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Solution of a mixed sensitivity problem with optimized weights and time performance index for a robust servo velocity and position controller

***Abstract**. Robustness of servo control systems in terms of uncertainties and disturbances is still an open issue in feedback control. The concept of an uncertain LTI system is essential for robust control. Model uncertainty arises when system parameters are not precisely known, or can vary over a given range. The article presents a robust velocity and position controller for a servo system with a DC-motor. The aim of the work is to synthesize a feedback control structure with a controller in the direct branch, with which the robustness and stability performance will be ensured. For the design procedure a mixed-sensitivity approach with an additional time performance index is used. The time performance index introduces a subsequent criterion in the mixed sensitivity approach to ensure accurate dynamic performance of the feedback loop. For simplification of the controller structure the evolution optimization will be employed. The control strategy has been tested on real system with use of TI-DSP microcontroller.*

***Streszczenie.** W artykule opisano metodę sterowania pozycją i prędkością w zamkniętej pętli dla serwomechanizmu z silnikiem DC. Dla zapewnienia niezawodności i stabilności algorytmu zastosowano optymalizację typu mixed-sensitivity z dodatkowym wskaźnikiem skuteczności w czasie. W celu uproszczenia struktury sterowania zastosowana zostanie także optymalizacja ewolucyjna. Przeprowadzono badania eksperymentalne. (**Zastosowanie optymalizacji współczynników wagowych funkcji czułości oraz indeksu skuteczności w czasie w sterowaniu serwomechanizmem**).*

Keywords: robust stability, timer performance system, uncertainty models, mixed-sensitivity approach, weight optimization
Słowa kluczowe: wysoka stabilność, działanie system zegarowego, modele niepewności, podejście Mixed-sensitivity, optymalizacja wag.

Introduction

The purpose of robust closed-loop control is to ensure efficient functioning of the controlled system, so that it functions within the set design criteria, performs disturbance rejection and compensates for potential structural changes in the controlled system. Most robust controller design procedures are based on a mathematical model of a real system. Most often the mathematical model is presented as a linear time invariable system – *LTI*, for which it is much easier to develop general design procedures or set the qualitative control criteria. Linear mathematical models are mostly the result of identification methods [1] or procedures of linearization of non-linear models and are merely approximations of real systems. Model uncertainties mostly occur because of imprecise identification procedures, poor knowledge of the process's physical background, model simplifications and the inability to include external influences, which are mostly random in nature. LTI models thus describe the dynamics of the real system only under one operation condition and with certain parameter uncertainty [2],[3].

It often occurs that the controller, designed by classic design methods, in reality does not fulfil the set control conditions. Standard design methods are mostly based on graphic approaches (Bode plot, Nichols plot, Root-locus etc.). The mentioned methods do not directly consider robustness criteria and thus provide no guarantee that the system will have satisfactory performance in the presence of disturbances or uncertainties. Robust control design methods ensure this. The synthesis of closed-loop system can be carried out so that we consider robustness criteria in the design procedure [2].

This article will present the design procedure for a robust velocity and position controller of a servo system with the mixed sensitivity approach-MxSA. The mixed sensitivity approach is a closed-loop system synthesis procedure, in which the model's uncertainty, external disturbances and performance characteristics are described by weight functions.

The choice of weight functions is crucial, as they influence the efficiency of the closed-loop system and consequently the controller structure [4],[5].

Robust stability in the mixed sensitivity approach is assessed on the basis of uncertainty models [6], where the weight function represents the possible deviation of the model from the nominal value. Uncertainty models describe different uncertainty types, e.g. high or low frequencies uncertainties, parameter uncertainties, etc. Exact uncertainty weights can be derived directly from uncertainty models, where proper and stable functions must be ensured [7],[8],[9]. Direct derivation of weight functions from the uncertainty model often results in weights which do not fulfil the stated conditions. This article will present a method for transformation of weights which have the same frequency characteristic as directly derived weights, they are stable, proper and of low order. To this end we will use the optimisation algorithm Differential Evolution – *DE* with spectral objective function [8],[10]. Additional weight functions, representing the desired system dynamics or other limitations of the closed-loop system, can be determined following the recommendations [2]. Following the recommendations is very non-transparent and requires many iterations and experience on the side of the designer. For this purpose we will also discuss in this article the time performance index, which we will determine using the *DE* algorithm. With the time performance index we will precisely describe the dynamics of the system, and we will consider it in the mixed sensitivity approach synthesis as an additional performance criterion.

