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Small Signal Stability Analysis of Six-phase Synchronous Generator

Abstract. *In the present work, a higher order linearized model has been developed to assess the small signal excursion characteristics of an asymmetrical six-phase synchronous generator system connected to infinite bus using eigenvalue criterion. The stability of the system under small perturbation of different variables has been examined from the placement of eigenvalues. The work establishes an association between generator and loading parameters with pair of eigenvalues, thus the system stability. Further a participation factor analysis has also been carried out to identify the effect of dynamic state variable on a given mode or eigenvalue.*

Streszczenie. *W artykule zaprezentowano linearyzowany model wyższego rzędu umożliwiający analizę asymetrycznego sześciofazowego generatora asynchronicznego podłączonego do nieskończonej linii. Badano stabilność systemu przy małych zakłóceniach. (**Analiza stabilności małego sygnału sześciofazowego generatora synchronicznego**)*

Keywords: six-phase synchronous generator, small signal stability, eigenvalue, participation factor.
Słowa kluczowe: sześciofazowy generator synchroniczny, stabilnmość.

Introduction

A substantial body of work spread over last two decades indicates the technical and economic viability of using number of phases higher than three in multi-phase AC machines for application in marine ship, thermal power plant, electric vehicles and nuclear power plants etc. Multi-phase drive possesses several advantages over conventional three-phase drive system, such as reducing the amplitude and increasing the frequency of torque pulsations, reducing the rotor harmonic currents, reducing the current per phase without increasing the voltage per phase, lowering DC link current harmonics, increased power to weight ratio, better performance and higher reliability [1-3]. In recent times, because of several benefits of multi-phase AC machine, interest has re-emerged in the use of multi-phase generators in conjunction with various prime movers [4-6]. Permanent magnet multi-phase synchronous generators [7] may become a viable solution for the direct driven applications in wind powered plants, while multi-phase generators may have a prospect for application in stand-alone generating systems of rural areas [4-8]. It needs to be emphasized though there is no evidence at present of any industrial uptake of such solutions. Recent work shows the feasibility of six-phase synchronous generator for stand-alone renewable energy generation system in conjunction with hydro power plant [5, 6, 8]. In these studies, modeling and steady state performance analysis of six-phase synchronous generator for energy generation has been discussed. The six-phase generator can either be used as an autonomous energy source or supplying power to utility grid through an interconnecting six-phase to three phase transformer, which raises the issues of machine stability. Among stability, small signal stability is an important one. To the best of author's knowledge, the small signal stability analysis of six-phase synchronous generator connected to utility grid have not been carried out, although the recent development in electronic power convertors has overcome the limitation of connecting six-phase generator with conventional three-phase utility grid.

The steady state stability is and has been a concern of electrical machine designer, power system engineer. Out of various aspects of stability studies, an important one is small perturbation about the equilibrium point. It has long been observed that the electrical machines operating at constant nominal speed might display small signal instability. The well established standard procedures for analysis of small signal stability of some operating point is to examine the linearized model of electrical machine which describes small excursions from fixed operating condition. Literature is available on stability of three-phase synchronous machine using various methods like root locus, Nyquist criterion and eigenvalue technique. The small perturbation characteristics of a single three-phase machine supplying an infinite bus were explored using frequency response method [9]. In this, the effects of excitation system on stabilizing requirements were presented. A state space eigenvalue program was presented for the stability study of three-phase regulated synchronous machine [10]. A comparative study of eigenvalue analysis capabilities of two simulation software's was carried out in [11]. The effect of large synchronous motor starting on an isolated steel plant was investigated to understand the transient stability limits of the cogeneration system in [12]. In this, the effect of voltage fluctuations was not considered. A model for nine-phase salient pole synchronous machine is developed by vector space decomposition to reduce the machine into a usual *d-q* equivalent circuit model and seven non-torque producing circuit in [13]. Using eigenvalue method, small signal stability analysis of six-phase induction machine was investigated by considering the effect of common mutual leakage reactance [14]. Duran et al have studied the stabilizing effect of harmonic injection in five phase induction motor drive using bifurcation theory, and concluded that the technique enhances torque and stability of the machine [15].

The behavior of six-phase synchronous generator can be described by a set of non-linear differential equations. In this work, a thirteenth order linearized model for a six-phase synchronous generator with excitation system has been developed. In the proposed model, effect of mutual leakage reactance between the two three-phase stator winding sets have not been included as the two winding sets are at thirty electrical degree apart. The stability of the six-phase generator connected to infinite bus under small excursion of any machine or loading variable has been examined at a time from the placement of the eigenvalues. The participation factor analysis is also carried out to identify the effect of dynamic variable on a given mode or eigenvalue.

Mathematical model

(a) Synchronous Machine model

The stator of the six-phase synchronous machine has six uniformly distributed phase windings. These phases are sinusoidally distributed, and are configured to form two three-phase *wye* connections with neutral isolated to prevent the flow of physical fault and triplen harmonic current between them. The three-phases of a *wye* set are 120° apart from each other and the magnetic axes of two wye sets (i.e '*abc*' and '*xyz*') are at an angle of 30° electrical from each other. The basic two pole, salient pole, six-phase synchronous machine is shown in Fig.1. The rotor circuit has a field winding (*fr*) and a damper winding (K_d) along d-axis and two damper windings (K_{q1}) and (K_{q2}) along q-axis. The instant shown is the time when '*a*' phase of set '*abc*' coincides with *d*-axis. All quantities are referred to '*abc*' winding set.

