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Constructability and observability of standard and positive electrical circuits

***Abstract**. The constructability and observability of standard and positive electrical circuits composed of resistors, coils, condensators and voltage (current) sources are addressed. Necessary and sufficient conditions for the constructability and observability of the positive electrical circuits are established. Effectiveness of the conditions is demonstrated on examples of positive electrical circuits*

Streszczenie.** W pracy rozpatruje się standardowe i dodatnie obwody elektryczne złożone z rezystorów, cewek i kondensatorów oraz źródeł napięcia i prądu. Podano warunki konieczne i wystarczające odtwarzalności i obserwowalności tych obwodów elektrycznych. Warunki te zostały zlustrowane na przykładach dodatnich obwodów elektrycznych.* ***Odtwarzalność i obserwowalność standardowych i dodatnich obwodów elektrycznych

Keywords: controllability, constructability, reachability, positivity, electrical linear circuit.
Słowa kluczowe: sterowalność, odtwarzalność, osiągalność, dodatniość, obwód elektryczny.

Introduction

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs [1, 2]. Variety of models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc..

The notion of controllability and observability and the decomposition of linear systems have been introduced by Kalman [3, 4]. These notions are the basic concepts of the modern control theory [5-9]. They have been also extended to positive linear systems [1, 2]. The decomposition of the pair (A,B) and (A,C) of the positive discrete-time linear system has been addressed in [10]. The positive circuits and their reachability has been investigated in [11] and controllability and observability of electrical circuits in [12].

The reachability of linear systems is closely related to the controllability of the systems. Specially for positive linear systems the conditions for the controllability are much stronger than for the reachability [2]. Tests for the reachability and controllability of standard and positive linear systems are given in [2, 13]. Positivity and reachability of fractional electrical circuits have been addressed in [14, 11]. The finite zeros of positive discrete-time and continuous-time linear systems has been investigated in [16-18] and the decoupling zeros of positive discrete-time linear systems in [18].

In this paper conditions for the constructability and observability of positive electrical circuits will be established.

The paper is organized as follows. In section 2 some classes of positive electrical circuits will be given. Reachability of standard and positive electrical circuits will be analyzed in section 3. The main result of the paper is presented in section 4, where necessary and sufficient conditions for the constructability and observability of the positive electrical circuits are established. Concluding remarks are given in section 5.

The following notation will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ - the set of $n\times m$ matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, M_n - the set of $n\times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), I_n - the $n\times n$ identity matrix.

Positive electrical circuits

Positive *R*, *L*, *e* electrical circuits

Consider the linear continuous-time electrical circuit described by the state equations

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t) + Du(t)\end{aligned} \tag{1}$$

where $x(t)\in\Re^n$, $u(t)\in\Re^m$, $y(t)\in\Re^p$ are the state, input and output vectors and $A\in\Re^{n\times n}$, $B\in\Re^{n\times m}$, $C\in\Re^{p\times n}$, $D\in\Re^{p\times m}$.

Definition 1. [1, 2] The electrical circuit (1) is called (internally) positive if $x(t)\in\Re_+^n$ and $y(t)\in\Re_+^p,\ t\ge 0$ for any $x(0)=x_0\in\Re_+^n$ and every $u(t)\in\Re_+^m,\ t\ge 0$.

Theorem 1. [1, 2] The electrical circuit (1) is positive if and only if

$$A\in M_n,\quad B\in\Re_+^{n\times m},\quad C\in\Re_+^{p\times n},\quad D\in\Re_+^{p\times m} \tag{2}$$

It is well-known [5-9] that any linear electrical circuit composed of resistors, coils, condensators and voltage (current) sources can be described by the state equations (1). Usually as the state variables $x_1(t),...,x_n(t)$ (the components of the state vector $x(t)$) the currents in the coils and voltages on the condensators are chosen.

Theorem 2. The linear electrical circuit composed of resistors, coils and voltage sources is positive for any values of the resistances, inductances and source voltages if the number of coils is less or equal to the number of its linearly independent meshes and the direction of the mesh currents are consistent with the directions of the mesh source voltages.
Proof is given in [11].