The aim of this article is to present robust controller synthesis with the mixed sensitivity approach, and particularly to ensure robust stability of the system in terms of possible parameter deviations and good compensation for external disturbances in case of load change of the servo system. With weights optimisation we also want to ensure simple regulator structure, because such structure is more suitable for real time operation [8].

Uncertainty models

Uncertainty models describe the total set of possible uncertainties. Every uncertainty model has its characteristic properties, with which we can determine the uncertainties alongside the entire frequency characteristic of the control object. Let us assume a set of models $\mathcal{P}$, where P_0 is the nominal model and P_Δ is the highest uncertainty model, both belonging to the set $\mathcal{P}$. The model uncertainty is described with the weight function ΔW, which must be proper and stable.

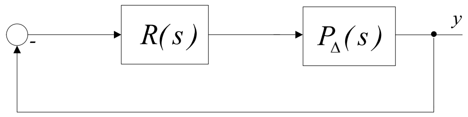

Fig.1. Feedback-loop with the uncertainty model P_Δ

Multiplicative uncertainty

With the multiplicative uncertainty model can be described high-frequency dynamics of the plant and the uncertainty of zeros P_0 in the area *Re*s>0.

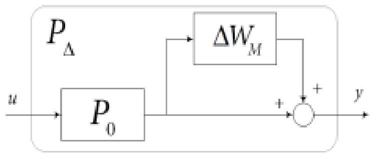

Fig.2. Multiplicative uncertainty model

(1) $$\Delta W_M(s) = (P_\Delta(s) - P_0(s))P_0(s)^{-1}$$

Additive uncertainty

Additive uncertainty model is appropriate for modelling the neglected high-frequency dynamics and the uncertainty of poles and zeros of the control plant P_0 in the areas *Re*s>0 and *Re*s<0.

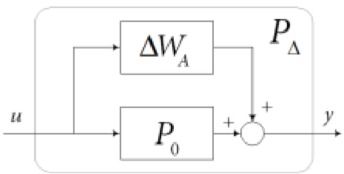

Fig.3. Additive uncertainty model

(2) $$\Delta W_A(s) = P_\Delta(s) - P_0(s)$$

Inverse uncertainty

Inverse uncertainty model is suitable for modelling uncertainty at the low frequencies and the uncertainty of the control plant P_0 model's poles in the area *Re*s<0.

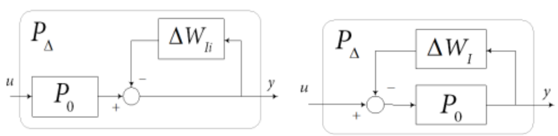

Fig.4. Inverse uncertainty model

(3) $$\begin{aligned}\Delta W_{Ii}(s) &= P_\Delta(s)^{-1}(P_0(s) - P_\Delta(s)),\\ \Delta W_I(s) &= P_\Delta(s)^{-1}(P_0(s) - P_\Delta(s))P_0(s)^{-1}.\end{aligned}$$

To determine the weight ΔW let us suppose the following. The nominal model P_0 has the same unstable poles as the uncertainty model P_Δ . The poles of P_0 cannot be cancelled with zeros of the weight function ΔW.

Robust stability assessments under uncertainty models

Controller *R* is robustly stable, if it ensures stability for all the models in set $\mathcal{P}$. System robustness is assessed on the basis of the weight function ΔW and the uncertainty models, where Nyquist curve is the stability limit. The distance between the key point -1 for system in Figure 1 and its closest point of the Nyquist curve of the transfer function is $1/\|S\|_\infty$ [11],[12] and the following holds true:

(4) $$\left|P_\Delta(j\omega)R(j\omega) - P_0(j\omega)R(j\omega)\right| \le \|S\|_\infty^{-1}$$

If condition (4) is fulfilled then controller *R* ensures stability for the entire set of models $\mathcal{P}$ [2],[13]. The conditions of robust stability for different uncertainty models:

- multiplicative uncertainty: $\|\Delta W_M T\|_\infty < 1$
- additive uncertainty: $\|\Delta W_A RS\|_\infty < 1$
- inverse uncertainty: $\|\Delta W_{Ii} S\|_\infty < 1$, $\|\Delta W_I P_0 S\|_\infty < 1$

where transfer functions *S* and *T* are sensitivity and complementary sensitivity of the closed loop system in Fig.1.

