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# The estimation of radio-electronic devices reliability on the basis of interval data analysis

***Abstract**. Estimation the radio-electronic device's reliability by the efficiency probability has been research in this paper. The approach based on interval data analysis proposed. Its low calculable complexity in comparing to the known methods has been shown.*

***Streszczenie.** Artykuł przedstawia sposób oszacowania bezawaryjnego czasu pracy urządzeń radioelektronicznych. Zaproponowana metoda bazuje na analizie interwałowej charakterystyk wyjściowych urządzeń. W artykule wykazano również niski koszt obliczeniowy prezentowanego podejścia w stosunku do innych metod. **(Określanie stopnia niezawodności urządzeń elektronicznych na podstawie analizy interwałowej)***

**Keywords**: Radio-Electronic Circuit, Reliability, Operational Suitability, Tolerance Domain of Parameters, Interval Data Analysis.
**Słowa kluczowe:** urządzenia radioelektroniczne, bezawaryjny czas pracy, interwałowa analiza danych

**Introduction**

Reliability of radio-electronic devices (RED) is their ability to perform their specified functions while maintaining, for a required period of time, operational performance within acceptable limits, and to resume their operation that was lost due to some reasons [1]. The concept of reliability is directly related to the estimation of the probability of operational suitability of RED in the course of its development and operation. In general, reliability of RED is directly determined by reliability and operational suitability of its components which are radio-electronic circuits (REC). While designing a REC one should find the vector of nominal parameters of the elements, which provides nominal values of output characteristics. In the process of production and operation of the equipment the real values of the output characteristics differ from the nominal ones. Moreover, the significant deviations of the real values from the nominal ones leading to the loss of operational suitability of REC, i.e. the output characteristics fall outside specified intervals what leads to lower reliability. One of the ways of increasing the reliability is taking into account, at the design stage, the evolution of the dispersion area of the random values of the elements' parameters in the course of RED operation. This fact dictates topicality and timeliness of the reported research.

**Statement of the problem**

The requirements for the operational suitability of REC may be written in the form of such a system

(1) $$y_i^- \le g_i(\vec{b}) \le y_i^+ ,$$

where $[y_i^-, y_i^+]$ are the intervals of output characteristics $y_{0i} \in [y_i^-, y_i^+]$ that are allowed in terms of operational suitability of REC. Let us apply to the nonlinear (in general case) REC characteristics $g_i(\vec{b})$ the linearization on logarithmic values of the parameters in the point neighborhood $(ln(b_{01}),\ldots,ln(b_{0m}))$ and proceed in the linearized system to the tolerances for the *i*-th REC characteristic $\delta y_i^- = y_i^- - y_{0i}$, $\delta y_i^+ = y_i^+ - y_{0i}$. We obtain [2]:

(2) $$\delta y_i^- \le \sum_{j=1}^{m} S_{ij} \cdot \delta b_j \le \delta y_i^+ , \quad i = 1,...,N,$$

where $S_{ij} = b_j \cdot \frac{\partial y_i(\vec{b})}{\partial (b_j)}\Big|_{\vec{b}=\vec{b}_0}$ is the sensitivity of the *i*-th REC characteristic to the variation of the *j*-th elements' parameter; $\delta b_j = \ln(b_j) - \ln(b_{0j})$.

The advantage of the expansion in logarithmic variables lies in the fact that for small values $\delta b_j$ the expansion is close to the relative deviation of the parameter $b_j$ from the nominal one and is normally distributed.

The solution of the system (2) in the space of the elements' parameters $\vec{b} \in R^m$ is the domain of REC efficiency which is also a tolerance domain $\tilde{\Omega}$ [2,3].

Under these conditions the task of evaluating the reliability of REC is the task of estimation of the efficiency probability $P_d$ - as the probability of "entering" a random vector $\delta\vec{b} = (\delta b_1,...,\delta b_m)^T$ into the efficiency domain $\tilde{\Omega}$. While considering the REC production the efficiency probability may be described in terms of the percentage of suitable REC and be calculated by the Eq. (3):

(3) $$P_d = \int \ldots \iint_{\tilde{\Omega}} W_\delta(y_1,...,y_N)dy_1...dy_N ,$$

where $W_\delta(y_1,...,y_N)$ is the probability density of random deviations $\delta y_1(\vec{b})$ of the REC characteristics.

Apparently, this problem is quite complicated, because the values $\delta y_i(\vec{b})$ are correlated. Its solution is obtained by means of approximate methods, including Monte Carlo method. The number of tests, depending on the desired accuracy of calculation, cans more than several thousand, which is the main drawback of this approach.