The voltage equations [6, 16, 17] in the rotor reference (considering generator convention) frame are given by:

(1) $v_{d1} = -r_1 i_{d1} - (\omega_r/\omega_b)\psi_{q1} + (1/\omega_b)p\psi_{d1}$

(2) $v_{q1} = -r_1 i_{q1} + (\omega_r/\omega_b)\psi_{d1} + (1/\omega_b)p\psi_{q1}$

(3) $v_{d2} = -r_2 i_{d2} - (\omega_r/\omega_b)\psi_{q2} + (1/\omega_b)p\psi_{d2}$

(4) $v_{q2} = -r_2 i_{q2} + (\omega_r/\omega_b)\psi_{d2} + (1/\omega_b)p\psi_{q2}$

(5) $v_{Kq1} = r_{Kq1} i_{Kq1} + (1/\omega_b)p\psi_{Kq1}$

(6) $v_{Kq2} = r_{Kq2} i_{Kq2} + (1/\omega_b)p\psi_{kq2}$

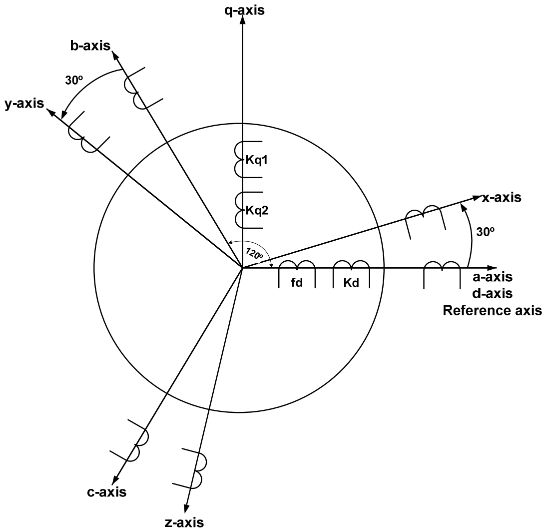

Fig1. Schematic diagram of six-phase synchronous generator

(7) $v_{Kd} = r_{Kd} i_{Kd} + (1/\omega_b)p\psi_{kd}$

(8) $v_{fr} = r_{fr} i_{fr} + (1/\omega_b)p\psi_{fr}$

Where flux equations are

(9) $\psi_{d1} = -X_{l1} i_{d1} + X_{lm}(-i_{d1} - i_{d2}) - X_{ldq} i_{q2} + X_{md}(-i_{d1} - i_{d2} + i_{Kd} + i_{fr})$

(10) $\psi_{q1} = -X_{l1} i_{q1} + X_{lm}(-i_{q1} - i_{q2}) + X_{ldq} i_{d2} + X_{mq}(-i_{q1} - i_{q2} + i_{Kq1} + i_{Kq2})$

(11) $\psi_{d2} = -X_{l2} i_{d2} + X_{lm}(-i_{d1} - i_{d2}) + X_{ldq} i_{q1} + X_{md}(-i_{d1} - i_{d2} + i_{Kd} + i_{fr})$

(12) $\psi_{q2} = -X_{l2} i_{q2} + X_{lm}(-i_{q1} - i_{q2}) - X_{ldq} i_{d1} + X_{mq}(-i_{q1} - i_{q2} + i_{Kq1} + i_{Kq2})$

(13) $\psi_{Kq1} = X_{lKq1} i_{Kq1} + X_{mq}(-i_{q1} - i_{q2} + i_{Kq1} + i_{Kq2})$

(14) $\psi_{Kq2} = X_{lKq2} i_{Kq2} + X_{mq}(-i_{q1} - i_{q2} + i_{Kq1} + i_{Kq2})$

(15) $\psi_{Kd} = X_{lKd} i_{Kd} + X_{md}(-i_{d1} - i_{d2} + i_{Kd} + i_{fr})$

(16) $\psi_{fr} = X_{lfr} i_{fr} + X_{md}(-i_{d1} - i_{d2} + i_{Kd} + i_{fr})$

(17) $p(\omega_r/\omega_b) = (1/2H)(T_{em} - T_{sh})$

where, X_{lm} is common mutual leakage reactance and X_{ldq} is the cross-saturation coupling between the two stator windings, and is given by:

(18) $X_{lm} = 0.866 * (X_{lax} + X_{lay})$

(19) $X_{ldq} = 0.5 * (X_{lax} + X_{lay} + 2 * X_{laz})$

Torque equation is

(20) $T_{em} = [\psi_{md}(i_{q1} + i_{q2}) - \psi_{mq}(i_{d1} + i_{d2})]$

where ψ_{md} and ψ_{mq} are mutual flux linkages along d- and q-axes respectively

(b) IEEE type-1 Excitation system model

Excitation model [18] considered for the purpose of this work is of IEEE type-1.

