Differential evolution with random scale factor for economic dispatch considering prohibited operating zones

Abstract. This paper addresses a novel technique to solve non-convex economic load dispatch (NCELD) problem. Generator constraints, such as valve point loading, ramp rate limits and prohibited operating zones are taken into account in the problem formulation of NCELD. Few Variants of Differential Evolution (DE) and Differential Evolution with Random Scale Factor (DE-RSF) is applied for solving the above problem. The technique is tested with IEEE standard test systems. It is shown that, the presented technique for solving NCELD problem generates quality solutions reliably.

Keywords: Differential Evolution, economic dispatch, prohibited operating zones, ramp-rate limits, valve-point effect.

Introduction

Economic operation of electric energy generating systems has been given proper attention in the electric power system industry. The objective of economic load dispatch problem (ELD) of electric power generation, whose characteristics are complex and highly nonlinear, is to schedule the committed generating unit output so as to meet the required load demand at minimum operating cost while satisfying all unit and system equality and inequality constraints [1].

In traditional ELD the cost function of each generator is approximately represented by a simple quadratic function and is solved using conventional mathematical programming [2] based on several optimization techniques, such as dynamic programming [3], linear programming [4], homogenous linear programming [5], and nonlinear programming technique [6, 7]. However, real input-output characteristics display higher-order non-linearities and discontinuities. Power plants usually have multiple valves that are used to control the power output of the unit. When steam admission valves in thermal units are first opened, a sudden increase in losses is observed. This leads to ripples in the cost function, which is known as the valve-point loading. The ELD problem with valve-point effects is represented as a non-smooth optimization problem [8].

The traditional algorithms can solve the ELD problem if the incremental fuel-cost curves of the generating units are monotonically increasing piece-wise linear functions. But, a practical ELD must include ramp rate limits, prohibited operating zones, valve-point effects and multi-fuel options.

The resultant ELD is a challenging non-convex optimization problem, which is hard to solve by the traditional methods. Modern heuristic optimization techniques such as Simulated Annealing [9], Evolutionary Algorithms (EAs) [10], Particle Swarm Optimization [11], Neural Networks, and Tabu Search have been given much attention by many researchers due to their ability to find an almost global optimal solution. These methods have drawbacks such as premature convergence and after some generations the population diversity would be greatly reduced. Differential evolution [12] is a stochastic optimization method. The fittest of an offspring competes one by one with that of the corresponding parent, which is different from the other evolutionary algorithms. This competition implies that the parent is replaced by its offspring if the fitness of the offspring is better than that of its parent. On the other hand, the parent is retained in the next generation if the fitness of the offspring is worse than that of its parent. This one by one competition gives rise to a faster convergence rate. However this faster convergence leads to a higher probability of obtaining a local optimum because the diversity of the population seconds faster during the solution process. To overcome this drawback, the mutation operator is made dynamic throughout the run process to maintain the diversity of the population, which guarantees a high probability of obtaining the global optimum.

This paper considers different types of non-convex ELD problem, namely prohibited operating zones, ramp rate limits and valve-point loading effects. The performance of the presented method in terms of solution quality and computational efficiency has been compared with other variants for four IEEE standard test systems including equality, inequality, thermal and dynamic constraints.

Problem formulation

The aim of ELD problem is to make the generators fuel consumption or the operating cost of the whole system minimal by determining the power output. Each generating unit under the constraint condition of the system load demand, power losses as well as some generating power constraints for all units should be satisfied. The objective of the economic dispatch is to minimize the total generation cost of a power system over some appropriate period while satisfying various constraints. The mathematical model of real power economic dispatch with primary constraints can be written as follows,

\[ \text{Min } F = \sum_{i \in \text{NG}} F_i( P_i ) \]

Subject to

\[ \sum_{i \in \text{NG}} P_i - P_{D} - P_{L} = 0 \]

where

\[ F_i = \min \sum_{i \in \text{NG}} F_i( P_i ) = \min \sum_{i \in \text{NG}} ( a_i + b_i P_i + c_i P_i^2 ) \]

where \( F \) is the total fuel cost of all generating units, \( i \) is the index of dispatchable units; \( F_i(P_i) \) is the cost function of the unit \( i \), \( P_i \) is the power output of the unit \( i \), \( NG \) is the set of all dispatchable units and \( a_i, b_i, c_i \) are the fuel cost coefficients of the unit \( i \). The general ELD problem consists in minimizing \( F_i \) subject to following constraints:
Power Balance Constraint:
\[ \sum_{i \in NG} P_i = P_D + P_L \] (4)

