

The Application of the Exact State Estimation Method in Electric Power Systems

Abstract. A large share of distributed, renewable, intermittent energy resources resulted in the increased dynamics of the electricity grid and made it less predictable. Electric Power Systems (EPS), which used to be considered quasi-static systems of high order, with the presence of Distributed Energy Resources (DERs) at the distribution level, are constantly changing into active dynamic systems. This implies the application of the Dynamic State Estimation (DSE) which can estimate voltage and phase in real-time as well as can be used for the diagnostic purposes, hardware maintenance and control. In this paper Exact State Estimator for the EPS, sometimes referred also as Integral State Observer (ISO) is designed. Furthermore the comparison study with the classical Kalman-Bucy filter is made showing the advantages of the ISO over classical methods.

Streszczenie. Znaczący udział rozproszonych oraz odnawialnych źródeł energii o dostawach nieciągłych spowodował zmianę dynamiki sieci elektroenergetycznej, utrudniając przewidywalność dostaw energii. System elektroenergetyczny traktowany wcześniej jako quasi-statyczny system wysokiego rzędu, ewoluje obecnie w stronę systemu dynamicznego ze względu na obecność rozproszonych źródeł energii na poziomie dystrybucji. Implikuje to konieczność dynamicznej obserwacji stanu systemu do celów określenia wartości napięcia i fazy w czasie rzeczywistym oraz na potrzeby systemów diagnostyki oraz systemów sterowania. Poniższy artykuł prezentuje metodę Dokładnego Odtwarzania Stanu, znaną również pod nazwą metody Obserwatorów Całkowych. W artykule dokonano porównania metody dokładnego odtwarzania stanu z klasyczną metodą opartą o Filtr Kalmana-Bucy, wskazując zalety tej pierwszej. **Metoda dokładnego odtwarzania stanu dla systemów elektroenergetycznych**

Keywords: exact state observer, integral observer, electric power system, dynamic state estimator, Kalman filter

Słowa kluczowe: dokładny obserwator stanu, obserwator całkowy, system elektroenergetyczny, dynamiczny estymator stanu, filtr Kalmana

1 Introduction

The smart grid concept has driven lots of attention in recent years as hope for a change in the energy management in every area from generation level to end-point customers. Following the IEC Electropedia definition [15], the Smart Grid is "electric power system that utilizes information exchange and control technologies, distributed computing and associated sensors and actuators, for purposes such as:

- the integration of the behaviour and actions of the network users and other stakeholders,
- efficient delivery of sustainable, economic and secure electricity supplies."

Moreover, over the last decade there has been a lot of changes in Electric Power Systems (EPS) around the world. A large share of distributed, renewable, intermittent energy resources resulted in the increased dynamics of the electricity grid and made it less predictable. EPS, which used to be considered quasi-static systems of high order, with the presence of Distributed Energy Resources (DERs) at the distribution level, are constantly changing into active dynamic systems. This implies the rethinking of the control and observation of the EPS.

The state estimation is crucial for the real-time operation, control and security of the EPS. The most common approach introduced over 40 years ago are static state estimators which are based on the iterative weighted least square (WLS) method. Their main advantage over Dynamic State Estimation (DSE) is no need for the dynamic model of the system which usually is full of uncertainties and simplifications. However, they do not consider past measurement data and needs to be restarted every sampling period. Therefore DSE is a tool which can be applied not only to voltage and phase estimation but also for diagnostic purposes for the hardware maintenance and control. In the future DSE may become an integral part of automatic power system control, contributing towards on-line stability analysis and power station control [7].

Only in the last decade several algorithms for the DSE in EPS were proposed. Huang *et. al.* [8] conducted the feasibility studies of applying Extended Kalman Filter to EPS. EKF with unknown inputs was examined by Ghahremani and Kamwa in [6]. In [5, 17] various Unscented Kalman Filters

were successfully applied to EPS state estimation with good computational and numerical results. Observers were also utilised in fault diagnostics in EPS in both centralised and distributed way e.g in [14, 16, 18]. One can find more extensive survey on the EPS tracking and dynamic estimation in [9]. All the authors stress that DSE is a crucial part of the Energy Management System and Distribution Management System.