**Proposed solving methodology**

Let us consider the case when the number of REC characteristics and the elements' parameters is the same, i.e. $N = m$. Then the sensitivity matrix $S$ in the system (2) is square ($m\times m$), and the efficiency domain in the space of the parameters forms a parallelotope $\tilde{\Omega}_m$. Let us assume that $rang(S) = m$. In this case the efficiency domain can be estimated by a tolerance ellipsoid:

(4) $$Q_m^- = \left\{\delta\vec{b} \in R^m \middle| \ (\delta\vec{b} - \bar{\bar{\delta\vec{b}}})^T \cdot S^T \cdot \tilde{E}^{-2} \cdot S \cdot (\delta\vec{b} - \bar{\bar{\delta\vec{b}}}) \le 1\right\},$$

where $\tilde{E}$ is diagonal matrix of tolerances $0{,}5\left(\delta y_i^+ - \delta y_i^-\right)$ the deviations of REC characteristics.

If $N < m$, on the basis of physical reasons we can supplement the system (2) with inequalities $\delta b_j^- \le \delta b_j \le \delta b_j^+$ which define tolerances for the parameters' deviations. The obtained system of $N = m$ inequalities (2) remains linear.

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If $N > m$ then system (2) is over-determined. To transfer it to the case of $N = m$ in-equalities we should exclude uninformative inequalities that do not affect or substantially influence the size of the efficiency domain. This procedure can be performed, for example, using the method of localization with a distinguished filled block [4].

Let's consider the case of discrete technology of REC production. In this case a random value $\delta b_j$ is normally distributed and tolerance values for deviations of REC parameters from nominal are set by confidence intervals:

(5) $$-\vec{\sigma} \cdot u(\alpha) \le \delta\vec{b} \le \vec{\sigma} \cdot u(\alpha),$$

where $\vec{\sigma} = (\sigma_1,...,\sigma_m)^T$ is a vector of the known standard deviations the radioelement's parameters; $u(\alpha)$ is a tabulated value (quantile) of the normalized normal distribution; $\alpha$ is confidence probability.

Quite often in the process of REC design the intervals of output characteristics $[y_i^-, y_i^+]$ are symmetric in relation to nominal values $y_{0i}$, then $\delta y_i^+ = -\delta y_i^-$ and accordingly the center of tolerance ellipsoid (5) coincides with a «zero» point, as it is shown in Fig. 1:

(6) $$\delta\bar{\vec{b}} = (0,...,0)^T .$$

In this case the estimation of efficiency probability $P_d$ on bound of tolerance area (5) determine by substitution instead of $\delta\vec{b}$ in the Eq. (5) the confidence interval $[-\vec{\sigma} \cdot u(\alpha); \vec{\sigma} \cdot u(\alpha)]$. With condition (6) considering will get:

(7) $$u^2(\alpha) \cdot [\vec{\sigma}^T] \cdot S^T \cdot \tilde{E}^{-2} \cdot S \cdot [\vec{\sigma}] = 1 .$$

Now, taking into account symmetry of normal distribution, finally will get:

(8) $$u^2(\alpha) \cdot \vec{\sigma}^T \cdot S^T \cdot \tilde{E}^{-2} \cdot S \cdot \vec{\sigma} = 1 .$$

Hence:

(9) $$u(\alpha) = 1 / \left(\vec{\sigma}^T \cdot S^T \cdot \tilde{E}^{-2} \cdot S \cdot \vec{\sigma}\right)^{\frac{1}{2}} .$$

Using the tables of the normalized normal distribution and on the basis of the calculated quantile $u(\alpha)$ will find confidence probability $\alpha$ and accordingly estimation of efficiency probability $P_d$ :

(10) $$P_d^- > 2\alpha .$$

However in the operating process the parameters of radio elements will change under the action of external factors, for example change of temperature, humidity etc. Obviously, that these changes will have group character. In case the normal distribution of the probable parameter's deviations in the operating process the random dispersions of radioelement's parameters under the action of external factors can be described by a ellipsoid in kind:

(11) $$Q(\alpha, m) = \left\{ \delta\vec{b} \in R^m \middle| \delta\vec{b}^T \cdot D(\delta\vec{b}) \cdot \delta\vec{b} \le \chi^2(\alpha, m) \right\},$$

where $D^{-1}(\delta\vec{b})$ is given covariance matrix of probable deviations the radioelement's parameters in the operating process; $\chi^2(\alpha, m)$ is a quantile of $\chi^2$ distribution. Notice that in the certain cases the change of radioelement's parameters under the influence of external factors can increase general efficiency probability $P_d$ , if these changes have opposite directionality in relation to technological deviations of REC parameters from nominal values. However mainly in the process of design these facts is impossible to detect, that is why the estimation of efficiency probability $P_d^+$ , related to the changes of radioelement's parameters in the operating process, follows to search by considering the worst case.