Weight preparation and optimization

Direct derivation of weights $\Delta W'$ from uncertainty models (1),(2),(3) often results in weights that do not fulfil the condition of stability or are not a proper function. In such a case weights must be designed so that such weights ΔW are found, which have the same frequency characteristic as the directly derived weights $\Delta W'$ and which fulfil the abovementioned criteria. To determine the weights we used the evolution algorithm called Differential Evolution – *DE* [14], which is appropriate for optimisation of multi-objective functions. The starting point of the problem is the determination of the *DE* objective function, which enables approximation of weight's frequency characteristic. The property of the objective function influences the solution and the convergence of the approximation. This article discusses the approach with an ISE (integral square error) type of the convex objective function,

(5) $$f_1 = \int_0^{\omega_{aprox}} \left(\Delta W'(\omega^2) - \Delta W(\omega^2)\right)^2 d\omega = \int_0^{\omega_{aprox}} \left(e_1(\omega^2)\right)^2 d\omega,\ \exists\left\|\Delta W \Delta W'^{-1}\right\|_\infty < 1,$$

where $\Delta W'$ is the derived weight, ΔW is the approximated weight and ω_{aprox} is the approximated frequency window. The solution of the problem is $min_{e1}(f_1)=0$. To the objective function f_1 criterion f_2 is added. The criterion of the function f_2 ensure the stability of the approximated weight ΔW (6),[15]:

(6) $$f_2 = \int_0^{\omega_{aprox}} \left(\Delta W(j\omega) + \Delta W(-j\omega)\right)^{-1} d\omega = \int_0^{\omega_{aprox}} \left(e_2(\omega^2)\right)^2 d\omega > 0,$$

The composed objective function is equals to f_{spet},

(7) $$f_{spet} = \min_{e_1}(f_1) \wedge (f_2 > 0).$$

The same approach is used to determine the weight of the time performance index, where the weight W_{time} of minimal structure from the given time response is assessed. The weight function is equals to:

(8) $$f_3 = \int_0^{t_1} \left(H(t) - W_{time}(t)\right)^2 dt = \int_0^{t_1} \left(e_3(t)\right)^2 dt,$$

where *H(t)* is the time reference characteristic and W_{time} is the sought performance weight. To the objective function f_3 the stability criterion (6) is added. The entire objective function is equals to:

(9) $$f_{time} = \min_{e_3}(f_3) \wedge (f_2 > 0).$$

The approximate weight $W_{time}(j\omega)$ is used further as a robustness criterion $min_\omega\|W_{time}S\|_\infty$ [2],[3],[6].

Mixed-sensitivity approach for the robust controller

Using uncertainty weights ΔW_A, ΔW_M, ΔW_I and the time performance index W_{time}, the robust controller synthesis can be determinate on the basis of the mixed sensitivity problem [2],[3].

With weights $\Delta W_I, \Delta W_A, \Delta W_M$ Fig. 5. we can describe uncertainties for individual models or simply describe the frequency characteristic of the sensitivity function e/v → z_1/v, the controller output u/v→ z_2/v or complementary sensitivity y/v→ z_3/v, where the corresponding weights are marked as $\Delta W_I \leftrightarrow W_{time}$, $\Delta W_A \leftrightarrow W_2$, $\Delta W_M \leftrightarrow W_3$, and $\|W_{time}\Delta W_{Ii}\|<1$, $\|W_2\Delta W_A\|<1$, $\|W_3\Delta W_M\|<1$ is hold true. The solution of the mixed sensitivity problem is the optimisation of the norm $\mathcal{H}_\infty$, where:

$$
\min_{R\in\mathcal{RH}_\infty}\left\|\begin{matrix}\Delta W_M T \vee W_3 T\\ \Delta W_A RS \vee W_2 S\\ \Delta W_{1i} S \vee W_{time} S\end{matrix}\right\|_\infty^2 \le \gamma^2,\quad \begin{matrix}\because\|\Delta W_M\|_\infty<\|W_3\|_\infty \Rightarrow \Delta W_M T,\\ \because\|\Delta W_M\|_\infty>\|W_3\|_\infty \Rightarrow W_3 T,\\ \because\|\Delta W_A\|_\infty<\|W_2\|_\infty \Rightarrow \Delta W_A RS,\\ \because\|\Delta W_A\|_\infty>\|W_2\|_\infty \Rightarrow W_2 RS,\\ \because\|\Delta W_{1i}\|_\infty<\|W_{time}\|_\infty \Rightarrow \Delta W_{1i} S,\\ \because\|\Delta W_{1i}\|_\infty>\|W_{time}\|_\infty \Rightarrow W_{time} S.\end{matrix} \tag{10}
$$

