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SURE-Based Projections Onto Convex Sets for Image Restoration

***Abstract**. Projections onto convex sets (POCS) algorithms have been widely used for image restoration problem. However, the relaxation parameter (λ) of POCS is strongly data-dependent and difficult to tune. In this work we focus on optimally selecting such parameter in POCS algorithm for image restoration. A stein's unbiased risk estimate (SURE) based POCS (SPOCS) for image restoration algorithm is proposed, in which SURE is used to determine an optimal λ value. Finally, the effectiveness of the optimality of the proposed parameter selection is tested by image restoration experiments.*

***Streszczenie.** W artykule przedstawiono metodę optymalnego doboru parametru relaksacji dla algorytmu POCS, służącego do odtwarzania obrazów. W proponowanym rozwiązaniu (SPOCS) zastosowano estymator Stein'a (SURE), służący do wyznaczenia optymalnej wartości współczynnika lambda. Działania algorytmu zostało zbadane eksperymentalnie. (**Zastosowanie estymatora SURE w metodzie POCS do odtwarzania obrazów**).*

Keywords: Image restoration, Projections onto convex sets, stein's unbiased risk estimate.
Słowa kluczowe: odtwarzanie obrazu, POCS, estimator Stein'a SURE.

Introduction

During acquisition or transmission, the obtained images often suffer from a blurring effect and noise due to defocusing, atmospheric distrubance, relative motion, etc. If we assume that the blur is global and translation invariant, then the observation process can be modeled as $y(m,n)=h(m,n)\otimes x(m,n)+\eta(m,n)$, where $h(m,n)$ is the blurring kernel, $\eta(m,n)$ is the additive noise, and $\otimes$ denotes two dimensional convolution.

These sources of image degradation have a direct bearing on the visual quality of image. Undoing these imperfections to remove the image degradation is crucial for many subsequent image-processing tasks, such as feature extraction, object detection, and pattern classification. Image restoration aims to restore the true image $x(m,n)$ from its degraded and noisy observation $y(m,n)$. Image restoration is a typical ill-posed inverse problem. Regularization terms that incorporate prior information about the true image are required for reasonable solution. Over the past decade, numerous regularization methods have been proposed, such as Tikhonov regularization [1], total variation regularization [2], wavelet regularization [3], and sparsity regularization [4] etc.

On the other hand, the POCS methods have been widely used for image restoration problem [5-7]. It is known that a main problem in image restoration is how to use more priori information about the objective to improve quality of the restored image. A feasible solution of the restored image is an intersection of sets, if all priori information can be considered and defined as closed and convex sets in signal space. Typical priori information convex sets include sets of images restricted by spatial extend, the known part of the spectrum, band limitedness, the known part of the image, nonnegativity, limited amplitude bound, energy, ect.

Row-Action Projection (RAP) [8] as a variant of POCS was proposed to achieve iteration parallelization, which has been successfully used to image restoration. However, the relaxation parameters of RAP are selected empirically, which leads to an exhaustive search for that one that gives the best restoration result. In [7], Papa proposed to use particle swarm optimization algorithm to find optimal/quasi-optimal relaxation parameter by maximizing or minimizing reference image quality measures. However, this method does not apply because the true image may not be available.

In this paper, we focus on the data-driven optimal relaxation parameter selection in POCS for image restoration using Stein Unbiased Risk Estimator (SURE). The proposed method can estimate an optimal relaxation parameter value λ . Experimental results demonstrate the effectiveness of the proposed method.

POCS-based image restoration

The POCS-based method is an iterative algorithm for find an image $\hat{x}$ in the intersection of L closed convex sets:

$$\hat{x}\in C_0=\bigcap_{i=1}^{L} C_i \tag{1}$$

where the set $C_i\in\Re$ denotes the ith constraints or a priori information on $\hat{x}$ and L is the number of those sets. When all the sets $C_i(i=1,...,L)$ are closed and convex, and their intersection C_0 is non-empty, the vector which belongs to C_0 can be found by successive projection onto the sets via corresponding projecting operator P_{C_i} .

$$\hat{x}^{(k+1)}=P_{C_L}P_{C_{L-1}}...P_{C_1}\hat{x}^{(k)} \tag{2}$$

where P_{Ci} is a projection operator that projects an kth $\hat{x}^{(k)}$ on the closed and convex set $C_i\in\Re$.