Thermal constraints:
The transmission Loss \( P_L \) may be expressed using B-coefficients as,
\[ P_L = \sum_{i \in NG} \sum_{j \in NG} P_{ij} B_{ij} + \sum_{i \in NG} B_{i0} P_i + B_{00} \] (5)
where \( P_{ij} \) is the total load demand; \( P_i \) is the power losses and \( B_{ij} \) is the power loss coefficient.

Generator Capacity Constraints:
The power generated by each unit lies within their lower limit \( P_i^{\text{min}} \) and upper limit \( P_i^{\text{max}} \). So that
\[ P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}} \] (6)

When a steam admission valve starts to open in thermal units, a sudden increase in losses occurs, which produce a rippling effect on the unit's cost function. The generator cost function is obtained from a data point taken "heat run" tests, when input and output data are measured as the function is obtained from a data point taken "heat run" tests. To consider the valve-point effects in the cost model, the rectified sinusoidal function is represented by a more complex formula along with (3), (4) and (5) as,
\[ f_i(x) = a_i x_i^2 + b_i x_i + c_i + \left| e_i \sin(f_i(P_{i}^{\text{min}} - P_i)) \right| \]
where \( e_i \) and \( f_i \) are the fuel cost coefficients of generator 'i', reflecting variable cost effects.

Dynamic Constraints:
The operating range of units is restricted by their ramp rate limits during each dispatch period. The power generated by each of the unit in certain interval may not exceed that of previous interval \( P_{ao} \) by more than a certain amount \( UR_i \), the upper ramp limit and neither less than a certain amount \( DR_i \), the lower ramp rate limit of the generator. Consequently the power output of a practical generator cannot be varied instantaneously beyond the range along with (4), (5), (6) and (7) as it is shown in the following expression:
As generation increases,
\[ P_i - P_{ao} \leq UR_i \] (8)
As generation decreases,
\[ P_{ao} - P_i \leq DR_i \] (9)
and
\[ \max(P_i^{\text{min}}, P_{ao} - DR_i) \leq P_i \leq \min(P_i^{\text{max}}, P_{ao} + UR_i) \] (10)

The prohibited operating zones(POZ) are considered due to the vibrations in the shaft bearing caused by the steam valve or due to the associated auxiliary equipment such as boiler or feed pumps. The range of output power of a generator was defined previously to inhibit as POZ. Mathematically, the feasible operating zones of unit can be described in addition to the constraints as in equations(4), (5), (6), (7) and (10) as follows:
\[ P_{i}^{\text{min}} \leq P_i \leq P_{i}^{\text{max}} \] (11)
\[ P_{i,j-1}^{\text{min}} \leq P_i \leq P_{i,j}^o, j = 2, ..., n_i \]
where \( P_{i,j}^o \) is the lower bound of the prohibited zone 'j' of unit 'i', \( P_{i,j}^o \) is the upper bound of the prohibited zone 'j' of unit 'i', \( n_i \) be the number of prohibited zones in unit 'i'.

Differential Evolution
Differential Evolution (DE) algorithm is a population based algorithm like genetic algorithm using the similar operators; crossover, mutation and selection. The main difference in constructing better solutions is that genetic algorithms rely on crossover while DE relies on mutation operators. This main operation is based on the differences of randomly sampled pairs of solutions in the population.

The algorithm uses mutation operation as a search mechanism and selection operation to direct the search towards the prospective regions in the search space. The DE algorithm also uses a non uniform crossover that can take child vector parameters from one parent more often than it does from other [13]. By using the components of the existing population members to construct trial vectors, the recombination (crossover) operator efficiently shuffles information about successful combinations, enabling the search for a better solution space.

Initialization
Initialization generates initial population \( P_0 \) which contains \( N_p \) individuals \( x_i^j \), \( 1 \leq i \leq N_p \).
\[ X_j^{i}(G) = X_j^{i}(G) + F \left( X_j^{i}(G) - X_j^{i}(G) \right), i = 1, ..., N_p \]
where, \( [X_j^{i}(G)] \) is the search space of the jth optimization parameter, \( d_j \) is a real random number but not necessarily uniform in the range [0, 1].