Positive R, C, e electrical circuit

Theorem 3. The linear electrical circuit is not positive for almost all values of its resistances, capacitances and source voltages if each its branch contains resistor, condensator and voltage source .
Proof is given in [11].

Theorem 4. The electrical circuit shown in Figure 1 is positive for any values of the conductances

G_k, $k = 0,1,...,n$; capacitances C_j, $j = 1,...,n$ and source voltage e.

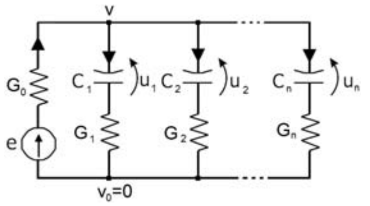

Fig. 1. Electrical circuit

Proof is given in [11].

Positive R, L, C, e electrical circuits

Theorem 5. The R, L, C, e electrical circuits are not positive for any values of its resistances, inductances, capacitances and source voltages if at least one its branch contains coil and condensator.

Proof is given in [11].

Theorem 6. The linear electrical circuit of the structure shown in Figure 2 for $n_1 = n_2 = 4$ is positive for any values of its resistances R_k, $k = 1,2,\ldots,n$ inductances L_k, $k = 2,4,\ldots,n_2$ and capacitances C_k, $k = 1,3,\ldots,n_1$.

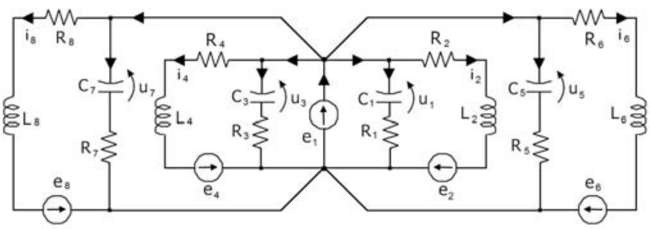

Fig. 2. Electrical circuit

Proof is given in [11].

Reachability of standard and positive electrical circuits

Consider the positive electrical circuit described by the equations (1).

Definition 2. The electrical circuit described by the equations (1) (or the pair (A,B)) is called reachable in time t_f if for any given final state $x_f \in \Re^n$ there exists an input $u(t) \in \Re^m$, for $t \in [0, t_f]$ that steers the state of the circuit from zero initial state $x(0) = x_0 = 0$ to the state x_f, i.e. $x(t_f) = x_f$.

Theorem 7. The electrical circuit (1) (or the pair (A,B)) is reachable if and only if one of the following conditions is satisfied:

i) $\mathrm{rank}[B \quad AB \quad ... \quad A^{n-1}B] = n$,

ii) $\mathrm{rank}[I_n s - A \quad B] = n$ for $s \in \delta_A = \{s_1,...,s_n\}$

where $s_1,...,s_n$ are the eigenvalues (not necessary distinct) of the matrix A.

Proof is given in [4-6, 9].

Definition 3. The positive electrical circuit (1) (or the positive pair (A,B)) is called reachable in time t_f if for any given final state $x_f \in \Re_+^n$ there exists an input $u(t) \in \Re_+^m$, for $t \in [0, t_f]$ that steers the state of the circuit from zero initial state $x(0) = 0$ to the state x_f, i.e. $x(t_f) = x_f$.

A real square matrix is called monomial if each its row and each its column contains only one positive entry and the remaining entries are zero.

Theorem 8. The positive electrical circuit (1) is reachable if the matrix

(3) $$R_f = \int_0^{t_f} e^{A\tau} BB^T e^{A^T \tau} d\tau, \; t_f > 0$$

is monomial. The input that steers the state of the electrical circuit in time t_f from $x(0) = x_0 = 0$ to the state x_f is given by the formula

(4) $$u(t) = B^T e^{A^T (t_f - t)} R_f^{-1} x_f \text{ for } t \in [0, t_f].$$

The proof is given in [2].

Consider the n-mesh electrical circuit with given resistances R_k, $k = 1,...,q$, inductances L_i, $i = 1,...,n$ and m-mesh source voltages e_{jj}, $j = 1,...,m$. It is assumed that to each linearly independent mesh belongs only one inductance [11]. By Theorem 7 the standard electrical circuit is reachable since $\det B \neq 0$.

