Petro STAKHIV[1], Yuriy KOZAK[2], Oksana HOHOLYUK[2]

Technical University of Łódz (1), Lviv Polytechnic National University (2),

Discrete mathematical macromodel of electric transmission line

Abstract. *In the paper the problem of development of mathematical models and macromodels of transmission lines is discussed. Mathematical discrete macromodel of single-phase transmission line in the form of state variables using a "black box" approach was developed. Description of procedure of its creation and verification of obtained results are presented.*

Streszczenie. *W artykule przedstawiono makromodel energetycznej linii przesyłowej. Prezentowany model został zapisany w formie zmiennych stanu i dotyczy linii jednofazowej. Dodatkowo w artykule opisano proces tworzenia modelu, sposób jego walidacji wraz z kryterium poprawności, a na końcu zamieszczono wyniki.**(Makromodel linii elektroenergetycznej)***

Keywords: *mathematical model, macromodel, transmiision line, optimization*
Słowa kluczowe: *model matematyczny, linia przesyłowa, optymalizacja*

Introduction

Over the last decade a considerable progress in creation of mathematical models of electric systems intended for the analysis of transient and periodic processes and further prognosis of electric power equipment operation was achieved. Improvement of the modeling process in order to obtain adequate simulation results of electric systems containing single- and three phase transmission lines is an actual problem till now.

Development of electric power transmission line models with a high level of adequacy using their equivalent schemes leads to their excessive complication, and as a result, there are significant difficulties during their adaptation to modern program tools intended for the transients' analysis. If a researcher is interested only in a reaction of the modeled object, it is possible to use the principles of macromodeling. Therefore it is expedient to create macromodels of specific objects instead of their precise mathematical models. Mathematical macromodels in the form of state variables (the "black box" approach) is the best type of macromodels which can solve the mentioned problem [1].

Two types of transmission line models are usually used for their analysis in time domain:
a) models with lumped parameters. Models of this type are created on the basis of equivalent electric circuit with lumped parameters which values are calculated at a certain frequency;
b) models with distributed parameters.

The first type of models is suitable for the calculation of steady state modes, although it can also be used for modeling of transient processes for frequency (and close to it) for which the model parameters were defined. However, the most accurate models intended for the calculation of transients include distributed parameters and their frequency dependence. Although some effective models are included into the most widely used programs, however, a new efforts aimed to develop more efficient models and enable their usage in modern software tools are observed.

The goal of the research is to develop a discrete macromodel of transmission line in the form of state variables using a "black box" approach (Fig. 1). The macromodel can be described by the following equation:

$$(1)\qquad \begin{cases} \mathbf{x}^{(k+1)} = \mathbf{F}\cdot\mathbf{x}^{(k)} + \mathbf{G}\cdot\mathbf{v}^{(k)} + \mathbf{\Phi}(\mathbf{x}^{(k)}, \mathbf{v}^{(k)}) \\ \mathbf{y}^{(k+1)} = \mathbf{C}\cdot\mathbf{x}^{(k+1)} + \mathbf{D}\cdot\mathbf{v}^{(k+1)} \end{cases}$$

where $\mathbf{F}$, $\mathbf{G}$, $\mathbf{C}$, $\mathbf{D}$ are matrices of corresponding dimensions; $\mathbf{\Phi}$ is some vector-function of several variables, $\mathbf{x}^{(k)}$, $\mathbf{v}^{(k)}$, $\mathbf{y}^{(k)}$ are vectors of internal, input and output variables respectively, *k* is the time discrete number.

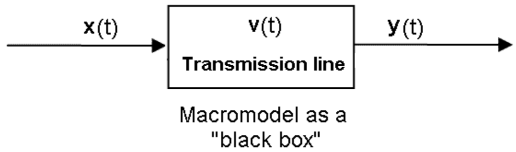

Fig. 1. A scheme of the object for marcromodel creation in the form of state variables

Construction of a macromodel of any dynamic object, and in this case it is a transmission line, is based on a set of characteristics which can be obtained during mathematical experiment or at field test.