Mutation
The mutation operator creates mutant vectors by perturbing a randomly selected vector \( x_s \), with the difference of two other randomly selected vectors \( x_a \) and \( x_b \),
\[ X_j^{i}(G) = X_j^{i}(G) + \alpha \left( b_j - a_j \right), i = 1, ..., N_p \]
where \( x_a, x_b \) and \( x_s \) are randomly chosen vectors among the \( N_p \) population, and \( a \neq b \neq c \). \( x_a, x_b, x_s \) and \( x_c \) are selected anew for each parent vector. The scaling constant "F" is a real random number but not necessarily uniform in the range [0, 1]. It is an algorithm control parameter used to adjust the perturbation size in the mutation operator and improve algorithm convergence.

Crossover
The crossover operation generates trial vectors \( x_t \) by mixing the parameters of the mutant vectors \( x_s \) with the target vectors \( x_t \) according to a selected probability distribution,
\[ X_j^{i}(G) = \begin{cases} X_j^{i}(G), & \text{if } \rho_j \leq C_h \text{ or } j=q \\ X_j^{i}(G), & \text{otherwise} \end{cases} \] (14)
where \( i=1, ..., N_p \) and \( j=1, ..., D; q \) is a randomly chosen index \( \in 1, ..., N_p \) that guarantees that the trial vector gets at least one parameter from the mutant vector; \( \rho_j \) is a uniformly distributed random number within [0, 1] generated a new for each value of \( j \). The crossover constant \( C_h \) is an algorithm parameter that controls the diversity of the population and aids the algorithm to escape from local minima. \( x_j^{i}(G) \) and \( x_j^{i}(G) \) are the jth parameter of the ith target vector, mutant vector, and trial vector at generation G, respectively.

Selection
The selection operator forms the population by choosing between the trial vectors and their predecessors (target vectors) those individuals that present a better fitness or are more optimal.
\[ X_j^{i}(G+1) = \begin{cases} X_j^{i}(G), & \text{if } f(X_j^{i}(G)) \leq f(X_j^{i}(G)) \\ X_j^{i}(G), & \text{otherwise} \end{cases} \] (15)
where \( i=1, ..., N_p \).
This optimization process is repeated for several generations, allowing individuals to improve their fitness as they explore the solution space in search of optimal values. DE has three essential control parameters, the scaling factor ($F$), the crossover constant ($C_r$) and the population size ($N_p$). The scaling factor is a value in the range $[0, 1]$ that controls the amount of perturbation in the mutation process. The crossover constant is a value in the range $[0, 1]$ that controls the diversity of the population. The population size determines the number of individuals in the population and provides the algorithm enough diversity to search the solution space.

**Differential Evolution with random scale factor (DE-RSF)**

In the original DE, the difference vector is scaled by a constant scaling factor "$F$". The usual choice for this control parameter is a number between 0.4 and 1. But a dynamic behavior to the scaling factor by varying the scaling factor [15] in a random manner in the range (0.5, 1) is applied by using the relation

$$F = 0.5 \times (1 + \text{rand}(0, 1))$$

Where $\text{rand}(0, 1)$ is a uniformly distributed random number within the range (0, 1). The mean of the scaling factor is 0.75. This allows for stochastic variations in the amplification of the difference vector and thus help retain population diversity as the search process, even when the tips of most of the population vectors point to locations clustered near the local optimum due to the randomly scaled difference vector, a new trial vector has fair chances of pointing at an even better location on the multimodal functional surface. Therefore, the fitness of the best vector in a population is much likely to get stagnant until a truly global optimum is reached.

**Implementation of DE-RSF for NCELD Problem**

**Step 1) Parameter Setup**

Initialize the number of generating units $N$ and Population size $N_p$. Specify minimum and maximum capacity of each generator $P_{min}$ and $P_{max}$ respectively. Initialize DE parameters like crossover probability $C_R$, scaling factors such as $\alpha$ and $\beta$. Set generation count, $G = 0$.

**Step 2) Initialization of the Population**

For a population size $N_p$ and dimension $D$, an initial vector $X_{0,G}$ is randomly generated. $D$ represents the number of decision variables to be optimized. In ELD problem $D$ is the number of generating units considered. $X_{0,G}$ is the real power value of $i^{th}$ unit of the $G^{th}$ population randomly generated within the operating limits using (12).

