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FDM based flux distribution analysis method for electrical machines

***Abstract**. The aim of this study is to compare the flux distributions obtained using the experimental method and calculated by FDM. For this purpose, first the design and manufacture of the test transformer which is to be used in the experimental study as well as the measuring coil which is to be used in measuring the prepared fluxes, and then the calculated flux values. Both proposed assessment methods can be used for in-service inspection of the structural integrity of ferromagnetic structures.*

***Streszczenie.** W artykule porównano rozkład strumienia otrzymany drogą eksperymentalną i poprzez obliczenia FDM. Zaprojektowano wykonano testowy transformator w dołączonymi cewkami pomiarowymi. (**Porównanie wyników analizy rozkładu strumienia magnetycznego w maszynach elektrycznych wykonanej eksperymentalnie i obliczeniowo**)*

Keywords: FDM, leakage flux, core-type voltage transformers.
Słowa kluczowe: strumień magnetyczny, metoda różniczki skończonej, transformator

Introduction

Estimation and analysis of leakage flux in electrical machines is a pretty serious issue. Today, several different methods are used for the field measurements. The primary examples of them are the finite difference method (FDM) and the finite element method (FEM). The FDM is a method used in drawing partially differentiable equations. These methods made significant contributions to the solution of problems in transformers [1] – [7].

In this study, the flux distributions obtained using FDM method are compared with the flux distributions obtained from the experimental measurements. For this purpose, a core-type voltage transformer and measuring coils were manufactured in order to obtain the test results. The matrix solution was developed to calculate the flux distributions using the numerical method. A polynomial mathematical equation obtained using Excel was used to calculate the flux values. Studies performed for these purposes are presented below. This paper is realized to develop another solution method to leakage flux calculation that is difficult and time consuming problems. Results obtained demonstrate sufficient accuracy prediction models were proposed.

Theoretical Studies

In the design of appliances, magnetic features of the relevant applicant are taken into consideration. In this stage, field analysis performed involving the designed appliances gives us information such as size appropriate for the conditions while also enabling us to get the useful and essential information to determine the features and behavior of the appliances.

Problems related to appliances differ due to the diversity of application fields. For instance, problems related to bus bars and cutters (such as surge protectors) differ from one another. The mathematical formulation of a studied physical phenomenon and the numerical solution of the formed model are required. It is also beneficial to consider simulations instead of experimental studies, in terms of time consuming. However, numerical studies require field computation in complex structures and accordingly understanding of scientific basics (e.g., the numerical solution) for that purpose.

The definition of a problem can be expressed by utilizing Maxwell equations. Generally, solutions to equations addressing the problems are difficult and time-consuming (except for exceptionally ordinary situations). Sometimes they cannot even be solved. Carrying out a study using numerical methods thus becomes inevitable in such situations.

In this study, the FDM was used to get to the solution of the flux distribution of the mono phase core-type voltage transformer. The experimental results were used as a basis to determine the success of using calculation method.

Some researchers have introduced first order algorithms for both the finite difference method and the finite element method. The result of the solution arrived at using any of these algorithms is a linear equation system which can be solved using various methods. The triangulated network block has a field that can be used more flexibly than a quadratic network division. It is possible to model a large-sized geometric field with triangular network divisions. A system organized orthogonally is enough to model a machine profile.

Finite Difference Method

The FDM is based on the use of finite difference equivalents instead of functions in differential equations. Easy application of this form with basic information, its similarity to the resistance simulation method, and its easy programmability in computers has enabled the method to spread and be understood more easily. The solution of the equation system is formed by written points in the domain where the field distribution will occur. FDM gives the potential values at these points. Equipotent lines can be drawn by using these values, thereby obtaining the distribution. FDM uses networks formed by homogeneous sections like hexagons. A solution of a problem using FDM consists of 4 stages: dividing the solution domain into finite sub-domains or sub-elements, writing the basic equations belonging to an element, assembling (combining) all elements in the solution domain, solving the obtained equation [8], [9]. Figure 1 shows the examples of the division of the solution domain and one-dimensional and two-dimensional finite elements.