(21) $T_e p(E_{fd}) = -(K_e + S_e(E_{fd}))E_{fd} + V_r$

(22) $T_a p(V_r) = -V_r + K_a R_f + (\frac{K_a K_f}{T_f})E_{fd} + K_a(V_{ref} - V_t)$

(23) $T_f p(R_f) = -R_f + \frac{K_f}{T_f}E_{fd}$

p is a differential operator

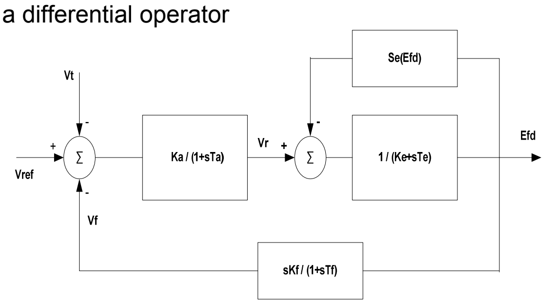

Fig. 2. IEEE type-1 excitation system

Linearized state space model

In this, a Taylor's expansion method has been implemented about a fixed steady state operating condition. According to this any variable may be written in its Taylor expanded form [17], as given by:

$$f(x_n) = f(x_{n0}) + f'(x_{n0})\Delta x_n + \left(\frac{f''(x_n)}{2!}\right)\Delta x_n^2 + \ ....$$

where $x_n = x_{n0} + \Delta x_n$ and x_{n0} is the fixed operating point. The higher order terms can be neglected to study small excursion about a fixed steady state operation point, that is

(24) $\Delta f(x_n) = f'(x_{n0})\Delta x_n$

Using equation (24), linearization of equations (1-23) with current as state variable can be written in the matrix form as:

(25) **E**p**X** = **FX + U**

where,

Input matrix

$\mathbf{U^t} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ T_l\ 0\ 0\ -\frac{KaVref}{Ta}\ 0]$

State variable matrix

$\mathbf{X^t} = \left[i_{q1}\ i_{d1}\ i_{q2}\ i_{d2}\ i_{Kq1}\ i_{Kq2}\ i_{fr}\ i_{Kd}\ \frac{\Delta\omega_r}{\omega_b}\ \delta\ V_f\ V_r\ E_{fd}\right]$

$$\mathbf{E} = \begin{bmatrix} W_p & Y_p & L_p \\ Q_p & S_p & M_p \\ X_p & Z_p & N_p \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} W_k & Y_k & L_k \\ Q_k & S_k & M_k \\ X_k & Z_k & N_k \end{bmatrix}$$

where

$$\mathbf{W_p} = \frac{1}{\omega_b}\begin{bmatrix} -(X_q + X_e) & 0 & -X_{mq} & 0 \\ 0 & -(X_d + X_e) & 0 & -X_{md} \\ -X_{mq} & 0 & -(X_q + X_e) & 0 \\ 0 & -X_{md} & 0 & -(X_d + X_e) \end{bmatrix}$$

$$\mathbf{Q_p} = \frac{1}{\omega_b}\begin{bmatrix} -X_{mq} & 0 & -X_{mq} & 0 \\ -X_{mq} & 0 & -X_{mq} & 0 \\ 0 & -X_{md} & 0 & -X_{md} \\ 0 & -X_{md} & 0 & -X_{md} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{Y_p} = \frac{1}{\omega_b}\begin{bmatrix} X_{mq} & X_{mq} & 0 & 0 & 0 & (X_q + X_e)i_{d10} + X_{mq}i_{d20} \\ 0 & 0 & X_{md} & X_{md} & 0 & -(X_d + X_e)i_{q10} - X_{md}i_{q20} \\ X_{mq} & X_{mq} & 0 & 0 & 0 & (X_q + X_e)i_{d20} + X_{mq}i_{d10} \\ 0 & 0 & X_{md} & X_{md} & 0 & -(X_d + X_e)i_{q20} - X_{md}i_{q10} \end{bmatrix}$$

$$\mathbf{S_p} = \frac{1}{\omega_b}\begin{bmatrix} X_{lKq1}+X_{mq} & X_{mq} & 0 & 0 & 0 & X_{mq}i_{d0} \\ X_{mq} & X_{lKq2}+X_{mq} & 0 & 0 & 0 & X_{mq}i_{d0} \\ 0 & 0 & (X_{lfr}+X_{md}) & X_{md} & 0 & -X_{md}i_{q0} \\ 0 & 0 & X_{md} & X_{lKd}+X_{md} & 0 & -X_{md}i_{q0} \\ 0 & 0 & 0 & 0 & 2H\omega_b & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_b \end{bmatrix}$$

where $i_{d0} = i_{d10} + i_{d20}$ and $i_{q0} = i_{q10} + i_{q20}$

$$\mathbf{L_p} = [0]_{3*3},\ \mathbf{M_p} = [0]_{6*3},\ \mathbf{Z_p} = [0]_{3*6}$$

$$\mathbf{N_p} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},\ \mathbf{X_p} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ M121 & M122 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$M121 = -\frac{K_a * v_{d1o} * X_e}{T_a * v_{to} * \omega_b},\quad M122 = -\frac{K_a * v_{q1o} * X_e}{T_a * v_{to} * \omega_b}$$

$$\mathbf{W_k} = \begin{bmatrix} -R & -k(X_d+X_e) & 0 & -kX_{md} \\ k(X_q+X_e) & -R & kX_{mq} & 0 \\ 0 & -kX_{md} & -R & -k(X_d+X_e) \\ kX_{mq} & 0 & k(X_q+X_e) & -R \end{bmatrix}$$

