6961.814	-6961.812
	Average	-6342.667	-6874.457	-6961.800	-6961.284	-6961.817	-6961.808
	Worst	-5473.982	-6352.867	-6961.800	-6960.482	-6960.945	-6961.803
	Std	7.4E+02	8.9E+01	0	5.3E-01	3.8E-03	3E-03
g07	Best	--	24.307	24.48	24.327	24.306	24.309
	Average		24.371	26.53	24.471	24.311	24.338
	Worst		24.638	28.46	24.833	24.324	24.412
	Std		2.8E-02	7.3E-01	6.5E-01	1.3E-01	3.2E-02
g08	Best	0.095825	0.095825	0.095825	0.095825	0.095825	0.095825
	Average	0.089156	0.095825	0.095825	0.095825	0.095825	0.095825
	Worst	0.029143	0.095825	0.095825	0.095825	0.095825	0.095825
	Std	4.5E-01	5.3E-11	3.7E-08	0	0	0
g09	Best	680.91	680.630	680.64	680.632	680.630	680.6301
	Average	681.16	680.656	680.72	680.643	680.632	680.6333
	Worst	683.18	680.763	680.87	680.719	680.671	680.6440
	Std	3.1E-01	2.1E-02	4.9E-02	1.7E-02	8.3E-03	4.4E-03
g10	Best	--	7054.316	7061.34	7051.903	7049.284	7049.376
	Average		7559.192	7627.89	7253.047	7049.289	7055.881
	Worst		8835.655	8288.79	7638.366	7049.291	7065.185
	Std		4.5E+02	3.9E+02	9.2E+01	1.7E-01	5.2
g11	Best	0.750	0.75	0.75	0.75	0.75	0.75
	Average	0.750	0.75	0.75	0.75	0.75	0.75
	Worst	0.750	0.75	0.75	0.75	0.75	0.75
	Std	3.4E-05	2.5E-08	5.2E-07	2.1E-15	0	0
g12	Best	1.000	1.000	1.000	1.000	1.000	1.000
	Average	0.9991	1.000	1.000	1.000	1.000	1.000
	Worst	0.9919	1.000	1.000	1.000	1.000	1.000
	Std	2.6E-01	0	0	0	0	0

Conclusions

This paper proposed a novel hybrid invasive weed optimization with a feasibility-based rule, which provides an effective alternative for solving constrained optimization problems to overcome the weakness of the penalty function methods. Simulation results and comparisons showed that our proposed HIWO is of good performances in terms of searching quality, efficiency and robustness. The future work is to study the adaptive HWIO and to solve the multi-objective constrained optimization problems.

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**Corresponding author: Yongquan Zhou, Ph.D.& Prof.*
College of Information Science and Engineering, Guangxi University for Nationalities, Nanning Guangxi, 530006, China
E-mail:yongquanzhou@126.com