RAP can achieve the image consistent with the measurement and minimize the sum-of-squares of the image intensities. In this paper, we choose RAP as projection technique. In that way, the projecting operator P_{C_k} is defined as

$$P_{C_k}\hat{x}^{(k)}(m,n)=\begin{cases}\hat{x}^{(k)}(m,n)+\lambda\dfrac{\xi(i,j)}{\|h(i,j)\|^2} & \text{if } \hat{x}^{(k)}(m,n)\in S_{h(i,j)}\\ \hat{x}^{(k)}(m,n) & \text{otherwise}\end{cases} \tag{3}$$

where $\xi(i,j)=\varepsilon(i,j)\cdot h(i-m,j-n)$, λ is the relaxation parameter, $S_{h(i,j)}$ denotes the support of the blurring kernel centered at pixel $y(i,j)$, and

(4) $$\varepsilon(i,j)=y(i,j)-\sum_{m,n\in S_{h(i,j)}} h(i-m,j-n)\hat{x}^{(k)}(m,n)$$

(5) $$\|h(i,j)\|^2=\sum_{m,n\in S_{h(i,j)}} h(i-m,j-n)^2$$

From Eq. (3), it can be seen that the relaxation parameter λ can be used to accelerate the rate of convergence of the algorithm. Although relaxation pararmeter of high value can turn the restoration task faster, the control of artifacts in the image becomes more difficult. On the other hands, low values result in poorly restored images. POCS technique requires appropriate selection of the relaxation parameter that controls the quality of the restored result.

SURE-based POCS (SPOCS) algorithm

The optimal λ should be data-driven and minimize the mean squared error (MSE) or corresponding risk. Fortunately, Stein has stated that the MSE can be estimated unbiasedly from the observed data [9]. In this letter, a SPOCS algorithm is presented to find the optimal λ that result in optimal restored images. Comparisons with other methods show that the proposed SPOCS achieves better performance.

The MSE $\mathrm{MSE}(\hat{x})=N^{-1}\|x-\hat{x}\|_2^2$ is commonly used to determine quality of restored image. However, $\mathrm{MSE}(\hat{x})$ cannot be directly used in practice due to its dependence on the true image x . SURE provides a means for unbiased estimation of the true MSE. It is specified by the following analytical expression [10]:

(6) $$\text{SURE}=N^{-1}\|y-\hat{x}\|^2-\sigma^2+2\sigma^2N^{-1}\mathrm{div}_y\{\hat{x}\}$$

where $\mathrm{div}_y\{\hat{x}\}$ is the divergence of the POCS algorithm, σ^2 is noise variance. The divergence term in (6) plays a crucial role in the experssion of SURE. For POCS algorithm, according to Eq. (3), the $\mathrm{div}_y\{\hat{x}\}$ is given by

(7) $$\mathrm{div}_y\{\hat{x}\}=\sum_m\sum_n\frac{\partial P_{C_i}\hat{x}^{(k)}(m,n)}{\partial\hat{x}^{(k)}(m,n)}$$

with

(8) $$\frac{\partial P_{C_i}\hat{x}^{(k)}(m,n)}{\partial\hat{x}^{(k)}(m,n)}=\begin{cases}1-\lambda\dfrac{\gamma(i-m,j-n)}{\|h(i,j)\|^2} & \text{if } \hat{x}^{(k)}(m,n)\in S_{h(i,j)}\\ 1 & \text{otherwise}\end{cases}$$

where $\gamma(i-m,j-n)=h(i-m,j-n)\cdot\sum_{m,n\in S_{h(i,j)}} h(i-m,j-n)$.

Now that we hold a complete expression of SURE for unbiased MSE estimation, the optimal λ can be chosen by minimizing it. Accordingly,

(9) $$\lambda^{opt}=\arg\min_{\lambda}\left\{\frac{\|y-\hat{x}\|^2+2\sigma^2\sum_m\sum_n\dfrac{\partial P_{C_i}\hat{x}^{(k)}(m,n)}{\partial\hat{x}^{(k)}(m,n)}}{N}-\sigma^2\right\}$$

Before calculating the λ^{opt} with Eq. (9), the noise variance σ^2 must be estimated. a good estimator for σ^2 is the median of absolute deviation using the highest level wavelet coefficients [11].