$$\mathbf{Q_k} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Qk41 & Qk42 & Qk41 & Qk42 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$k = \omega_{ro}/\omega_b$, $Qk41 = (X_{md}(-i_{d0} + i_{fr0}) + X_{mq}i_{d0})$,
$Qk42 = (-X_{md}i_{q0} + X_{mq}i_{q0})$

$$\mathbf{Y_k} = \begin{bmatrix} 0 & 0 & kX_{md} & kX_{md} & Yk14 & Yk15 \\ -kX_{mq} & -kX_{mq} & 0 & 0 & Yk24 & Yk25 \\ 0 & 0 & kX_{md} & kX_{md} & Yk34 & Yk35 \\ -kX_{mq} & -kX_{mq} & 0 & 0 & Yk44 & Yk45 \end{bmatrix}$$

where

$Yk15 = Ri_{d10} - k(X_d + X_e)i_{q10} - k(X_{md}i_{q20}) + v_{d10}$
$Yk25 = -Ri_{q10} - k(X_q + X_e)i_{d10} - k(X_{mq}i_{q20}) - v_{q10}$
$Yk35 = Ri_{d20} - k(X_d + X_e)i_{q20} - k(X_{md}i_{q10}) + v_{d20}$
$Yk45 = -Ri_{q20} - k(X_q + X_e)i_{d20} - k(X_{mq}i_{d10}) - v_{q20}$

$$\mathbf{S_k} = \begin{bmatrix} r_{Kq1} & 0 & 0 & 0 & 0 & 0 \\ 0 & r_{Kq2} & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{fr} & 0 & 0 & 0 \\ 0 & 0 & 0 & r_{Kd} & 0 & 0 \\ -X_{mq}i_{d0} & -X_{mq}i_{d0} & X_{md}i_{q0} & X_{md}i_{q0} & 0 & (Sk45) \\ 0 & 0 & 0 & 0 & -\omega_b & 0 \end{bmatrix}$$

$Sk45 = -i_{d10}[X_{mq}i_{d0} - X_{md}(i_{d0} - i_{fr0})] - i_{q10}[(X_{md} - X_{mq})i_{q0}] - i_{d20}[X_{mq}i_{d0} - X_{md}(i_{d0} - i_{fr0})] - i_{q20}[(X_{md} - X_{mq})i_{q0}]$

$$\mathbf{L_k} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\quad \mathbf{M_p} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{r_{fd}}{X_{md}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\quad \mathbf{Z_k} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K129 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{X_k} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ K121 & K122 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$K121 = -\frac{K_a}{T_a}\left(\frac{v_{d10}}{v_{t0}}R_e + \frac{v_{q10}}{v_{t0}}\frac{\omega_{r0}}{\omega_b}X_e\right)$$
$$K122 = -\frac{K_a}{T_a}\left(\frac{v_{q10}}{v_{t0}}R_e - \frac{v_{d10}}{v_{t0}}\frac{\omega_{r0}}{\omega_b}X_e\right)$$
$$K129 = -\frac{K_a}{T_a}\left(\frac{v_{q10}}{v_{t0}}\frac{X_e}{\omega_b}i_{d1o} - \frac{v_{q0}}{v_{t0}}\frac{X_e}{\omega_b}i_{q10}\right)$$

$$\mathbf{N_k} = \begin{bmatrix} -\frac{1}{T_f} & 0 & \frac{K_f}{T_f^2} \\ \frac{K_a}{T_a} & -\frac{1}{T_a} & -\frac{K_a K_f}{T_a T_f} \\ 0 & \frac{1}{T_f} & -\frac{K_e + S_e(E_{fr})}{T_e} \end{bmatrix}$$

$$S_e(E_{fr}) = -K_e + 0.0039 * \exp(1.555 * E_{fr0})/T_e$$

Equation (25) may be written in the standard form as:

(26) **pX = AX + BU**

where

A = $\mathbf{E^{-1}F}$ and **B** = $\mathbf{E^{-1}}$

Variables in rotor reference and synchronously rotating reference frame are related by

$$\begin{bmatrix} f_q \\ f_d \end{bmatrix} = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix}\begin{bmatrix} f_q^e \\ f_d^e \end{bmatrix}$$

Eigenvalues and machine parameters

The roots of characteristic matrix [**A**], referred to as characteristic roots, latent roots or eigenvalues, have been calculated using a computer program. As the state space model, given by equation (26), of six-phase synchronous generator with excitation system is described by thirteen states variables, thirteen eigenvalues were obtained. Out of these thirteen eigenvalues, three are in complex conjugate pairs and remaining are real negative numbers. Eigenvalue may be a real number or complex conjugate pair. A positive real eigenvalue corresponds to periodic instability, negative real value indicates forced back to the operating point, zero means stays at the position of operating point, complex number with positive real part represents oscillatory mode with increasing amplitude, complex number with negative real part means oscillations with decaying amplitude about the fixed operating point and only imaginary values represents oscillations with constant amplitude around the fixed operating point.