**Step 3) Evaluation of Fitness Function**

Evaluate the fitness value of each individual vector $X_{i,G}$. The evaluation function $F(P_i)$ is defined to minimize the fuel cost function given by (1) for a given load demand $P_d$ while satisfying the constraints given in equations (3), (7), (10) and (11).

$$F = \sum_{i=1}^{N} F_i(P_i) + \lambda \cdot \left[ \sum_{i=1}^{N} P_i - P_d \right]^2 + \gamma \cdot \left[ \sum_{i=1}^{N} v_i^R \right]$$

where $\lambda$ is the penalty parameter for not satisfying the load demand and $\gamma$ represents the penalty for a unit loading falling within a prohibited operating zone. $v_i^R$ is the violation of the prohibited zone constraint for the $i^{th}$ unit which is defined as

$$v_i^R = \begin{cases} 1 & \text{if } P_i \text{ violates the prohibited zones} \\ 0 & \text{otherwise} \end{cases}$$

**Step 4) Mutation Operation**

**Step 4.1) Perform cross over operation using**

$$U_{j,G} = \begin{cases} v_{j,G} & \text{if } \text{rand}(0,1) \leq \frac{G+1}{C_R} \\ X_{j,G} & \text{otherwise} \end{cases}$$

**Step 5) Recombination**

Recombination is employed to generate a trial vector $U_i$ by replacing certain parameters of $X_i$ with corresponding parameters of donor vector $V_i$. The trial vector by crossover operation is obtained using (19) and its fitness is evaluated using (1).

**Step 6) Selection**

Members to constitute the population of next generation $(G+1)$ are decided by (19). The new vector $X_{j,(G+1)}$ is selection based on the comparison of fitness value of both $X_i$ and $U_i$.

**Step 7) Verification of Stopping Criterion**

Set the generation count $G = G+1$. Go to step 3 until stopping criterion is reached. The stopping criterion considered is usually maximum generation count $G_{max}$.

**Simulation Results and Discussion**

In order to verify the effectiveness of the presented DE-RSF method, four test systems are considered. Table 1 shows the case studies with different type of practical constraints considered in solving a non-smooth economic load dispatch. The fuel cost coefficients and the operating limits for all the case studies are taken from [11] and [14].

In addition to the constraints given in Table 1, each test system is also subjected to the power balance constraint which is given in equation (4).

<table>
<thead>
<tr>
<th>Case study</th>
<th>Test system</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3-unit</td>
<td>valve point effects , ramp rate limits and prohibited operating zones</td>
</tr>
<tr>
<td>2</td>
<td>13-unit</td>
<td>valve point effects , ramp rate limits and prohibited operating zones</td>
</tr>
<tr>
<td>3</td>
<td>15-unit</td>
<td>valve point effects , ramp rate limits and prohibited operating zones</td>
</tr>
<tr>
<td>4</td>
<td>40-unit</td>
<td>valve point effects , ramp rate limits and prohibited operating zones</td>
</tr>
</tbody>
</table>

The results obtained for each case study using the DE-RSF method is compared with other types of DE variants. Each DE variant considered in this paper differs from the other based on the mutation factor. A brief outline of the different DE variants used in the comparison study is shown in Table 2.

<table>
<thead>
<tr>
<th>Table 2 Different Types of DE Variants</th>
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<tbody>
<tr>
<td>Strategy</td>
</tr>
<tr>
<td>DEV (1)</td>
</tr>
<tr>
<td>DEV (2)</td>
</tr>
<tr>
<td>DEV (3)</td>
</tr>
<tr>
<td>DEV (4)</td>
</tr>
<tr>
<td>DEV (5)</td>
</tr>
</tbody>
</table>

More detailed information on the types of DE strategy can be seen in [15]. The consistency in getting optimal solution and comparison of solution quality of the DE-RSF method with other techniques is carried out by taking 50 independent runs. The software is coded using MATLAB and executed on a personal computer.
Control Parameters in DE-RSF

The DE-RSF method consists of six control variables. They are $N_P$ - population size, $N_G$ - is the set of all dispatchable units, $\alpha$ and $\beta$ - scaling factors, $C_r$ - crossover rate, and $w$ - weight factor. The following control parameters have been chosen for all the DE variants for 50 trials was applied in this paper except the presented DE-RSF. Other parameters were, $N_P = 100$, $N_G = 1000$, $C_r = 0.8$. In this paper, the value of weight factor is started with 0 for all vectors and then increased up to 1 during the execution of algorithm. The scaling factor for DE-RSF was varied in between the random value (0,1) throughout the progress of its run and is dynamic in nature.