The procedure of the macromodel construction includes the following four steps:
- obtaining information which will be used for the macromodel construction;
- the selection of the macromodel form;
- the identification of the macromodel coefficients;
- the verification of the macromodel using independent test signal that was not used for its creation.

To test the adequacy of the macromodel it is necessary to conduct its criterion evaluation. The criterion of the macromodel accuracy is the minimizing of the goal function $\mathbf{Q}(\boldsymbol{\lambda})$, where $\boldsymbol{\lambda}$ is a vector of unknown coefficients including elements of matrices $\mathbf{F}$, $\mathbf{G}$, $\mathbf{C}$, $\mathbf{D}$ and coefficients of nonlinear function $\mathbf{\Phi}$. If we find the minimum of function $\mathbf{Q}$, we will have the best set of coefficients for the selected form of macromodel with minimization of the error. Here the goal function can satisfy the following expressions:

$$(2)\qquad Q(\lambda) = \sqrt{\frac{1}{k}\sum_k \left|\mathbf{y}-\mathbf{y}^*\right|^2}\ ,\ Q(\lambda) = \sum_k \left|\mathbf{y}-\mathbf{y}^*\right|$$

where $\mathbf{y}^*$ are transient characteristics calculated based on developed macromodel, $\mathbf{y}$ are transient characteristics of the real object obtained based on mathematical or field experiment.

Mathematical model of transmission line

A full-scale experiment for research of long lines transient processes when they are considered as components of high-voltage transmission systems is connected with considerable difficulties that involves using of complex measuring equipment, special hardware and software for the registration and further processing of obtained data. Usage of modern software, such as ATP or MATLAB/Simulink, in order to obtain adequate data arrays involves developing of mathematical models of electric system elements. The following transmission lines models, for example, are built into MATLAB/SIMULINK environment:

1) single-phase PI section transmission line with lumped parameters, 2) three-phase PI section transmission line, 3) N-phase distributed parameter transmission line model with lumped losses.

The models created as a PI section circuit are insufficiently adequate for fast transients research. Main faults of N-phase distributed parameter transmission line model are as follows: lumped resistors are eliminated out of distributed parameters sections and it is impossible to take into account initial conditions.

Therefore the mathematical model of three-phase transmission line was created in the form of differential equations in partial derivatives using coordinates of phase voltages and currents using travelling wave method in view of initial conditions [2]. The model was simplified and used for simulation of single-phase line and creation of its macromodel.

For infinitesimal element of transmission line the model was written down using the classic equations of long line:

$$-\frac{\partial u}{\partial x} = L\frac{\partial i}{\partial t} + Ri \,; -\frac{\partial i}{\partial x} = C\frac{\partial u}{\partial t} + Gu \,, \tag{3}$$

where u, i are phase voltage and current of the line respectively; R, G are wire resistance and conductance between wire and ground respectively; L and C are inductance of loop "wire–ground and capacitance of phase wire relatively to ground respectively.

Differential equations of the line (3) were written down in the operator form using the direct Laplace transform with considering of non-zero initial conditions as follows:

$$\begin{aligned} u_s - Z_c i_s &= [u_r - Z_c i_r]e^{-\gamma l} + \int_0^l [CZ_c u(0,x) - Li(0,x)]e^{-\gamma x}dx \\ u_r + Z_c i_r &= [u_s + Z_c i_s]e^{-\gamma l} + \\ &+ \int_0^l [CZ_c u(0,x) + Li(0,x)]e^{-\gamma(l-x)}dx \end{aligned} \tag{4}$$

where $u_s = u_s(p,0)$, $i_s = i_s(p,0)$, $u_r = u_r(p,l)$, $i_r = i_r(p,l)$ are operator voltage and current at sending and receiving end of the line respectively, $u(0,x), i(0,x)$ - are voltage and current in some point of the line in the moment $t = 0$ respectively, L, C, Zc, γ are inductance, capacitance, characteristic impedance and propagation constant of electromagnetic wave respectively, l – is a length of line. If there is no distortion line, then $\alpha = R/L = G/C$ (damping coefficient of the line), hence $\gamma = \lambda(p+\alpha)$, where $\lambda = \sqrt{LC}$.