For the purpose of validation of the developed model, the proposed approach was applied on two separate six-phase generators, and small signal stability analysis carried out. Similar behavior in both the cases was observed. Whereas the results presented in this paper are of prototype laboratory test machine-2 only. The parameters of the two machines and IEEE type-1 excitation system are given in Appendix. Different eigenvalues have been calculated by varying the generator parameters like resistance /reactance of stator, rotor and inertia constant of the machine about the quiescent operating point when (i) both the winding were subjected to full load at 0.8 lagging power factor and (ii) only one winding set is subjected to full load at 0.8 lagging power factor. This provides an association of the machine parameters and eigenvalues. The state space model given by equation (26) can be easily modified for a six-phase synchronous generator without excitation system by eliminating last three rows and columns of the characteristic matrix [A], accordingly the model is reduced to tenth order. The eigenvalue variation has also been determined with and without excitation system. Some results are tabulated in Table 1 & 2, and are plotted in Fig. 3(a-d) after normalizing the tabulated values with the corresponding nominal machine parameter (marked with *).

The normalized values are defined as

$$\%NV = \frac{EVCP}{EVRP} * 100$$

where

% NV = Percentage normalized value
EVCP = Eigenvalue with change in parameter
EVRP = Eigenvalue with reference (rated) parameter

Here, the complex conjugate pairs affected by change in stator parameter are called stator eigenvalue-I and stator eigenvalue-II as shown in Table 2. The real part of the stator eigenvalues becomes more negative with the increase in stator resistance or with the reduction in stator leakage reactance; this suggests lower time constant and higher system stability. It is also evident from the ratio comparison shown in Table 1, that the ratio of absolute real stator eigenvalue-I to imaginary stator eigenvalue-I follow the same trend as that of the ratio of stator resistance to stator leakage reactance and higher value of this ratio further suggests higher rate of decay and enhanced machine stability.