(10) $$\sigma=\frac{\mathrm{median}(|w|)}{0.6745},\ w\in HH_1$$

where $\mathrm{median}(\cdot)$ denotes the median operation, HH_1 is the highest frequency subband of wavelet coefficients.

Experimental results

In this section, we state the experiments performed in order to validate the proposed SPOCS for image restoration. The proposed method was tested on several natural images and compared with the Wiener Filter, Richardson-Lucy (R-L) image restoration algorithm and POCS method with a fixed parameter λ . For POCS based methods and R-L image restoration algorithm execution we used five iterations. We experimented on the standard 512×512 Lena, Boat, Peppers, and Barbara images with blur kernel with size 15×15 (PSF1), 9×9 Gaussian blur with standard deviation 2 (PSF2). In the experiments, four measures (ISNR, UIQI, MAE, MSE) are used to evaluate the performance of the methods. Given f , g and $\hat{f}$ be the original, degraded and restored images, respectively. The ISNR index is defined as

(11) $$\text{ISNR}=10\log_{10}\left\{\frac{\sum_{i,j}[g(i,j)-f(i,j)]^2}{\sum_{i,j}[\hat{f}(i,j)-f(i,j)]^2}\right\}$$

The UIQI measure index can be given by

(12) $$\text{UIQI}=\frac{4\sigma_{f\hat{f}}\,mean(f)mean(\hat{f})}{\left(\sigma_f^2+\sigma_{\hat{f}}^2\right)\left[(mean(f))^2+\left(mean(\hat{f})\right)^2\right]}$$

where $mean(\cdot)$ denotes the average function, σ_f^2 and $\sigma_{\hat{f}}^2$ are the variance of the original and restored images, respectively, and $\sigma_{f\hat{f}}$ is the correlation coefficient between the original and restored images. The MAE and MSE measures are defined as

(13) $$\text{MAE}=\frac{\sum_{i=1}^M\sum_{j=1}^N\left|f(i,j)-\hat{f}(i,j)\right|}{MN}$$

(14) $$\text{MSE}=\frac{\sum_{i=1}^M\sum_{j=1}^N\left[f(i,j)-\hat{f}(i,j)\right]^2}{MN}$$

where M and N are, respectively, the size of image rows and columns.

Tables 1-4 display the numerical results in terms of ISNR UIQI,MSE and MAE. From Tables 1-4, it can be seen that the proposed method consistently gives the best value of ISNR UIQI,MSE and MAE compared to the other methods.

Table 1. Restoration results for Lena image degraded with PSF1 and PSF2

Exper.	Methods	ISNR	UIQI	MSE	MAE
PSF1	Wiener	-1.0831	0.9363	210.98	10.863
	R-L	0.1045	0.9488	160.50	8.127
	POCS	0.0334	0.9447	163.15	8.702
	SPOCS	0.0212	0.975	150.45	7.653
PSF2	Wiener	-1.6831	0.9161	250.93	16.953
	R-L	0.214	0.9318	176.51	9.227
	POCS	0.0935	0.9427	168.55	8.912
	SPOCS	0.0422	0.9656	157.33	7.923

Table 2. Restoration results for Boats image degraded with PSF1 and PSF2

Exper.	Methods	ISNR	UIQI	MSE	MAE
PSF1	Wiener	-1.205	0.9459	184.230	10.176
	R-L	0.1963	0.9580	133.421	7.319
	POCS	0.0856	0.9529	139.561	7.988
	SPOCS	0.0650	0.9729	124.650	6.871
PSF2	Wiener	-1.505	0.9351	189.331	12.173
	R-L	0.2953	0.9461	153.461	9.221
	POCS	0.0976	0.95019	142.571	8.998
	SPOCS	0.0710	0.9659	134.641	7.687