Let's a covariance matrix $D^{-1}(\delta\vec{b})$ of probable deviations the radioelement's parameters in the operating process is priori given. At first will assume that technological deviations in the process of radioelement production are absent, that is: $\vec{\sigma} = (\sigma_1,...,\sigma_m)^T = (0,...,0)^T$ and then will estimate the efficiency probability $P_d^+$ , related exceptionally with probable deviations of radioelement's parameters in the REC operating process. Taking into account given conditions, the unknown value of confidence probability $\alpha$ in Eq. (11) will find as solution the next task:

(12) $$\chi^2(\alpha, m) \xrightarrow{\alpha \in [0,1]} max , \; Q(\alpha, m) \subseteq \tilde{\Omega}_m .$$

From the geometrical point of view a task (12) is the task of inscribing the *m*-dimensional ellipsoid (11) $Q(\alpha, m)$ into the efficiency domain $\tilde{\Omega}_m$ with the choice of value $\alpha$ by such way, that the inscribed ellipsoid had a maximum volume.

As centers of ellipsoid (11) and efficiency domain $\tilde{\Omega}_m$ under condition (6) coincide, then for solving the task (12) will use the Eq. (13) [4]:

(13) $$H = S^T \cdot \tilde{E}^{-1} \cdot \Lambda^{-1} \cdot \tilde{E}^{-1} \cdot S ,$$

Eq. (13) set the configuration matrices of ensemble the possible ellipsoids inscribed into the efficiency domain $\tilde{\Omega}_m$.

Will replace in Eq. (13) the matrix $H$ on the matrix $D(\delta\vec{b}) / \chi^2(\alpha, m)$ of configuration the ellipsoid (11) reduced to the single radius and will get:

(14) $$D(\delta\vec{b}) / \chi^2(\alpha, m) = S^T \cdot \tilde{E}^{-1} \cdot \Lambda^{-1} \cdot \tilde{E}^{-1} \cdot S .$$

Will solve this system in relation to a matrix $\Lambda' = \Lambda / \chi^2(\alpha, m)$:

(15) $$\Lambda' = \tilde{E}^{-1} \cdot S \cdot D^{-1}(\delta\vec{b}) \cdot S^T \cdot \tilde{E}^{-1}$$

There is considered in [4] that according to the properties 1, 2 of ellipsoids inscribed in a tolerance domain, for the diagonal elements $\Lambda_{ii}$ of matrix $\Lambda$ the condition $\Lambda_{ii} \le 1$, $\forall i = 1,...,m$ is executed, and at the same time $\Lambda_{ii} = 1$ if an ellipsoid is tangent to corresponding *i*-th pair of facets. Then for the diagonal elements of the system (15) solution the such conditions $\Lambda'_{ii} \le 1 / \chi^2(\alpha, m)$, $\forall i = 1,...,m$ will be executed, and at that ellipsoid of parameters dispersion will be tangent to *i*-th pair of facets when $\Lambda'_{ii} = 1 / \chi^2(\alpha, m)$.

With the above in this case it is follow the next algorithm of estimation the efficiency probability of REC:

Step 1. Calculation the elements $\Lambda'_{ii}$ of matrix $\Lambda'$ by Eq. (15).

Step 2. Calculation the quantile of $\chi^2$ distribution by Eq.:

(16) $$\chi^2(\alpha, m) = 1 / max \{\Lambda'_{ii}\}, \; i = 1,...,m .$$

Step 3. Finding in tables of $\chi^2$ distribution the confidence probability $\alpha$ for a value $\chi^2(\alpha, m)$.

Step 4. Calculation the estimation of efficiency probability by Eq.:

(17) $$P_d^+ = 1 - \alpha \ .$$

Aggregate estimation of efficiency probability $P_d$ with taking into account the technological domain of deviations the radioelement's parameters in the production process will find by Eq:

(18) $$P_d > P_d^- \cdot P_d^+ .$$

As evidently from Eq. (18) the reliability of REC in the operating process will decrease. Therefore it is important on the design stage to take into account probable deviations of radioelement's parameters in the REC operating process. These deviations caused by influence of external factors.