Theorem 9. The positive n-meshes electrical circuit with only one inductance in each linearly independent mesh is reachable if

(5) $$R_{ij} = 0 \text{ for } i \neq j, \; i, j = 1,...,n$$

where R_{ij} is the resistance of the branch belonging to two linearly independent meshes.

Proof is given in [11].

Consider the electrical circuit shown in Figure 3 with given conductances G_k, G'_k, G_{kj} $k, j = 1,...,n$; capacitances C_k, $k = 1,...,n$ and source voltages e_k, $k = 1,...,n$.

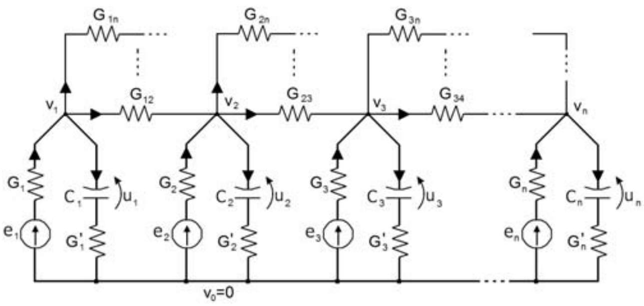

Fig. 3. Electrical circuit.

The electrical circuit shown in Figure 3 is positive for all values of the conductances, capacitances and source voltages [11].

Theorem 10. The electrical circuit shown in Figure 3 is reachable if and only if

(6) $$G_{k,j} = 0 \text{ for } k \neq j \text{ and } k, j = 1,...n.$$

Proof is given in [11].

Constructability and observability of positive electrical circuits

Consider a positive electrical circuit described by the state equations

(7a) $$\dot{x}(t) = Ax(t)$$

(7b) $$y(t) = Cx(t)$$

where $x(t) \in \Re_+^n$, $y(t) \in \Re_+^p$ and $A \in M_n$, $C \in \Re_+^{p\times n}$.

Definition 4. The positive electrical circuit (7) is called constructible if knowing the output $y(t) \in \Re_+^p$ and its derivatives $y^{(k)}(t) = \frac{d^k y(t)}{dt^k} \in \Re_+^p$, $k = 1,2,\dots,n-1$, it is possible to find the state vector $x(t) \in \Re_+^n$.

Theorem 11. The positive electrical circuit (7) is constructible if and only if the matrix

$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{8}$$

has n linearly independent monomial rows.

Proof. From (7) we have

$$\begin{bmatrix} y(t) \\ \dot{y}(t) \\ \vdots \\ y^{(n-1)}(t) \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} x(t). \tag{9}$$

It is possible to find from (9) the state vector $x(t) \in \Re_+^n$ for given $y^{(k)}(t) \in \Re_+^p$, $k = 1,2,\dots,n-1$ if and only if the matrix (8) has n linearly independent monomial rows since the inverse matrix has nonnegative entries if and only if the matrix is monomial [2]. □

Theorem 12. The positive electrical circuit (7) is constructible only if the matrix $\begin{bmatrix} C \\ A \end{bmatrix}$ has n linearly independent monomial rows.

Proof. It is easy to show that the matrix (8) has n linearly independent rows only if the matrix $\begin{bmatrix} C \\ A \end{bmatrix}$ has n linearly independent rows. □

Definition 5. The positive electrical circuit (7) is called observable if knowing the output $y(t) \in \Re_+^p$ and its derivatives $y^{(k)}(t) = \frac{d^k y(t)}{dt^k} \in \Re_+^p$, $k = 1,2,\dots,n-1$ it is possible to find the initial values $x_0 = x(0) \in \Re_+^n$ of $x(t) \in \Re_+^n$.

Theorem 13. The positive electrical circuit (7) is observable if and only if the matrix $A \in M_n$ is diagonal and the matrix (8) has n linearly independent rows.