According to physical assumptionss it may be assumed that charge of line is distributed uniformly, before line is being switched on, so $u(0,x)=u(0,0)=u(0)=const$, $i(0,x)=i(0)=0$.

As a result, we have symbolical equations for voltages and currents for sending and receiving end of the line:

$$\begin{aligned} u_s - Z_c i_s &= (u_r - Z_c i_r)e^{-\gamma l} + (CZ_c u(0) - Li(0))(1 - e^{-\gamma l})/\gamma \\ u_s + Z_c i_s &= (u_r + Z_c i_r)e^{-\gamma l} + (CZ_c u(0) + Li(0))(1 - e^{-\gamma l})/\gamma \end{aligned} \tag{5}$$

Using inverse Laplace transform we can find inverse transforms based on the shift theorem.

As a result, we obtained equations for voltages and currents for sending and receiving end of line in time domain:

$$\begin{aligned} u_s(t) - Z_c i_s(t) &= (u_r(t-\tau) - Z_c i_r(t-\tau))e^{-\alpha\tau} + \\ &+ U(0)(1 - 1(t-\tau))e^{-\alpha t} \end{aligned} \tag{6}$$

$$\begin{aligned} u_r(t) + Z_c i_r(t) &= (u_s(t-\tau) + Z_c i_s(t-\tau))e^{-\alpha\tau} + \\ &+ U(0)(1 - 1(t-\tau))e^{-\alpha t} \end{aligned} \tag{7}$$

where τ is a propagation time of electromagnetic wave along line; $1(t-\tau)$ is the Heaviside function.

The line was modeled as a part of the following electric circuit:

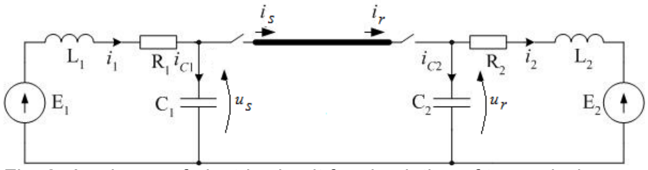

Fig. 2. A scheme of electric circuit for simulation of transmission line transients

Obtained system of transmission line equations (6,7) was supplemented by equations of electric circuit shown in the Fig. 2. It was implemented into MATLAB/Simulink in the form of structural blocks of mathematical operations describing the processes using finite and integral equations as a separate mathematical model:

$$u_s = C_1^{-1}\int_0^t (i_1 - i_s)dt + u_s(0) \tag{8}$$

$$u_r = C_2^{-1}\int_0^t (i_r - i_2)dt + u_r(0) \tag{9}$$

$$i_1 = L_1^{-1}\int_0^t (E_1 - R_1 i_1 - u_s)dt + i_1(0) \tag{10}$$

$$i_2 = L_2^{-1}\int_0^t (u_r - R_2 i_2 - E_2)dt + i_2(0) \tag{11}$$

$$\begin{aligned} i_s(t) &= Z_c^{-1}(u_s(t) - [u_r(t-\tau) - Z_c i_r(t-\tau)]e^{-\alpha\tau} - \\ &- u(0)[1 - 1(t-\tau)]e^{-\alpha t}) \end{aligned} \tag{12}$$

$$\begin{aligned} i_r(t) &= Z_c^{-1}(-u_r(t) + [u_s(t-\tau) + Z_c i_s(t-\tau)]e^{-\alpha\tau} + \\ &+ u(0)[1 - 1(t-\tau)]e^{-\alpha t}) \end{aligned} \tag{13}$$

Based on this accurate enough structural model (8-13), the original data were obtained and used for construction of the line macromodel with the following parameters: $E1=Unom=220$ kV, $l=100$ km, $L0=951,7$mkH/km, $C0=11.9$ nF/km, $Zc=282.8$ Ohm, $E2=0$. For this purpose switching-on processes with zero initial conditions in no-load, short circuit and loaded regimes have been analyzed.