The optimal solution is equals to:

$$
|\Delta W_{M,3}(j\omega)T(j\omega)|^2 \le \gamma^2 \wedge |\Delta W_{A,2}(j\omega)R(j\omega)S(j\omega)|^2 \le \gamma^2 \wedge |\Delta W_{I,time}(j\omega)S(j\omega)|^2 \le \gamma^2 \;;\; \forall\omega. \tag{11}
$$

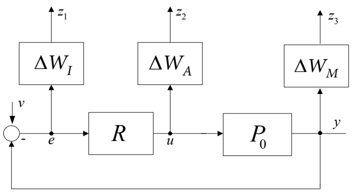

Fig.5. Mixed sensitivity problem

The synthesised controller R fulfils the condition $R\in\mathcal{RH}_\infty$. In case of a high order controller structure we carry out order reduction on the basis of Hankel singular value decomposition-HSV [2],[4],[15], where $R_{low}\in\mathcal{RH}_\infty$ is also hold true.

Synthesis of the robust velocity controller

The nominal transfer function of the DC servo engine **Escap 28D11-219E**:

$$
P_{0\omega}(s)=\frac{\omega(s)}{U_{Voltage}(s)}=\frac{\frac{k_m}{LJ_n}}{s^2+\left(\frac{RJ_n+BL}{LJ_n}\right)s+\left(\frac{BR+k_e k_m}{LJ_n}\right)} =\frac{1}{2.78\cdot 10^{-8}s^2+0.0002257s+0.0196}
$$

The uncertainty weights are determined by deviation the engine coefficients: the change of resistance *R(±15%)*, the change of load which results in the change of inertia *J_n(±30%),* the change of *k_e, k_m (±10%)* because of nonlinear magnet characteristic, etc.

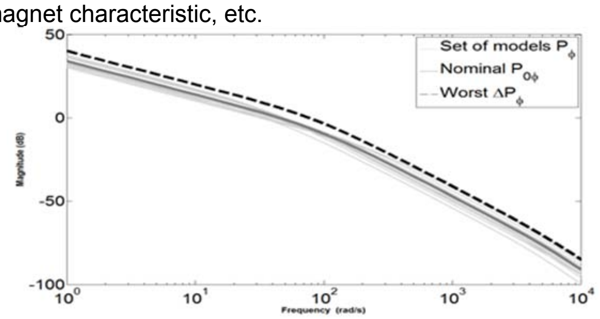

Fig.6. Frequency characteristic $P_{0\omega}$ with changing parameters *R, J_n, k_e, k_m*

The transfer function of the highest uncertainty ΔP_ω:

$$
\Delta P_\omega(s)=\frac{1.8}{2.68\cdot 10^{-8}s^2+0.00023s+0.023}
$$

The uncertainty weights are determinates according to criteria (1),(2),(3).

$$
\Delta W_A(s)=\frac{2.995\cdot 10^7}{s^2+83342s+7.286\cdot 10^5},\ \Delta W_M(s)=0.8,
$$

$$
\Delta W_I{}'(s)=-1.2\cdot 10^{-8}s^2-0.0001s-0.008747,\ \Delta W_{Ii}(s)=-0.444.
$$

Both weights, except for $\Delta W_I'$, are stable and proper. Weight $\Delta W_I'$ is transformed into a stable and proper weight ΔW_I with the algorithm *DE* and objective function (5),(6),(7) and frequency window *ω_{aprox} =0.01-10^5rad*,

$$
\Delta W_I(s)=\frac{1\cdot 10^{-5}s^2+1.3\cdot 10^{-16}s+9.4\cdot 10^{-16}}{s^2+1\cdot 10^{-4}s+7.2\cdot 10^{-11}}.
$$