Table 3 shows the results for DE variants and DE-RSF for the case study 1. Table 4 shows the convergence result for 10 thermal unit system considering constraints. The system data are considered from [14].

Third case study has been taken from [11]. In this case, the load demand expected to be determined was $P_D = 2630$ MW. The data for 15 unit system is available in [14]. Fig. 1 clearly shows that the presented method outperforms than other DE variants. Fig. 2 shows the presented method produces minimum fuel cost. Fourth case study consist of 40 thermal units of generation with the effects of valve-point loading, Ramp rate limits, Prohibited operating zones, equality and inequality constraints as referred. In this case, the load demand expected to be determined was $P_D = 10500$ MW. The individual power generation of units for DE variants and DE-RSF are shown in Fig. 3. Minimum fuel cost obtained by presented method and various DE variants were also shown in Fig. 4. It is clear from Fig 4 that the presented technique implies better results.

Table 3 Convergence Results for 3 Generator System Load demand=850MW; Maximum iteration=1000 and NP=100

<table>
<thead>
<tr>
<th>Unit</th>
<th>DE Variants</th>
<th>DE-RSF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DEV (1)</td>
<td>DEV (2)</td>
</tr>
<tr>
<td>Unit 1(MW)</td>
<td>174.960</td>
<td>182.140</td>
</tr>
<tr>
<td>Unit 2(MW)</td>
<td>400.000</td>
<td>400.000</td>
</tr>
<tr>
<td>Unit 3(MW)</td>
<td>489.460</td>
<td>487.812</td>
</tr>
<tr>
<td>$P_{loss}$(MW)</td>
<td>214.420</td>
<td>213.984</td>
</tr>
<tr>
<td>FuelCost($/hr)</td>
<td>10124.620</td>
<td>10120.562</td>
</tr>
<tr>
<td>Total power output(MW)</td>
<td>1064.420</td>
<td>1064.420</td>
</tr>
</tbody>
</table>

Table 4 Convergence Results for 13 Generator System Load demand=1800MW; Maximum iteration=1000 and NP=100

<table>
<thead>
<tr>
<th>Unit</th>
<th>DE Variants</th>
<th>DE-RSF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DEV (1)</td>
<td>DEV (2)</td>
</tr>
<tr>
<td>Unit 1(MW)</td>
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<td>Unit 2(MW)</td>
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<tr>
<td>Unit 9(MW)</td>
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<tr>
<td>Unit 10(MW)</td>
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<tr>
<td>Unit 11(MW)</td>
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<tr>
<td>Unit 12(MW)</td>
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</tr>
<tr>
<td>Unit 13(MW)</td>
<td>55.000</td>
<td>55.000</td>
</tr>
<tr>
<td>Fuel cost($/hr)</td>
<td>17932.470</td>
<td>17931.948</td>
</tr>
<tr>
<td>Total power output(MW)</td>
<td>1800.000</td>
<td>1800.000</td>
</tr>
</tbody>
</table>

Conclusion

This paper presents new approach combining to solve the ELD problem of electric energy with the valve-point effect. The DE-RSF algorithm has the ability to find the better quality solution and has better convergence characteristics, computational efficiency, and robustness. Many realistic and nonlinear characteristics constraints of the generator such as ramp rate limits, prohibited operating zones and generation limits are considered for practical uses in the presented method. It is clear from the results obtained by different trials that the presented method has good convergence property and can avoid the shortcoming.
of premature convergence of other optimization techniques to obtain better quality solution. Four case studies have been used and the simulation results indicate that this optimization method is very accurate and converges very rapidly so that it can be used in the practical optimization problems. Due to these properties, the DE-RSF method future can be tried for solution of complex unit commitment, dynamic ELD problems in the search of better quality results.

Fig. 3. Individual unit power generation for case study 4

Fig. 4. Fuel Cost Distribution for Different DE Variants

REFERENCE

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