Mathematical macromodel of electric transmission line

Based on transient characteristics obtained during energization of the line in no-load and loaded regimes ($R_2=Zc$, $R_2=3*Zc$, $R_2=Zc/3$) the mathematical macromodel in the form (1) was under development based on several sets of transient characteristics (low, normal and high input voltage). Voltage u_1 and current i_2 were input variables (vector $\mathbf{v}$), and current i_1 and voltage u_2 were output variables (vector $\mathbf{y}$).

The practical construction of the macromodel of the line was conducted using optimization approach [3,4]. The main problem during long line modeling is a principal impossibility of its mathematical description based on simple electric circuit with lumped parameters. Due to this fact in order to simulate the line the following form of mathematical description was select based on the expert analysis:

$$(14) \qquad \begin{cases} \mathbf{x}^{(k+1)} = \mathbf{F}\cdot\mathbf{x}^{(k-m)} + \mathbf{G}\cdot\mathbf{v}^{(k)} + \mathbf{\Phi}(\mathbf{x}^{(k-m)}, \mathbf{v}^{(k)}) \\ \mathbf{y}^{(k+1)} = \mathbf{C}\cdot\mathbf{x}^{(k-m)} + \mathbf{D}\cdot\mathbf{v}^{(k+1)} \end{cases}$$

It made it possible to select randomly a set of coefficients of the macromodel and use the original, actually available data, without going into a detailed analysis of the physical processes. As a result, the line macromodel different from the classical form of the macromodel in the form of state variables due to the presence of $\mathbf{x}^{(-35)}$ variable was obtained:

$$(15) \qquad \begin{cases} \mathbf{x}^{(k+1)} = \begin{pmatrix} 0 & 0.99 \\ -0.99 & 0 \end{pmatrix}\mathbf{x}^{(k-35)} + \begin{pmatrix} 0.001 & 0 \\ 0 & 0.2832 \end{pmatrix}\mathbf{v}^{(k)} \\ \mathbf{y}^{(k+1)} = \begin{pmatrix} 0 & 6.999 \\ 1977 & 0 \end{pmatrix}\mathbf{x}^{(k-35)} + \begin{pmatrix} 0.003535 & 0 \\ 0 & -282.5 \end{pmatrix}\mathbf{v}^{(k+1)} \end{cases}$$

Verification of the results was carried out by comparing of the results obtained during modeling of transients using detailed mathematical model and macromodel of the line.

