Electromagnetic scattering by a moving object

Abstract. Here the problem is analysed whether a scatterer moving in space with a constant velocity, generates the electromagnetic field distribution which is a moving map of the distribution obtained for the same scatterer being at rest. A perfectly conducting half-plane is chosen as a test scatterer. It is shown that the distributions are different and both geometrical optics and diffracted field are affected.

Problem formulation

The problem considered is 2D. In the laboratory frame of reference \( \{x, y, z\} \) the electromagnetic, harmonic in both space and time, plane wave

\[
E^i = E_0 e^{i(k \cdot r - ct)} \quad cB^i = cB_0 e^{i(k \cdot r - ct)}
\]

is propagating. The incident angle is \( \xi \). The corresponding wave number \( k \) and the radius vector are given by

\[
k = [\cos \xi, 0, \sin \xi], \quad r = [x, y, z].
\]

This wave is scattered by a perfectly conducting half-plane, which is moving along the \( x \) direction with the constant velocity \( v \) (see Fig. 1). This velocity may take values from zero to relativistic ones.

We also introduce the \textit{stationary} frame of reference \( \{x', y', z'\} \), wherein the half-plane is at rest. In this frame the half-plane is described by \( z' \leq 0, x' = 0 \). Our goal is to find the resulting EM field in the laboratory frame.

Solving procedure

It is convenient to make transformation between laboratory and stationary frames of reference, as the solution in the latter frame is already known. Thus,

- We represent the incident field in the laboratory frame of reference as a sum of fields of type E and type H with respect to the \( y \) axis. Since in 2D geometry those fields are disconnected, further analysis can be carried out for each independently [8].
- With the use of Lorentz transformation and the demand of equal phases in both frames of reference [9] we transform the incident fields to the laboratory frame.
- Given the incident field, we find total field in this frame.

Fig. 1. Geometry of the problem. The incident plane wave is propagating under the angle \( \xi \) with respect to the \( x \) axis. The half-plane is moving with the velocity \( v \) in the \( x \) direction.

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i.e. the solution of the scattering problem wherein the scatterer is at rest.

- We finally transform the total field to the laboratory frame.

This procedure was first used by Einstein [10].

**Lorentz transformation**

For the geometry considered here the Lorentz transformation formulas can be simplified to the following form:

- for the coordinates in both frames of reference:
  \[ x' = \gamma(x - \beta ct), \quad t' = \gamma(ct - \beta x), \]
  \[ y' = y, \quad z' = z, \]
  \[ \beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} \]

- for the electromagnetic fields:
  \[ E_{\|}' = E_{\|}, \quad cB_{\|}' = cB_{\|}, \]
  \[ E_{\perp}' = \gamma(E_{\perp} + \beta \times cB_{\perp}), \quad cB_{\perp}' = \gamma(cB_{\perp} - \beta \times E_{\perp}), \]

where the symbols \( \| \) \( \perp \) denote, respectively, the field components in the direction of the half-plane movement, and in the plane perpendicular to that direction. The vector \( \beta \) is defined as \( \beta = k\beta \), where \( k \) is a unit vector in the \( x \) direction.

**Diffraction in the stationary frame of reference**

We shall stick to the convention that the quantities in the laboratory frame are not primed and in the stationary frame are primed.

**Type E field** : The incident field in the stationary frame of reference takes the form

\[ E^i = \hat{y}A_x e^{i(k'r - ct')}, \quad cB^i = \frac{1}{ik'} \nabla' \times E^i, \]

and results from Lorentz transformation of the incident field

\[ E^i = \hat{y}A_x e^{i(k'r - ct)}, \quad cB^i = \frac{1}{ik'} \nabla \times E^i. \]

in the laboratory frame of reference. Here, \( A_x' = A_1\gamma(1 - \beta \cos \xi) \). Thus the plane wave transforms from the laboratory to the stationary frame of reference also as a plane wave, but with modified amplitude and the direction of propagation.