Proof. Substituting of the solution

$$x(t) = e^{At}x_0 \tag{10}$$

of the equation (7a) into (7b) yields

$$y(t) = Ce^{At}x_0. \tag{11}$$

From (11) we have

$$\begin{bmatrix} y(t) \\ \dot{y}(t) \\ \vdots \\ y^{(n-1)}(t) \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} e^{At}x_0. \tag{12}$$

It is possible to find from (12) $e^{At}x_0 \in \Re_+^n$ if and only if the matrix (8) has n linearly independent monomial rows. From the equality $e^{At}e^{-At} = I_n$ it follows that the matrix $e^{At} \in \Re_+^{n\times n}$ for $A \in M_n$ if and only if it is diagonal. Therefore, it is possible to find $x_0 \in \Re_+^n$ from the equation (12) if and only if the matrix $A \in M_n$ is diagonal and the matrix (8) has n linearly independent rows. □

Remark 1. From comparison of Theorem 11 and 13 it follows that the necessary and sufficient conditions for the observability are more restrictive then than for the constructability.

Theorem 14. If the positive electrical circuit (7) is observable then it is also constructible.

Proof follows immediately from comparison of the conditions of Theorems 11 and 13.

Theorem 15. The positive electrical circuit (7) is observable if the matrix

$$O_p = e^{A^T t}C^T Ce^{At} \tag{13}$$

is monomial.

Proof. Premultiplying (11) by $e^{A^T t}C^T$ we obtain

$$e^{A^T t}C^T Ce^{At}x_0 = e^{A^T t}C^T y(t). \tag{14}$$

If the matrix (13) is monomial then $O_p^{-1} = [e^{A^T t}C^T Ce^{At}]^{-1} \in \Re_+^{n\times n}$ and from (14) we have

$$x_0 = [e^{A^T t}C^T Ce^{At}]^{-1} e^{A^T t}C^T y(t) \in \Re_+^n \tag{15}$$

since $e^{A^T t}C^T y(t) \in \Re_+^n$ for $y(t) \in \Re_+^p$.

Note that the matrix (13) can be monomial only if the matrix C is monomial.

Remark 2. Note that Theorem 15 is an analog for the observability of Theorem 8 for reachability.

Theorem 16. Every standard linear electrical circuit is constructible (observable) if the corresponding positive linear electrical circuit is constructible (observable).

Proof follows immediately from comparison of the conditions for constructability and observability of standard and positive electrical circuits.

Example 1. Consider the electrical circuit shown in Figure 4 with given resistances R_1, R_2, R_3, inductances L_1, L_2 and source voltages e_1, e_2.

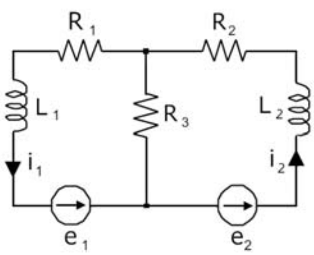

Fig. 4.1. Electrical circuit.

Using the Kirchhoff's laws we can write the equations

$$e_1 = R_3(i_1 - i_2) + R_1 i_1 + L_1 \frac{di_1}{dt}$$
$$\text{(16)} \qquad e_2 = R_3(i_2 - i_1) + R_2 i_2 + L_2 \frac{di_2}{dt}$$

which can be written in the form

$$\text{(17a)} \qquad \frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = A\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + B\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

where

$$\text{(17b)} \qquad A = \begin{bmatrix} -\frac{R_1 + R_3}{L_1} & \frac{R_3}{L_1} \\ \frac{R_3}{L_2} & -\frac{R_2 + R_3}{L_2} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \end{bmatrix}.$$

The electrical circuit is positive since the matrix A is Metzler matrix and the matrix B has nonnegative entries. Note that by Theorem 12 the standard pair (17b) is reachable since $\det B \neq 0$.

We shall show that the positive electrical circuit is reachable if $R_3 = 0$. In this case

$$\text{(18)} \qquad A = \begin{bmatrix} -\frac{R_1}{L_1} & 0 \\ 0 & -\frac{R_2}{L_2} \end{bmatrix}$$

and

$$\text{(19)} \qquad e^{A\tau} = \begin{bmatrix} e^{-\frac{R_1}{L_1}\tau} & 0 \\ 0 & e^{-\frac{R_2}{L_2}\tau} \end{bmatrix}$$

from (3) we obtain

$$\text{(20)} \quad R_f = \int_0^{t_f} e^{A\tau} B B^T e^{A^T \tau} d\tau = \int_0^{t_f} \begin{bmatrix} \frac{1}{L_1^2} e^{-\frac{2R_1}{L_1}\tau} & 0 \\ 0 & \frac{1}{L_2^2} e^{-\frac{2R_2}{L_2}\tau} \end{bmatrix} d\tau .$$

The matrix (20) is monomial and by Theorem 8 the positive electrical circuit is reachable if $R_3 = 0$.