The coordinates related to the three corner points of triangular elements must be known in order to determine the coefficients of this polynomial. The appearance of a triangular element called (e) in the coordinate system is shown in Figure 2. (The alignment of the nodes numbered (1, 2, and 3 is counter clockwise.)

Solution acquired through the finite difference method utilize finite difference operations where operators like a Δ forward finite difference, ∇ backward finite difference and δ central finite difference are used. For an f(x) function, the forward difference of first degree is as follows:

(1) $$\Delta f_i = f_{i+1} - f_i$$

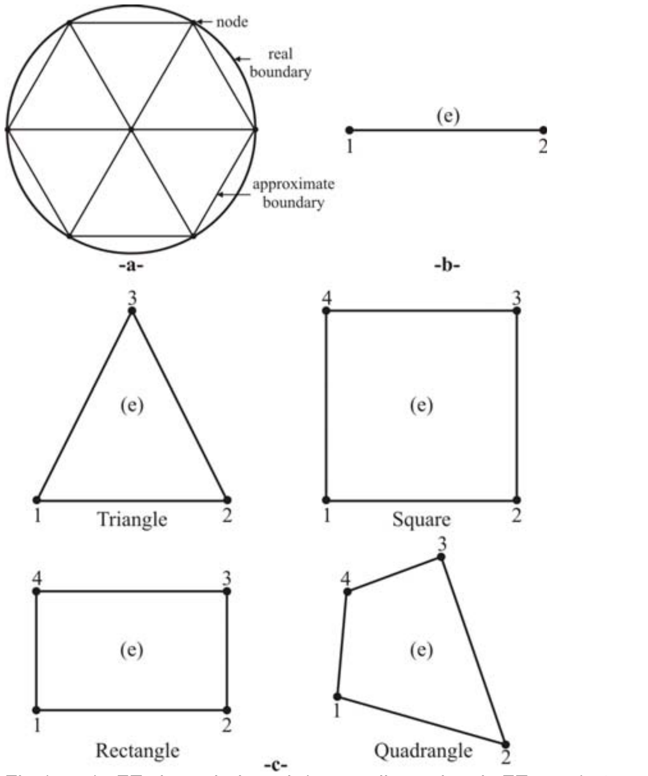


Fig.1. a-) FE boundaries, b-) one-dimensional FEs, c-) two-dimensional FEs.

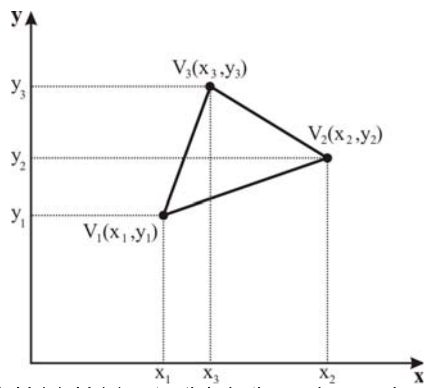


Fig.2. V_1(e), V_2(e), V_3(e) potentials in the nodes numbered 1, 2, and 3 of a triangular element called (e).

In the equation above, if "h" represents the distance (or difference range) instead of the script, we get (2).

$$\begin{aligned} f_i &= f(x) \\ f_{i+1} &= f(x+h) \\ f_{i-1} &= f(x-h) \\ f_{i+1/2} &= f(x+h/2) \\ f_{i-1/2} &= f(x-h/2) \end{aligned} \tag{2}$$

The FDM holds a prominent role in solution field problems, and especially in designing the machines. The fundamental equation related to the design of the machines is Poisson's equation. These observations can be made with Poisson's equation in accordance with the condition of the problem. Laplace's equation is an uncommon condition of Poisson's equation in which the current density is zero. In a domain covered by a network formed by $h \times h$ sized sections, the two-dimensional Laplace's equation (1) can be expressed in Cartesian coordinates if the values of the nodal points are V.