The total field resulting from diffraction of \( E'^i \) on the motionless half-plane was found by Sommerfeld, and it takes the form

\[ E' = \hat{y}[u(\rho', \varphi') - u(\rho', 2\pi - \varphi')], \]

where

\[ u(\rho', \varphi') = A_1 e^{i(k'\rho' - ct')} G(w'), \]

\( \hat{y} \) is the unit vector along the \( y' \) axis, and \( \rho', \varphi' \) are polar coordinates in the stationary frame of reference. The function \( G(a) \) is expressed via the Fresnel integral

\[ G(a) = e^{-ia^2} \int_a^\infty e^{-\tau^2} d\tau \]

and

\[ w' = \sqrt{2k'\rho' \sin \frac{\varphi' - \xi'}{2}}. \]

The angle \( \xi' \) is related to \( \xi \) through

\[ \cos \xi' = \frac{\cos \xi - \beta}{1 - \beta \cos \xi}, \quad \sin \xi' = \frac{\sin \xi}{\gamma(1 - \beta \cos \xi)}. \]

**Type H field**:

\[ cB^i = \hat{y}A_x e^{i(k'r - ct')}, \quad E^i = -\frac{1}{ik'} \nabla' \times cB^i, \]

and the total field is equal

\[ cB^i = \hat{y}[u(\rho', \varphi') + u(\rho', 2\pi - \varphi')]. \]

Since the angular coordinate \( \varphi' \) in the second term in square brackets in (7) and (13) is simply replaced by \( 2\pi - \varphi' \), it is just enough to examine the transform of \( u(\rho', \varphi') \) to the laboratory frame of reference.

**Exact solution in the laboratory frame of reference**

The Lorentz transformation of the total field from the stationary to the laboratory frame of reference yields

\[ E = \hat{y} \gamma \left( E'_{\|} + \frac{\beta}{ik'} \frac{\partial E'_{\rho'}}{\partial x'} \right) \]

\( a^1 \)

\[ E = \hat{y} \gamma B_1 e^{-i\pi/4} / \sqrt{\pi}. \]

**Asymptotic solution in the laboratory frame of reference**

The representation of the total field in (15) does not give us a possibility to interpret it physically. However, such a possibility appears if we expand it asymptotically. It is enough to expand the function \( G(w') \) for large values of \( w' \):

\[ G(a) = H(-a)\sqrt{\pi} e^{i\pi/4} e^{-ia^2} + \frac{i}{2a} + O(a^{-2}) \]

\( a \to \infty, \)

where \( H(\cdot) \) is a Heaviside (unit step) function. Our restriction to large values of \( w' \) is equivalent to the assumption, that the asymptotic field will be valid sufficiently away from the edge \( \rho' = 0 \) of the screen, and from the shadow boundary \( \varphi' = \xi' \) of the incident wave. The asymptotic solution takes the following form:

\[ E \sim \hat{y} \gamma B_1 e^{i(k'r - ct')} \left[ H(-w') \sqrt{\pi} e^{i\pi/4} e^{ik'r - ik'\rho'} (1 + \beta \cos \xi') \right] + \frac{i}{2w'} (1 + \beta \cos \varphi') \]

This asymptotic representation of the field in the laboratory frame is for brevity described in coordinates specific to the stationary frame. The coordinates in both frames are related through (3).

The first term in (16), multiplied by the common factor, describes the geometric optics contribution to the total field:

\[ E^i = H(-w') \hat{y}A_1 e^{i(k'r - ct)}. \]

(Here, we took advantage of the fact, that plane wave phases are equal in both frames of reference.) Equation (18) describes the incident wave, non-vanishing in its illuminated region. The shadow boundary, separating the illuminated and
The asymptotic representation of the propagation of incident wave rays. Similar conclusion applies to the shadow boundary, is defined by the equation \( \phi' = \xi' \). Unlike the stationary frame of reference, the shadow boundary in the laboratory frame is not parallel to the direction of propagation of incident wave rays. Similar conclusion applies to the asymptotic representation of the solution (in both stationary and laboratory frames) and do not appear in the exact solution. The reason is that the assumption of large \( \alpha \) in (16) is not satisfied in vicinities of the shadow boundaries.

In addition to the fact that unlike the stationary frame the shadow boundaries in the laboratory frame are not parallel to the rays of incident and reflected wave, the radiation pattern of the diffracted wave in the laboratory frame is also modified by the inclusion in (19) of the term proportional to \( \beta \).

**Conclusions**

The problem of plane wave scattering by a half-plane in motion was used to demonstrate that the scattered field distribution in case of the moving object is not a moving map of the corresponding field distribution obtained for the object at rest. The differences appear in both the geometrical optics component and in the edge diffracted component. They affect the illuminated and shadow regions of the geometrical optics field and the radiation pattern of the diffracted field. The differences increase with growing values of the velocity of the object, and are clearly noticed at relativistic velocities.

**BIBLIOGRAPHY**


**Author:** Ph.D. Adam Ciarkowski, Faculty of Applied Informatics and Mathematics, Warsaw University of Life Sciences, ul. Nowoursynowska 159, 02-776 Warszawa, Poland, email: adam_ciarkowski@sggw.pl