**Example**

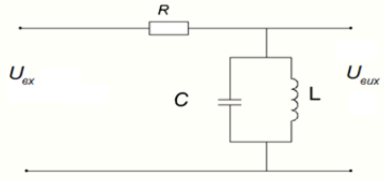


Fig.1. Scheme of band filter

A scheme of band filter is shown in Fig. 1. It will exemplify for evaluation its reliability by the efficiency probability $P_d$ . Amplitude frequency characteristic of band filter for the nominal values of radio-element's parameters will have kind:

(19) $$y_{0i} = K_{0i} = K_0(f_i) = \frac{1}{\sqrt{R_0^{\,2}(\frac{1}{2\pi \cdot f_i \cdot L_0} - 2\pi \cdot f_i \cdot C_0)^2 + 1}} .$$

Will set the nominal values of parameters: $L_0$ =10 mH, $C_0$ =0,5 μF, $R_0$ =1 kΩ. The nominal values the modulus of transmission coefficient are obtained for three frequencies: $f_1$ =1410 H, $f_2$ =2110 H, $f_3$ =2810 H. Thus from Eq. (16) will get three nominal values of REC's characteristic ($N$=3): $K_{01}$ =0,1441, $K_{02}$ =0,7354, $K_{03}$ =0,3020. Will set requirements to the operating suitability of band filter (Fig.2): on given frequencies the modulus of transmission coefficient should not deviate more than 30% relative to nominal values, that is $\delta K_i^- = K_{0i} - 0{,}3 \cdot K_{0i}$ ; $\delta K_i^+ = K_{0i} + 0{,}3 \cdot K_{0i}$ .

After applying amplitude-frequency characteristic linearization on the radioelement's parameters, which set by Eq. (16), in the nominal values neighborhood and after defining the sensitivity of this characteristic on different frequencies to the changes of radioelement's parameters, then will get system of interval equations (2) in matrix kind:

(17) $$\delta\vec{K}^- \le S \cdot \delta\vec{b} \le \delta\vec{K}^+ ,$$

where $\delta\vec{b} = (\delta R, \delta C, \delta L)^T$ is a vector of relative deviations of values from nominal resistance, capacity and inductance accordingly in a band filter;

$$\delta\vec{K}^- = \{\delta K_i^-, i = 1,...,3\} = (-0{,}0432 \quad -0{,}2206 \quad 0{,}0906\ )^T \ ;$$

$$\delta\vec{K}^+ = \{\delta K_i^+, i = 1,...,3\} = (0{,}0432 \quad 0{,}2206 \quad 0{,}0906\ )^T \ ;$$

$$S = \{S_{ij}, i = 1,...,3, j = 1,...,3\} = \begin{pmatrix} -0{,}1411 & 0{,}0910 & 0{,}2321 \\ -0{,}3377 & 2{,}4282 & 2{,}7658 \\ -0{,}2745 & -0{,}7672 & -0{,}4927 \end{pmatrix} .$$

Let's in the process of radioelement's production for band filter the technological deviations of their values from nominal have normal distribution and at that according to Eq. (6) have following confidence intervals: $|\delta R| \le 0{,}02 \cdot u(\alpha)$ ; $|\delta C| \le 0{,}02 \cdot u(\alpha)$; $|\delta L| \le 0{,}02 \cdot u(\alpha)$. Notice that in got confidence intervals the deviations are given in relative units at the level of 2% deviations from nominal values.

After calculations by Eq. (9) will get $u(\alpha) = 1{,}78$ . Therefore according to table of normalized normal distribution and by Eq. (10) will get: $P_d^- = 0{,}935$ .

Now will consider the probability of operating suitability related to deviations the radioelement's parameters of band filter in the operating process. Let's the covariance matrix of ellipsoid (11) of the predicted probable deviations the radioelement's parameters in the operating process have following kind:

$$D^{-1}(\delta\vec{b}) = 10^{-4} \cdot \begin{pmatrix} 4 & 1 & 2.5 \\ 1 & 4 & 1 \\ 2.5 & 1 & 4 \end{pmatrix} .$$

It is necessary to notice that the diagonal elements of matrix set the squares of dispersions the relative deviations of REC parameters from nominal values at the level of 2% like in a previous example. Then calculate a matrix $\Lambda'$ by Eq. (15) (step 1) and quantile $\chi^2(\alpha, m = 3)$ by Eq. (16) (step 2): $\chi^2(\alpha, m = 3) = 7{,}89$ . Using tables of $\chi^2$ distribution (step 3) will get: $P_d^+ = 0{,}95$ . Reliability (efficiency probability) of band filter considering technological domain of deviations the radioelement's parameters in the production process and deviations of parameters in the operating process will be not less then calculated one's by Eq. (18): $P_d > 0{,}89$ . As you can see, the aggregate reliability of REC substantially decrease taking into account possible random change of parameters under the influencing of external factors.

**Conclusion**

The mathematics to estimate the REC's efficiency probability has been derived in this paper. Offered approach has substantially less calculable complexity in comparing to the known methods, as gives possibility to find a solution in an analytical kind. Application the offered approach on an example showed importance on the stage of design to consider the possible deviations of REC's parameters in the operating process. In opposite case reliability of REC will be substantially understated.

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