Let

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} R_1 i_1(t) \\ R_2 i_2(t) \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix},$$
$$\text{(21)} \qquad C = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}.$$

The positive circuit described by (17) and (21) is constructible for all nonzero values of R_1 and R_2 since

$$\text{(22)} \qquad \det C = \det\begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} = R_1 R_2 \neq 0 .$$

We shall show that the positive circuit is also observable. Using (13), (18) and (21) we obtain

$$O_p = e^{A^T t} C^T C e^{At}$$
$$\text{(23)} \qquad = \begin{bmatrix} e^{-\frac{R_1}{L_1}t} & 0 \\ 0 & e^{-\frac{R_2}{L_2}t} \end{bmatrix}\begin{bmatrix} R_1^2 & 0 \\ 0 & R_2^2 \end{bmatrix}\begin{bmatrix} e^{-\frac{R_1}{L_1}t} & 0 \\ 0 & e^{-\frac{R_2}{L_2}t} \end{bmatrix}$$
$$= \begin{bmatrix} R_1^2 e^{-\frac{2R_1}{L_1}t} & 0 \\ 0 & R_2^2 e^{-\frac{2R_2}{L_2}t} \end{bmatrix}.$$

Therefore, by Theorem 15 the positive circuit is also observable, since the matrix (23) is monomial.

Example 2. Consider the electrical circuit shown in Figure 5 with given conductances $G_1, G'_1, G_2, G'_2, G_{12}$, capacitances C_1, C_2 and source voltages e_1, e_2.

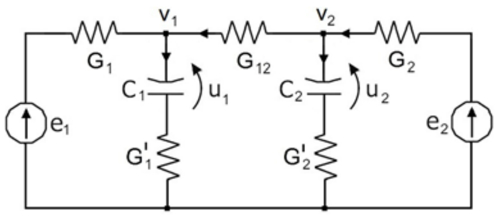

Fig. 4.2. Electrical circuit

Using the Kirchhoff's laws we can write the equations

$$\text{(24)} \qquad \frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{G'_1}{C_1} & 0 \\ 0 & \frac{G'_2}{C_2} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} \frac{G'_1}{C_1} & 0 \\ 0 & \frac{G'_2}{C_2} \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

and

$$\text{(25a)} \begin{bmatrix} -G_{11} & G_{12} \\ G_{12} & -G_{22} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} G'_1 & 0 \\ 0 & G'_2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} - \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

where

$$\text{(25b)} \qquad G_{11} = G_1 + G'_1 + G_{12}, \quad G_{22} = G_2 + G'_2 + G_{12}.$$

Taking into account that the matrix

$$\text{(26)} \qquad \begin{bmatrix} -G_{11} & G_{12} \\ G_{12} & -G_{22} \end{bmatrix}$$

is nonsingular and

$$\text{(27)} \qquad -\begin{bmatrix} -G_{11} & G_{12} \\ G_{12} & -G_{22} \end{bmatrix}^{-1} \in \Re_+^{2\times 2}$$

from (25) we obtain

$$\text{(28)} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -G_{11} & G_{12} \\ G_{12} & -G_{22} \end{bmatrix}^{-1}\left\{\begin{bmatrix} G'_1 & 0 \\ 0 & G'_2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} - \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}\right\}.$$

Substitution of (28) into (24) yields

$$\text{(29)} \qquad \frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + B\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

where

(30a)

$$A=-\begin{bmatrix}\frac{G_1}{C_1} & 0\\ 0 & \frac{G_2}{C_2}\end{bmatrix}\begin{bmatrix}-G_{11} & G_{12}\\ G_{12} & -G_{22}\end{bmatrix}^{-1}\begin{bmatrix}G_1 & 0\\ 0 & G_2\end{bmatrix}-\begin{bmatrix}\frac{G_1}{C_1} & 0\\ 0 & \frac{G_2}{C_2}\end{bmatrix}\in M_2,$$