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \tag{3}$$

The central finite difference approximate values of the functions of first degree for any (x, y) point in this domain is (4). If (4) is used instead of (3), the finite difference form of Laplace's equation for the potential at a point (node) becomes (5). It is (6) in subscript [10], [11].

$$\begin{aligned} \frac{\partial^2 V}{\partial x^2} &\cong \frac{V(x+h,y) - 2V(x,y) + V(x-h,y)}{h^2} \\ \frac{\partial^2 V}{\partial y^2} &\cong \frac{V(x,y+h) - 2V(x,y) + V(x,y-h)}{h^2} \end{aligned} \tag{4}$$

$$\begin{aligned} &V(x+h,y) + V(x-h,y) + V(x,y+h) + V(x,y-h) - 4V(x,y) = 0 \\ &V(x,y) = \frac{1}{4}\left[V(x+h,y) + V(x-h,y) + V(x,y+h) + V(x,y-h)\right] \end{aligned} \tag{5}$$

$$V(i,j) = \frac{1}{4}\left[V_{i+1,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1}\right] \tag{6}$$

Drawing the Flux Lines

When equipotent lines are drawn, instead of calculating the potential value of a point on the elements in the solution domain, the coordinates of the points whose lines will be drawn at the edge of the elements and which are equal to the potential value determined beforehand are looked up. The reason that the potential values of the flux lines must be determined in advance with either of two different methods. These methods are as follows:

In the drawing where the total number of lines is taken as the basis, the difference between the lowest value and the highest value in the solution domain is divided by the determined number of lines and gives us the difference between the flux lines. This difference is defined as the amount of increase between the lines. In this method, other lines are drawn with increases (or decreases) equivalent to the difference starting from the lowest or highest value line. When the drawing process is finished, the obtained number of lines becomes equal to the number of lines determined beforehand.

Taking the potential difference (increase or decrease) between the flux lines as a basis: If the differences between the flux lines, that is to say the increases (or decreases) are requested to be a certain value, drawing starts from the lowest or highest value. The value of the next flux line is different from the previous flux line at a value equal to the amount of increase (or decrease). In this method, the total number of lines is equal to the amount of increase (or decrease) in the difference between the highest value and the lowest value.

The potential value of the line, which will be drawn, is compared with the potential value of the two adjacent nodes of each element. If the value to be drawn is equal to the value of either of the two adjacent nodes, or is between these two values, there is certainly a point on the edge which combines these two adjacent nodes and which is equivalent to the potential value to be drawn. After this comparison, the coordinate of the equipotent point can be calculated by utilizing the coordinates of the adjacent nodes. An equipotent line crossing over a triangular element is shown in Figure 3.

If the A_K value in Figure 3 is between A_1 and A_2, the coordinates of $A_K(X_{K1}, Y_{K1})$ whose value are equal to A_K become (7) considering that the potential value on the A_1 - A_2 edge changes linearly.

$$\begin{aligned} x_{K1} &= \frac{(A_1 - A_2)(x_1 - x_2)(A_1 - A_2)x_1}{A_2 - A_1} \\ y_{K1} &= \frac{(A_1 - A_2)(y_1 - y_2)(A_1 - A_2)y_1}{A_2 - A_1} \end{aligned} \tag{7}$$

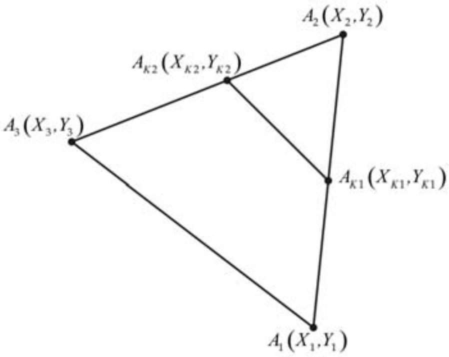


Fig.3. An equipotent line crossing over a triangular element (magnetic field line).

If the potential value of a point on the edge of an element is equal to the potential value of the flux line to be drawn, there is certainly a point whose potential value is equal to the potential value of the flux line to be drawn on one of the other edges of this element. The line which combines the $A_{K2}(X_{K2},Y_{K2})$ point and the $A_{K1}(X_{K1},Y_{K1})$ point obtained with the same method are the equipotent line. A potential line is obtained from the combination of equipotent lines which are obtained by repeating this operation for all elements (provided that the elements are small enough).