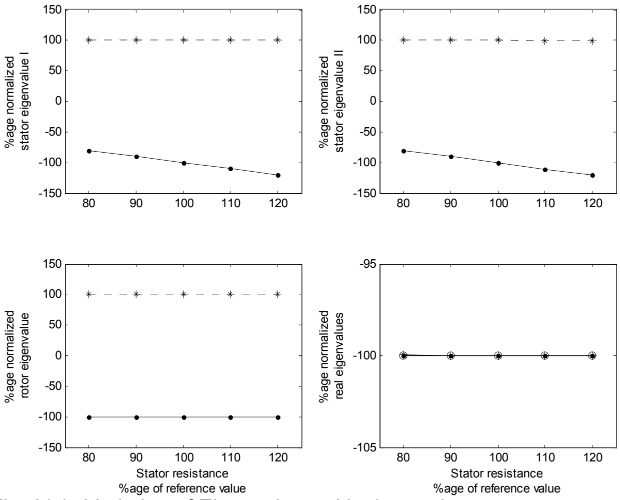

Fig. 3(a) Variation of Eigenvalues with change in stator resistance

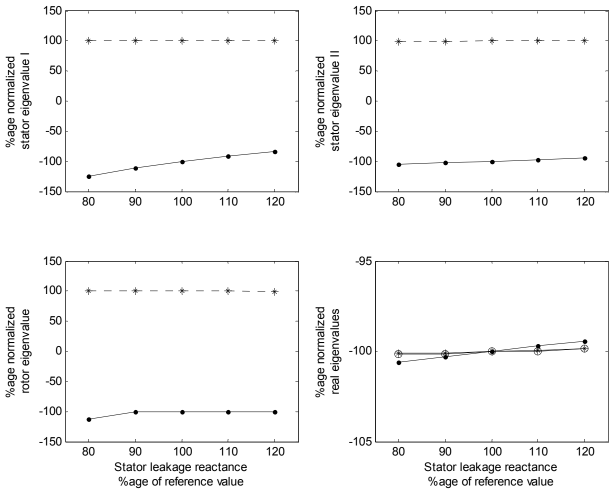

Fig. 3(b) Variation of Eigenvalues with change in stator leakage reactance

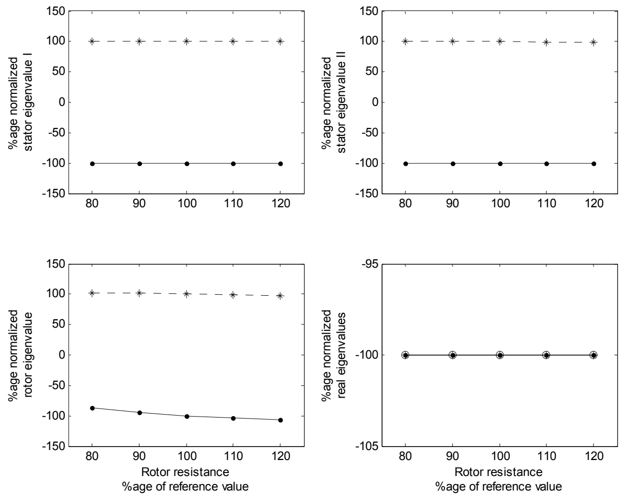

Fig. 3(c) Variation of Eigenvalues with change in rotor resistance only

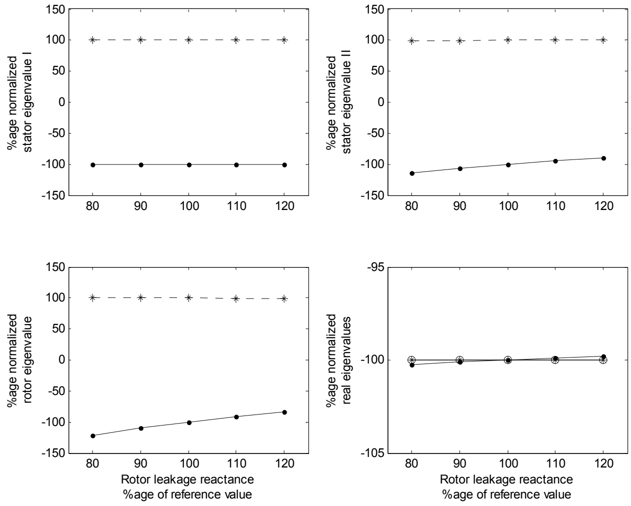

Fig. 3(d) Variation of Eigenvalues with change in rotor leakage reactance only

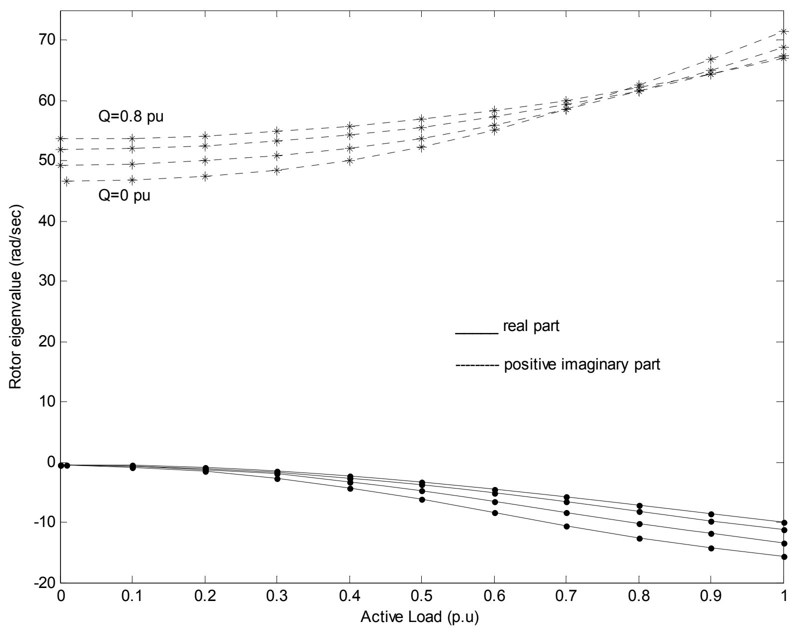

Fig. 4 Variation of rotor eigenvalues with change in reactive load on the generator with excitation system

Table 1. Ratio comparison of ratio of stator resistance to stator reactance with the ratio of |real eigenvalue-I| to |Imaginary eigenvalue-I| without excitation system

Stator Resistance, 'r_s'(p.u)	Stator leakage Reactance, 'X_{ls}'(p.u)	Ratio, r_s/X_{ls}	Stator Eigenvalue-I	Ratio, |real| /|Imag|
0.0108*	0.009*	1.2	-375±j314	1.19420
0.0097	0.0081	1.197	-375±j314	1.19420
0.0119	0.0099	1.2	-375±j314	1.19420
0.0119	0.009*	1.322	-394±j314	1.25477
0.0097	0.009*	1.077	-357±j314	1.13690
0.0108*	0.0099	1.0909	-358±j314	1.14030
0.0108*	0.0081	1.333	-395±j314	1.25796

Table 2. Variation of eigenvalues with the change in various generator parameters without excitation system when both the winding set are at full load 0.8 power factor lagging

Equal change in stator resistance 'r_s' of both winding sets (p.u)	Stator Eigenvalue-I (rad/s)	Stator Eigenvalue-II (rad/s)	Rotor Eigenvalue (rad/s)	Real Eigenvalue (rad/s)
0.0086	-300±j314	-45±j304	-8±j62	-27408, -2395, -602, -39
0.0097	-338±j314	-50±j303	-8±j62	-27409, -2396, -602, -39
0.0108*	-375±j314	-56±j301	-8±j62	-27411, -2396, -602, -39
0.0119	-413±j314	-62±j299	-8±j62	-27412, -2396, -602, -40
0.0130	-450±j314	-67±j297	-8±j62	-27413, -2396, -602, -40
Equal change in stator leakage reactance of both winding sets, 'X_{ls}' (p.u)				
0.0072	-469±j314	-59±j299	-9±j62	-27579, -2399, -603, -42
0.0081	-417±j314	-57±j300	-8±j62	-27492, -2398, -603, -40
0.009*	-375±j314	-56±j301	-8±j62	-27411, -2396, -602, -39
0.0099	-341±j314	-55±j302	-8±j62	-27334, -2395, -602, -38
0.0108	-313±j314	-53±j302	-8±j61	-27262, -2393, -601, -37
Change in rotor resistance, 'r_{fd}' (p.u)				
0.00128	-375±j314	-56±j303	-7.13±j63.31	-27410, -2396, -602, -30
0.00144	-375±j314	-56±j302	-7.73±j62.52	-27410, -2396, -602, -34
0.00160*	-375±j314	-56±j301	-8.19±j61.69	-27411, -2396, -602, -39
0.00176	-375±j314	-56±j300	-8.52±j60.84	-27411, -2396, -602, -45
0.00192	-375±j314	-55±j299	-8.72±j59.99	-27411, -2396, -602, -50
Change in rotor leakage reactance, 'X_{lfd}' (p.u)				
0.19217	-375±j314	-56±j296	-9.97±j62.09	-27477, -2396, -602, -46
0.21619	-375±j314	-56±j299	-9.01±j61.91	-27441, -2396, -602, -42
0.24021*	-375±j314	-56±j301	-8.19±j61.69	-27411, -2396, -602, -39
0.26423	-375±j314	-56±j303	-7.48±j61.44	-27384, -2396, -602, -37
0.28825	-375±j314	-55±j304	-6.87±j61.18	-27361, -2396, -602, -35
Change in inertia constant, 'J' (Kg/m^2)				
0.4224	-375±j314	-56±j301	-8.65±j70.09	-27411, -2396, -602, -38
0.4752	-375±j314	-56±j301	-8.43±j65.54	-27411, -2396, -602, -39
0.5280*	-375±j314	-56±j301	-8.19±j61.69	-27411, -2396, -602, -39
0.5808	-375±j314	-56±j301	-7.96±j58.39	-27411, -2396, -602, -40
0.6864	-375±j314	-56±j301	-7.72±j55.51	-27411, -2396, -602, -40