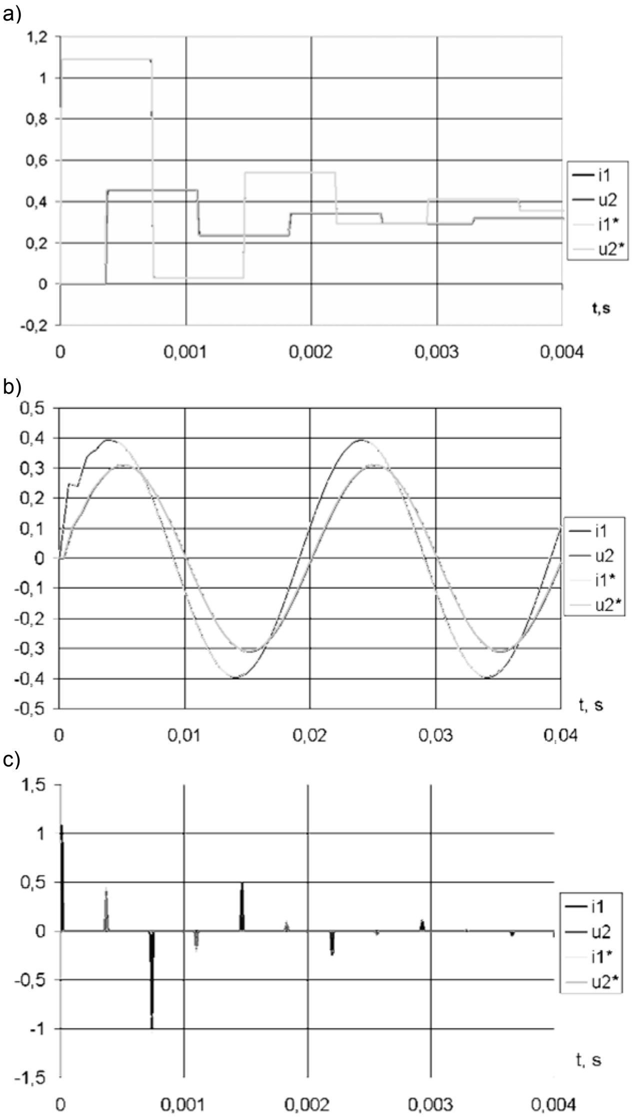

Fig. 3. A reaction of the macromodel of the transmission line on test signals

In the Fig. 3 a reaction of the detailed model and macromodel on the test signal (here just output variables are shown) is presented: reaction of the model on applied DC voltage (a), on sine wave (b) and on jump impulse with duration equal to one discrete time (c). In this figures transient characteristics are presented in p. u. values: currents in kA, and voltages in MV.

Table 1 shows the accuracy of the simulation of different modes of the transmission line operation using developed macromodel. Based on the obtained results we can conclude that accuracy is satisfactory for engineering calculations.

Table 1. Comparative characteristics of the macromodel accuracy at simulation of different operation modes

Type of the input signal applied to the line (E1)	Load	Tolerance
Pulse signal	3*Zc*	6,3%
DC voltage	*Zc*	0,4%
DC voltage	3*Zc*	0,3%
DC voltage	100 kOhm (no-load regime)	10,4%
Sine wave	*Zc*	0,4%
Sine wave	*Zc*/3	1,3%
Sine wave	3*Zc*	0,1%
Sine wave	100 kOhm (no-load regime)	0,9%

Conclusions

The discrete macromodel of transmission line in the form of state variables was developed. The verification of obtained results was conducted based on comparing of simulation results obtained due to verification of detailed mathematical model and macromodel of the same line. Conclusions about the adequacy of derived models are presented.

Due to a minor change in the form of the macromodel the process of its creation was successful and obtained model of the line is enough adequate and simple at the same time.

The expedience of constructing of macromodels for transmission line transient analysis is undeniable because it allows reducing the computer time during the further use of macromodels as separate components intended for the analysis of electric power systems transients.

REFERENCES

[1] R. Kalman, P. Falb, and M. Arbib, “Topics in Mathematical System Theory”, McGraw Hill, New. York, 1969.
[2] Stakhiv P., Hoholyuk O. Mathematical Modeling of transmission Line Transients in MATLAB/Simulink Environment// Acta Technica CSAV 49, 2004, pp. 89-105.
[3] P. Stachiw, J. Kozak, B. Melnyk Zastosowanie równoległych algorytmów w syntezie makromodelów wielobiegunników dynamicznych Pracy XXVI Międzynarodowa Konferencja z Podstaw Elektrotechniki i Teorii Obwodów IC-SPETO 2003, Gliwice-Niezica, Polska, 28-31.05.2003, Tom II, str. 263-266.
[4] Yu. Kozak, B. Melnyk, P. Stakhiv “Simplification of Optimization Process during Mathematical Models Creation”, Przegląd Elektrotechniczny, Nr. 12/2008, 281-283.

***Authors**: Prof. Petro Stakhiv, Institute of Computer Sciences, Technical University of Łódz, ul. Wolczanska 215, Łódz, 90-924, Poland, spg@ics.p.lodz.pl*
Assoc. Prof. Yuriy Kozak. Assoc. Prof. Oksana Hoholyuk - Department of Theoretical Electrical Engineering, Institute of Electric Power and Control Systems, Lviv Polytechnic National University, 12 S. Bandera str, Lviv, 79013, Ukraine, ykozak@mail.ru, oph@polynet.lviv.ua