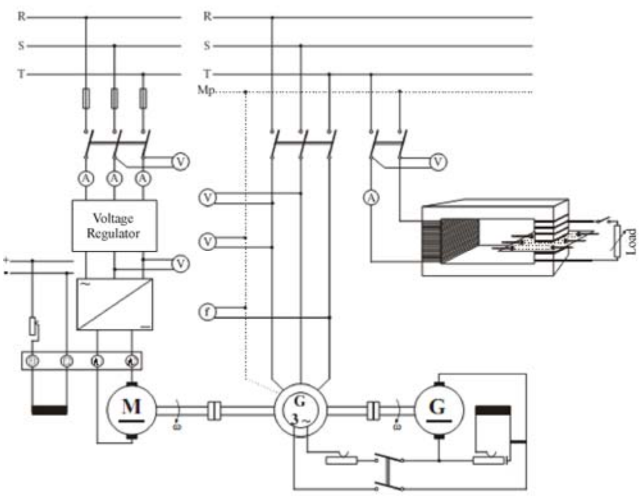


Fig.4. Experimental setup connection diagrams.

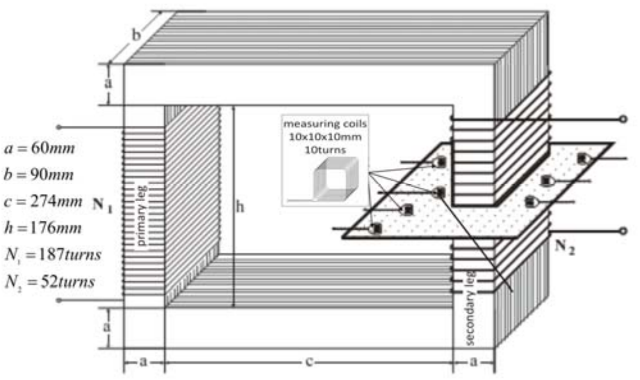


Fig.5. Appearance of the transformer and positioned perspective appearance of the measuring coils.

Experimental Studies

In experimental studies, Constant voltage (220V) for on the primary coil provided by a regulator and a synchronous generator also supplied the necessary energy. Digital measurement gadgets have been used in the measurements, and measurements have been performed on the secondary leg. The measurements (measurements of the potential values) have been performed at eight different heights (55 mm - 125 mm) starting from the inner corner of the leg, and at two different currents (2.1A – 4.1A). Figure 4 and Figure 5 show the connection diagram prepared for the experimental studies and perspective view of the transformer with dimensions of measuring coils respectively.

Determining the Flux Distribution of the Transformer with FDM

Superficial flux distribution in the secondary leg of the experiment voltage transformer has been drawn for eight different heights using a computer program utilizing FDM. For the drawing process, first the surface on which the flux distribution was divided into 3232 elements with this program as shown in Figure 6. Then equipotent points were calculated and superficial flux distributions obtained as shown in Figure 7 (includes samples for 4 heights).