(30b) $$B=\begin{bmatrix}\frac{G'_1}{C_1} & 0\\ 0 & \frac{G'_2}{C_2}\end{bmatrix}\begin{bmatrix}-G_{11} & G_{12}\\ G_{12} & -G_{22}\end{bmatrix}^{-1}\begin{bmatrix}G_1 & 0\\ 0 & G_2\end{bmatrix}\in\Re_+^{2\times 2}.$$

Therefore, the electrical circuit is positive since the matrix (30a) is a Metzler matrix and the matrix (30b) has nonnegative entries. Note that the standard pair (30) is reachable since the matrix (30b) is nonsingular for all nonzero conductances.

In this case for $G_{12}=0$, the matrices (30) are diagonal matrices

(31a)

$$A=-\begin{bmatrix}\frac{G'_1}{C_1} & 0\\ 0 & \frac{G'_2}{C_2}\end{bmatrix}\begin{bmatrix}\frac{1}{G_1+G'_1} & 0\\ 0 & \frac{1}{G_2+G'_2}\end{bmatrix}\begin{bmatrix}G'_1 & 0\\ 0 & G'_2\end{bmatrix}-\begin{bmatrix}\frac{G'_1}{C_1} & 0\\ 0 & \frac{G'_2}{C_2}\end{bmatrix}$$

$$=\begin{bmatrix}a_1 & 0\\ 0 & a_2\end{bmatrix}\in M_2$$

(31b)

$$B=\begin{bmatrix}\frac{G'_1}{C_1} & 0\\ 0 & \frac{G'_2}{C_2}\end{bmatrix}\begin{bmatrix}\frac{1}{G_1+G'_1} & 0\\ 0 & \frac{1}{G_2+G'_2}\end{bmatrix}\begin{bmatrix}G_1 & 0\\ 0 & G_2\end{bmatrix}$$

$$=\begin{bmatrix}b_1 & 0\\ 0 & b_2\end{bmatrix}\in\Re_+^{2\times 2}.$$

and

(32) $$e^{A\tau}=\begin{bmatrix}e^{a_1\tau} & 0\\ 0 & e^{a_2\tau}\end{bmatrix}.$$

Using (32) and (31) we obtain

(33) $$R_f=\int_0^{t_f}e^{A\tau}BB^Te^{A^T\tau}d\tau=\int_0^{t_f}\begin{bmatrix}b_1^2e^{2a_1\tau} & 0\\ 0 & b_2^2e^{2a_2\tau}\end{bmatrix}d\tau.$$

The matrix (33) is monomial and by Theorem 8 the positive electrical circuit is reachable if $G_{12}=0$.

Let

(34) $$y(t)=\begin{bmatrix}y_1(t)\\ y_2(t)\end{bmatrix}=\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix},\ C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

The electrical circuit described by (29) and (34) is positive. The positive circuit is constructible since $\det C=1$. It is also observable since by Theorem 15 the matrix

(35)

$$O_p=e^{A^Tt}C^TCe^{At}=\begin{bmatrix}e^{a_1t} & 0\\ 0 & e^{a_2t}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}e^{a_1t} & 0\\ 0 & e^{a_2t}\end{bmatrix}$$

$$=\begin{bmatrix}e^{2a_1t} & 0\\ 0 & e^{2a_2t}\end{bmatrix}$$

is monomial

Concluding remarks

Necessary and sufficient conditions for constructability and observability have been established for positive electrical circuits composed of resistors, coils, condensators and voltage (current) sources. It has been shown that the conditions for observability are more restrictive than for constructability of the positive electrical circuits. The considerations have been illustrated by two examples of positive electrical circuits. An open problem is and extension to positive electrical circuits the well-known Kalman decomposition Theorem [3, 4]. An other open problem is an extension of decoupling zeros introduced in [18] to discrete-time linear systems for positive linear electrical circuits and to fractional linear systems [19].

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