From control requirements: settling time *t_s<0.09s*, overshoot *M_p<2%* and steady-state error *e_s≈0*, we determine the proper and stable weight *W_{time},* where *$||W_{time}\Delta W_{Ii}||<1$* is hold true (8),(9).

$$
W_{time}(s)=\left(\frac{s^2+184s+1.265\cdot 10^4}{s^2+184s+1\cdot 10^{-5}}\right)^{-1}.
$$

The solution of MxSA with weights ΔW_A, ΔW_M, ΔW_I, W_{time} and HSV:

$$
R_\omega(s)=\frac{30.66s^3+1.619\cdot 10^6 s^2+1.139\cdot 10^{10}s+9.912\cdot 10^{11}}{s^4+8.98\cdot 10^4 s^3+3.99\cdot 10^9 s^2+7.326\cdot 10^{11}s+3.982\cdot 10^4}.
$$

Conditions of robust stability and the time performance index: $\|\Delta W_M T\|_\infty<0.799$, $\|\Delta W_A RS\|_\infty<0.786$, $\|\Delta W_I P_0 S\|_\infty<0.437$, $\|W_{time}S\|_\infty<0.455$. The values of the norms show that the closed-loop system with velocity controller R_ω fulfils the robust stability and performance conditions.

Synthesis of the robust position controller

Nominal transfer characteristic of a DC servo motor for position control:

$$
P_{0\varphi}(s)=\frac{\varphi(s)}{U_{Voltage}(s)}=\frac{\frac{k_m}{LJ_n}}{s^2+\left(\frac{RJ_n+BL}{LJ_n}\right)s+\left(\frac{BR+k_e k_m}{LJ_n}\right)}\cdot\frac{1}{Ks} =\frac{1}{2.78\cdot 10^{-8}s^2+0.0002257s+0.0196}\cdot\frac{1}{0.85s}
$$

Determination of the highest uncertainty transfer function ΔP_φ with deviation of DC-engine parameters *R(±13%), J_n(±35%), k_e, k_m (±12%)*:

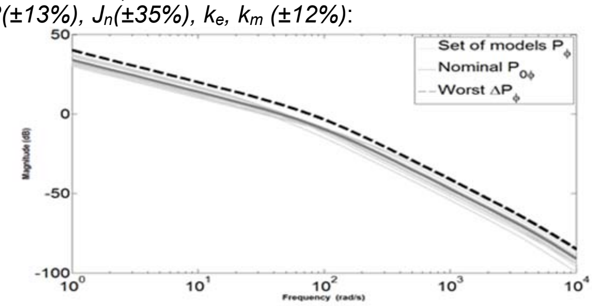

Fig.7. Frequency characteristic $P_{0\varphi}$ with changing parameters *R, J_n, k_e, k_m*

The transfer function of the highest uncertainty ΔP_φ:

$$
\Delta P_\varphi(s)=\frac{2}{2.98\cdot 10^{-8}s^3+0.000207s^2+0.024s}.
$$

Uncertainty weights determined according to criteria (1),(2),(3):

$$
\Delta W_A{}'(s)=\frac{3.693\cdot 10^7}{s^3+8334s^2+7.268\cdot 10^5 s},\ \Delta W_M(s)=1,
$$

$$
\Delta W_I{}'(s)=-1.354\cdot 10^{-8}s^2-0.000112s-0.00984,\ \Delta W_{Ii}(s)=-0.5.
$$

Weights $\Delta W_A'$, $\Delta W_I'$ are transformed into proper and stable functions ΔW_A, ΔW_I with the algorithm *DE* (5),(6),(7) and frequency window *ω_{aprox} =0.02-10^4rad*.

$$
\Delta W_A(s)=\frac{3.411\cdot 10^7}{s^3+8234s^2+7.12\cdot 10^5 s+0.0012}
$$

$$
\Delta W_I(s)=\frac{1.5\cdot 10^{-5}s^2+1.27\cdot 10^{-16}s+1.02\cdot 10^{-15}}{s^2+0.9\cdot 10^{-4}s+6.992\cdot 10^{-11}}.
$$

From control requirements; settling time *t_s<0.05s*, overshoot *M_p<1%* and steady-state error *e_s<5%*, we determine the proper and stable weight W_{time}, where $||W_{time}\Delta W_{Ii}||<1$ is hold true (8),(9).

$$W_{time}(s)=\left(\frac{s^2+920s+2.612\cdot 10^5}{s^2+920s+1\cdot 10^{-5}}\right)^{-1}$$