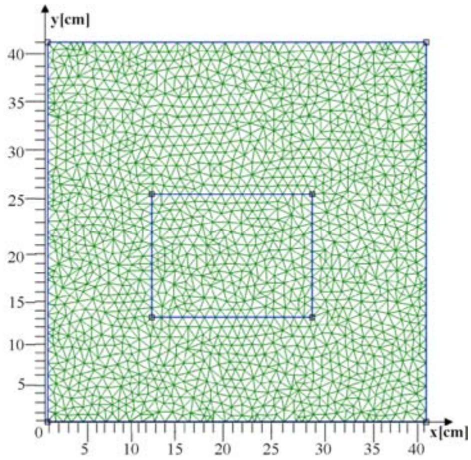
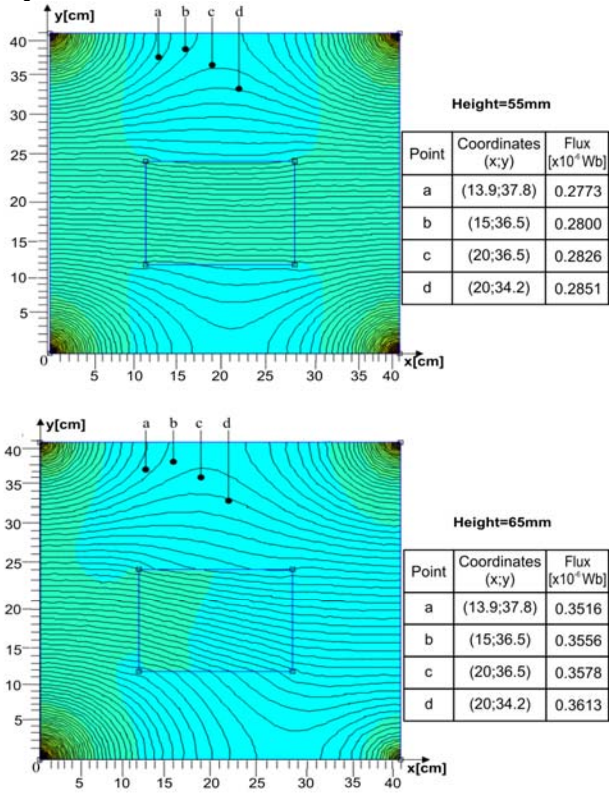
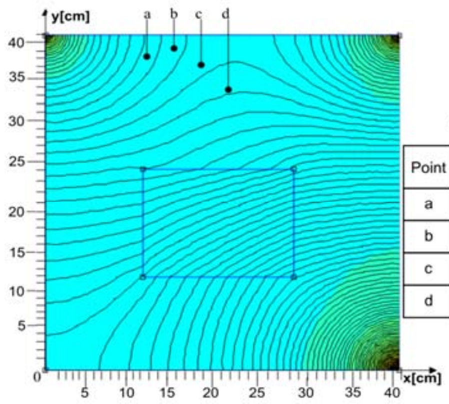


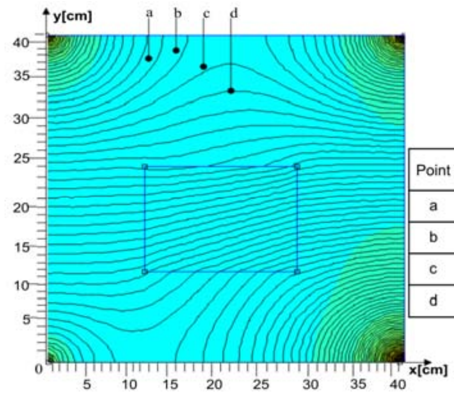
Fig.6. 3232 finite elements for flux distribution measurement.





Height=115mm

Point	Coordinates (x;y)	Flux [x10^{-6}Wb]
a	(13.9;37.8)	0.2323
b	(15;36.5)	0.2343
c	(20;36.5)	0.2370
d	(20;34.2)	0.2388



Height=125mm

Point	Coordinates (x;y)	Flux [x10^{-6}Wb]
a	(13.9;37.8)	0.2141
b	(15;36.5)	0.2158
c	(20;36.5)	0.2186
d	(20;34.2)	0.2202

Fig.7. Flux distributions at different heights (55mm, 65mm, 115mm, 125mm) on the leg obtained using a computer program in accordance with the measurement results.

The boundary values must be known to calculate the voltages of each frame in Figure 5–b by FDM and to compare with measurement results. Experimental measurements provided these values. Accordingly, finite difference equations for internal nodes in the region can be arranged as:

$$
\begin{aligned}
&V_1 = \frac{1}{4}\left(V_{1523} + V_{40} + V_{1681} + V_2\right) \\
&V_2 = \frac{1}{4}\left(V_{1524} + V_{41} + V_1 + V_3\right) \\
&\ldots \\
&V_{1483} = \frac{1}{4}\left(V_{1641} + V_{1444} + V_{1643} + V_{1484}\right)
\end{aligned}
\tag{8}
$$