Table 3. Restoration results for Peppers image degraded with PSF1 and PSF2

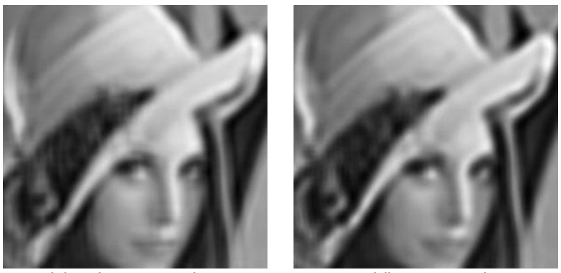
Exper.	Methods	ISNR	UIQI	MSE	MAE
PSF1	Wiener	-0.6445	0.9338	218.148	11.083
	R-L	-0.1908	9.382	196.504	9.804
	POCS	0.0131	0.9369	187.949	9.878
	SPOCS	0.0101	0.9639	170.212	9.654
PSF2	Wiener	-0.7844	0.9213	258.214	16.108
	R-L	-0.4191	9.2181	212.514	12.180
	POCS	0.0513	0.9226	199.994	10.987
	SPOCS	0.0310	0.9433	182.231	9.765

Table 4. Restoration results for Barbara image degraded with PSF1 and PSF2

Exper.	Methods	ISNR	UIQI	MSE	MAE
PSF1	Wiener	-0.7465	0.9446	196.180	10.525
	R-L	-0.2997	0.9450	177.003	9.368
	POCS	-0.0028	0.9446	165.303	9.221
	SPOCS	-0.0013	0.9578	158.561	8.732
PSF2	Wiener	-0.9746	0.9244	219.118	13.125
	R-L	-0.6219	0.9314	187.120	12.113
	POCS	-0.0112	0.9456	176.312	10.232
	SPOCS	-0.0093	0.9523	168.125	9.413

For visual evaluation, two examples using the standard "Lena" and "Barbara" images with PSF1 are given. Figure 1 and figure 2 show restored results on a cropped subregion of Barbara and Lena. Fig.1(a) and Fig.2(a) show an original Lena and Barbara images, respectively. (b) is a blurry image with PSF1. (c) and (d) are restored images obtained by conventional Wiener Filter and R-L method, respectively. (e) and (f) are restored images by POCS and SPOCS. From the figures we can find that the proposed method yields the best restored results and can recover the fine-scale details more efficiently.

(a) Original (b) degraded with PSF1



(c) Wiener result (d) R-L result

(e) POCS result (f) SPOCS result

Fig.1. Experimental results on Lena with PSF1

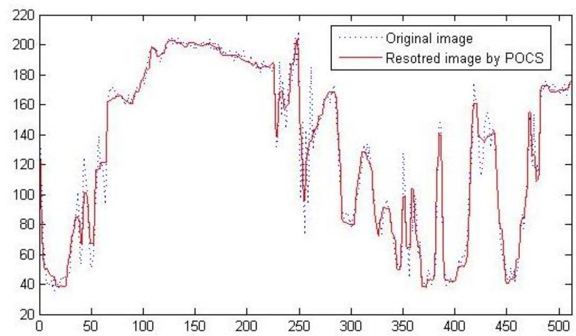
(a) Original (b) degraded with PSF1

(c) Wiener result (d) R-L result

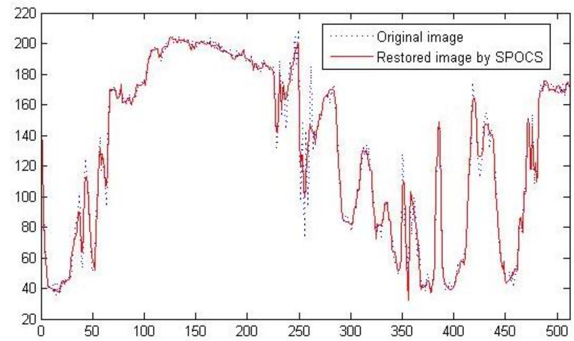
(e) POCS result (f) SPOCS result

Fig.2. Experimental results on Barbara with PSF1

In order to visually see the improvements generated by the proposed method, we have included in figure 3 a cross-section of the 128th line (from column 1 to 512) of the original Lena image along with the POCS restored estimate, and the recovered image by SPOCS.



(a) cross-section of the original Lena along with POCS



(b) cross-section of the original Lena along with SPOCS

Fig.3. Comparison of the corss-section

Conclusions

In this paper, we improve the POCS using the Stein's Unbiased Risk Estimate (SURE). Compared with POCS, the proposed method can determine an optimal relaxation parameter, which controls the projection. Our experimental results indicate that it produces both higher objective evaluation criterions (such as ISNR, UIQI, MAE, and MSE) and better visual quality than Wiener, R-L, and POCS methods.

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