With the change in rotor parameters and inertia constant, a change in the real eigenvalues of one complex pair has been observed. Here, this complex conjugate pair is called as rotor eigenvalue. Also, Table 2 indicates that the real part of the rotor eigenvalues is more negative with the increase in rotor resistance and one of the real eigenvalue becomes less negative with the decrease in rotor reactance thus suggests more system stability. However with the increase in stator/rotor resistance, machine losses will increase and with the decrease in stator/rotor reactance, short circuit fault current of the machine will increase. Thus, at design level a careful selection of machine parameters is required. It is worth to mention here that the change in stator parameters has negligible effect on the rotor eigenvalues. The decrease in the magnitude of imaginary part of the rotor eigenvalues with the increase in inertia constant illustrates the less oscillatory nature of the rotor system.

Effect of machine loading

A correlation between the active load for a fixed reactive load and the rotor eigenvalues is depicted in Table 3. It illustrates that the rotor eigenvalues becomes positive at a load of 1.76 per unit, means unstable behavior of the machine. This value is very near to the experimental result (173 %) given in [5] which further validate the proposed model. The effect on the eigenvalues was also studied by varying the active power for different values of reactive power. It is evident that the change in stator and real eigenvalues is not very much significant; however, the variation of rotor eigenvalues is plotted in Fig. 4.

The real power was varied from 0 to 1.0 per unit for reactive power of 0, 0.3, 0.6 and 0.8 per unit respectively. There is a slight increase in positive magnitude of imaginary rotor eigenvalue with the increase in active power for a given reactive power output. It illustrates higher oscillation frequency of the rotor system with the increase in active and reactive power output. The linearized model is further utilized to determine the transfer function of ΔQgen/ΔEf at a given operating point where both the winding sets (i.e. '*abc*' and '*xyz*') have been fully loaded at 0.8 power factor lagging. The root locus plot of ΔQgen/ΔEfr at the specified operating point is shown in Fig. 5. The root locus plot also suggests the machine is stable, as all the poles are lying in the negative half of the x-axis.

Table 3. Variation of rotor eigenvalues (other eigenvalues remains unchanged) with the change in equal load on both winding sets without excitation system

Total load on both winding sets (pu)	Rotor Eigenvalue (rad/s)
1.6	-0.117±j66.74
1.7	-0.044±j65.17
1.74	-0.013±j65.74
1.76	0.0013±j66.02
1.8	0.032±j66.59
2.0	0.19±j69.42

Participation factor

Participation factor is an excellent tool to identify the state variables that have significant participation in a selected mode. It is obvious, that the significant state variables for an eigenvalue are those that correspond to large values in eigenvector. But the elements of eigenvector are dependent on dimensions of state variables. Verghese et al. [19] have suggested a related but dimensionless measure of state variables participation called participation factor. It helps in the identification of how each dynamic variable effects a given mode or eigenvalue in a linear system. A participation factor is a sensitivity measure of an eigenvalue to a diagonal entry of the system matrix **[A]**. This is defined as [20];