If the known potentials in the boundaries and the unknown potentials in the inner points are separated from each side, we get (9)

$$
\begin{aligned}
&4V_1 - V_2 - V_{40} = V_{1523} + V_{1681} \\
&4V_2 - V_3 - V_{41} - V_1 = V_{1524} \\
&\ldots \\
&4V_{1483} - V_{1444} - V_{1484} = V_{1643} + V_{1641}
\end{aligned}
\tag{9}
$$

By solving this equation system, all potential values at the points which are on the surface at a height of 55 mm from the leg can be calculated. This study used Microsoft Excel, which does not use operations such as inverting a matrix. As is known, by using the iteration feature of this program, calculations can be performed faster and easier with FDM.

Drawing and Comparing the Flux Distributions

Drawing the flux distribution is the best method for emphasizing the measurement and calculation results, and comparing them with each other. Due to the fact that the difference between the lowest value and highest value in the solution gives the total amount of flux line and, lastly, the difference between the flux lines found by dividing this difference by the determined number of lines. Other equipotent points have been determined by increasing the value with an amount equal to the difference starting from the lowest value. The coordinates of all determined equipotent points at 8 different heights were found and the drawing of the flux distribution performed by combining the points at the same values. The measurement values and flux distributions drawn according to the $2.14 \cdot 10^{-7} Wb$ flux value, which is equal to a value of $0.4746 mV$ using FDM are given in Figure 8. It suggests that at a height of 55mm (from the leg), $0.4746 mV$ (or $2.14 \cdot 10^{-7} Wb$ flux value) is observed at a horizontal distance of 83mm according to the measurement results, at a horizontal distance of 77 mm according to FDM. At a height of 95 mm, the same potential value is observed at a horizontal distance of 92mm.

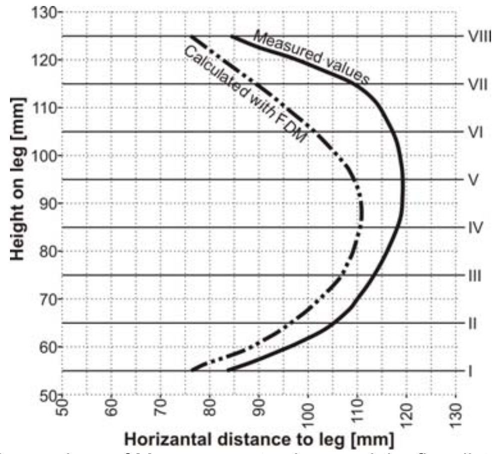


Fig.8. Comparison of Measurement values and the flux distributions drawn according to the $2.14 \cdot 10^{-7} Wb$ flux value which is equal to a value of $0.4746 mV$ using FDM.

Conclusion

In view of the difficulty and sometimes the impossibility of the experimental study that uses in determining the flux distribution, time consuming problems, and possible mistakes, it is wiser to get the results using numerical methods. For instance, it is possible to determine the flux distributions easily with FDM, which is used in the calculation of the measurement results taken from several points with field probes in order to obtain the flux distribution in large and powerful voltage transformers in which measurement processes are difficult to implement.

This study compares the results of experiments and obtained from numerical methods. It shows that if a good imitation and definition is in place, numerical methods are superior in terms of accuracy for detection of flux distributions.

As is seen in Figure 8, FDM offer close results (accuracy rate is approximately 80%) for a network that contains an equal number of unknowns. Moreover, the numerical method provides adequate close results in terms of the flux distributions compared to experimental measurements. In the study, the results obtained using triangular elements and the approximation function of first degree has been found to be adequate, and the degree of the polynomial has not been increased accordingly. The accuracy rate can be increased by increasing the polynomial used in FDM.

Thanks to the multifaceted and flexible structure of this method, environment and field relations in complex structures as well as cause-effect relationships in different problems can be constantly calculated efficiently. Elements with irregular shapes can be easily modeled with this method. The results obtained are show that both numerical methods can be applied effectively and easily in determining of leakage flux of voltage transformers. These methods can also be used for different transformer model future investigations.

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