The robust position controller R_φ with weights ΔW_A, ΔW_M, ΔW_I, W_{time} and HSV simplification:

$$R_\varphi(s)=\frac{9.5\cdot 10^5 s^4+8.904\cdot 10^9 s^3+8.847\cdot 10^{12} s^2+7.113\cdot 10^{14} s+3.634\cdot 10^6}{s^5+5.9\cdot 10^4 s^4+4.989\cdot 10^8 s^3+6.13\cdot 10^{11} s^2+1.95\cdot 10^{14} s+2.11\cdot 10^6}$$

Conditions of robust stability and the time performance index: $\|W_M T\|_\infty<0.89$, $\|W_A RS\|_\infty<0.94$, $\|W_I P_0 S\|_\infty<0.62$, $\|W_{time} S\|_\infty<0.63$. According to the norms values position controller R_φ fulfils the conditions of robust stability and performance.

Experimental results

The test of robust velocity and position controller was performed on the embedded system *TI-DSP TMS320F28335P* with sampling time $T_s=1.3ms$.

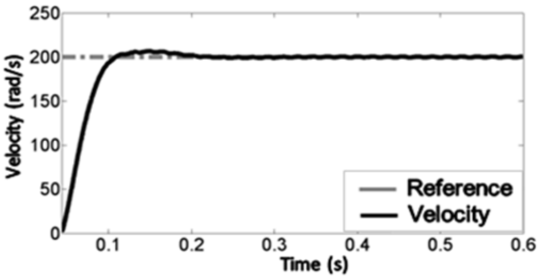

Fig.8. Angle speed control

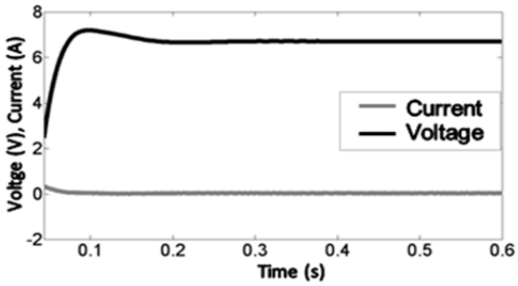

Fig.9. Velocity controller output, voltage (V), current (A)

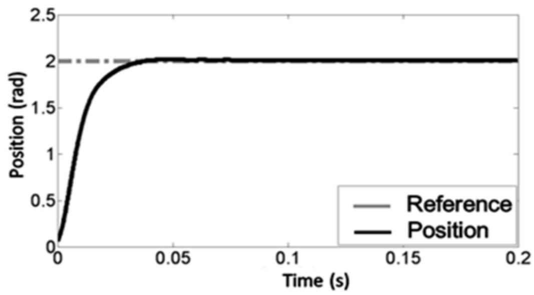

Fig.10. Position control

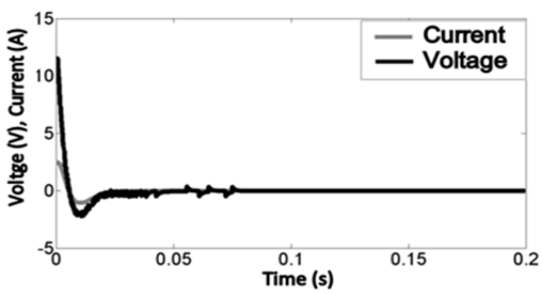

Fig.11. Position controller output, voltage (V), current (A)

The control results from Fig. 8–11 confirm that the designed velocity and position controllers, R_ω and R_φ respectively, fulfil the set robustness and control dynamics criteria.

Conclusions

This article presented the synthesis of a robust controller of a servo system by designing weights ΔW_A, ΔW_M, ΔW_I, W_{time}. The purpose of this article is to present the process of designing weights, which are determined directly from the uncertainties model, as only in this way we can keep the precision of the uncertainty description. Weights transformation was performed with the evolution algorithm *DE* and the composed objective function. We also added a performance weight W_{time} to weights ΔW_A, ΔW_M, ΔW_I , with which we have determined the dynamics of the closed-loop system. The controller synthesis was based on assessing the robustness and keeping the desired system dynamics with the mixed sensitivity method, where weight functions ΔW_I,ΔW_A,ΔW_M or W_1,W_2,W_2 can have different meanings.

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