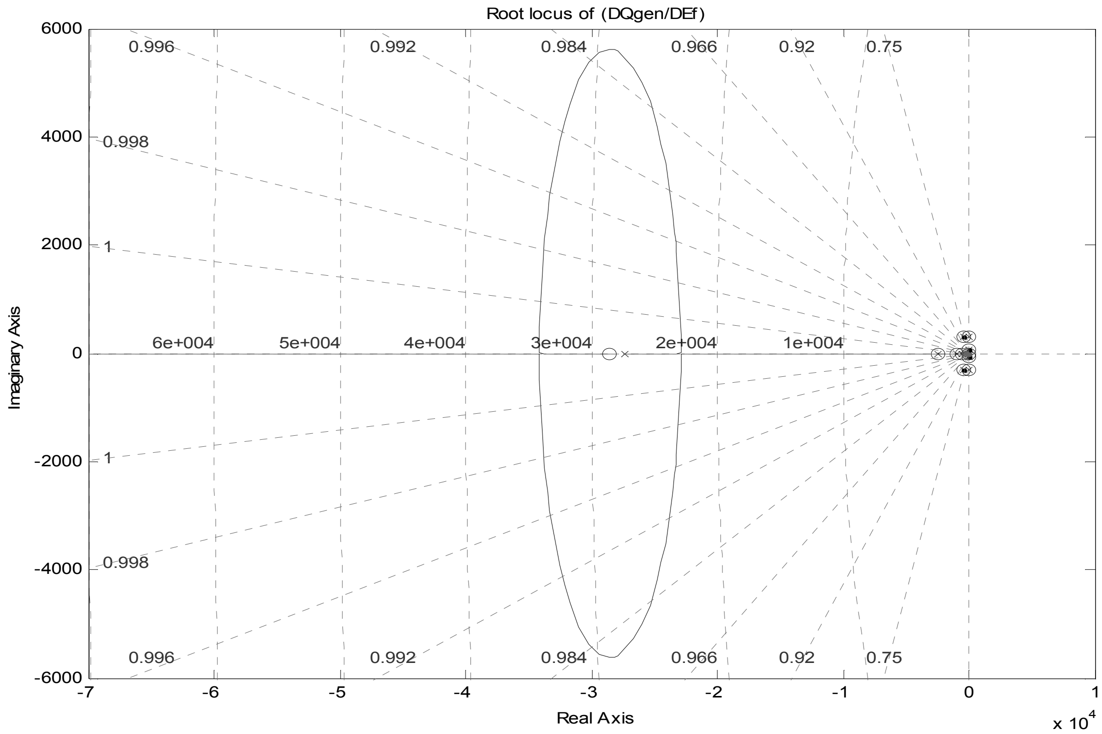

Fig. 5 Root locus plot of ΔQgen/ΔEf with excitation system

$$p_{ki} = \frac{\partial \lambda_i}{\partial a_{kk}}$$

where, λ_i is the ith system eigenvalue, a_{kk} is a diagonal entry in the system matrix **[A]**.

Table 4. Eigenvalues and their Participation factors

Mode	State Variable	Participation Factor
1	i_{kq1}	0.9952
2	i_{kd}	0.9998
3	ω_r	0.9932
4	i_{q1}, i_{d1}, i_{q2}	0.1261, 0.5486, 0.2485
5	i_{q1}, i_{d1}, i_{q2}, i_{d2}	0.25, 0.25, 0.25, 0.25
6	i_{d1}, i_{d2}	0.4623, 04623
7	i_{d1}, i_{d2}, i_{fr}	0.2467, 0.2467, 0.4723
8	i_{d1}, i_{d2}, i_{fr}, δ	0.1764, 0.1764, 0.3688, 0.1329
9	i_{d1}, i_{d2}, i_{fr}	0.2001, 0.2001, 0.4367
10	i_{q1}, i_{q2}, i_{fr}	0.2344, 0.2344, 04827
11	v_R, E_{fr}	0.5, 0.495
12	i_{q1}, i_{q2}, i_{fr}, v_f, E_{fr}	0.1341, 0.1341, 0.2753, 0.2317, 0.2125
13	v_f, E_{fr}	0.4795, 0.5

The participation factors associated with various eigenvalues are given in table 4. Only the participation factors whose value is greater than 0.1 are listed here.

Conclusions

The present work addresses the small signal perturbation stability characteristics of an asymmetrical six-phase synchronous generator connected to a constant voltage and frequency bus by means of eigenvalue approach. This work explores the correlation between generator /loading parameters with pair of eigenvalues about the given quiescent point of operation. From the results presented in this work, it is evident that two complex conjugate pair of eigenvalues corresponds to two three-phase stator winding sets of a six-phase machine, and are nearly unaffected by change in rotor electrical or mechanical parameters. One complex pair affected by variation of rotor parameters and inertia constant is related to rotor eigenvalue of machine. The investigation shows that there is an enhancement in the machine stability limit as stator/rotor resistance increases or decrease in leakage reactance. The increase in active power delivered by the machine for the given reactive power influences mainly the rotor eigenvalues, which indicates higher oscillation frequency of the rotor system with higher rate of decay. Through determination of participation factor, association of various state variables and eigenvalues has been correctly predicted. The obtained results provide basis for design and control circuitry of six-phase synchronous generator.

Appendix

Parameters of machine-I [7]:
$r_1 = r_2 = 0.21$, $X_{l1} = X_{l2} = 0.1758$, $X_{md} = 6.17$, $X_{mq} = 3.9$, $r_{Kq1} = 5.07$, $X_{Kq1} = 0.66097$, $r_{Kq2} = 1.06$, $X_{Kq2} = 0.5495$, $r_{fd} = 0.00160$, $X_{fd} = 0.24021$, $X_{Kd} = 1.54959$, , $X_{lm} = 0$, $X_{ldq} = 0$ (for 30° displacement) all values are in ohms, $J = 0.528$ Kg-m^2

Parameters of machine-II [17]:
$r_1 = r_2 = 0.0166$, $X_{l1} = X_{l2} = 0.0558$, $X_{md} = 1.128$, $X_{mq} = 0.521$, $r_{Kq} = 0.0025$, $r_{Kd} = 0.00237$, $X_{Kq} = 0.0676$, $X_{Kd} = 0.0529$, $r_{fd} = 0.0016$, $X_{fd} = 0.04684$, $X_{lm} = 0$, $X_{ldq} = 0$ (for 30° displacement) all values are in ohms, $J = 5.728$ Kg-m^2

Excitation system parameters:
$K_a = 300$, $K_e = 1$, $K_f = 0.001$, $T_a = 0.001$, $T_e = 1$ and $T_f = 0.1$

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