In this paper we design the Exact State Estimator for the EPS, sometimes referred also as Integral State Estimator, firstly proposed by Byrski and Fuksa in [2]. We compare its performance with the classical approach, the Kalman-Bucy filter. The main reason for this study is to examine the feasibility of such observers in the EPS and to show its advantages over classical, well-renowned methods.

2 Dynamic state estimation

One has the access to a real-world dynamic system via its output and input signals, while modern multi-dimensional control algorithms are based on a full-state vector which is not often measurable. Thus it is necessary to estimate a state vector of a system given limited above-mentioned data. If that is possible then such a system is observable. In more formal way, the linear dynamic system (1) is observable for $t \in [t_0, t_1]$ if and only if given the system's input and output vectors $u(t)$ and $y(t)$ respectively, the unique initial state x_0 can be determined. System observability can be confirmed by checking the non-singularity of the observability Gramian or the rank of the observability matrix [12].

There is given a linear, continuous, time-invariant and observable system model:

$$(1) \quad \begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w(t) \\ y(t) &= Cx(t) + v(t) \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is a state vector, $y(t) \in \mathbb{R}^m$ is the output vector, $w(t)$ and $v(t)$ denote process and output noise respectively. They are assumed to be zero mean Gaussian with covariance Q and R respectively, for $\forall t \geq 0$, $m < n$. Initial state is unknown.

2.1 Classical approach

As a classical observer we have chosen the well-renowned Kalman-Bucy filter (KBF) which is an optimal stochastic observer proposed by Kalman and Bucy in [11] as

a continuous-time version of the Kalman filter. Its main goal is the estimation of the state vector and the outputs of the system (1). Unlike the discrete Kalman filter, instead of predictor-corrector equations, Kalman-Bucy filter uses two differential equations for the state estimation calculation:

- (2) $\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K(t)(y(t) - C\hat{x}(t))$
- (3) $\dot{P}(t) = AP(t) + P(t)A^T + Q(t) - K(t)R(t)K^T(t)$
- (4) $K(t) = P(t)C^T R^{-1}(t)$,

where \hat{x} is a state estimate, P is an error covariance estimate and K is a gain or blending factor which minimises error covariance.

2.2 Integral State Observers

The scope of this paper is a special subclass of the Integral State Observers (ISO) - so-called "Moving Window Observers" (MWO) for an exact state reconstruction in a finite-time range. Basically speaking, the structure of the ISO is directly related to the definition of the observability and integral operators. Starting with a well-known formula (5) of the output of the system (1), with neglected noises $w(t)$ and $v(t)$, let us assume that both the control and process values are measured in a range $[0, T]$, where T is a fixed period of observation.

$$(5) \quad y(t) = Ce^{At}x(0) + C \int_0^t e^{A(t-s)} Bu(s) ds$$

In order to make the equation (5) dependent on the final state $x(T)$ after simple operations the relation (6) is obtained:

$$(6) \quad y(t) = Ce^{-A(T-t)}x(T) - C \int_t^T e^{A(t-s)} Bu(s) ds.$$

Moreover the final state reconstruction formula $x(T)$ takes the form of (7): [2].

$$(7) \quad x(T) = M_T^{-1} \int_0^T \left[e^{-A^T(T-t)} C^T y(t) + e^{-A^T(T-t)} C^T C \int_t^T e^{A(t-s)} Bu(s) ds \right] dt,$$

where M_T is the appropriate real Gram matrix (8), which according to the observability theorem is non-singular for any T :

$$(8) \quad M_T = \int_0^T e^{-A^T(T-t)} C^T C e^{-A(T-t)} dt$$

The transformation of the equation (7) and the change of the integration boundaries leads to the equation (9) for the online observer for $t \geq T$.

$$(9) \quad x(t) = M_T^{-1} \int_{t-T}^t \left[e^{-A^T(t-\tau)} C^T y(\tau) + \int_0^{T-t+\tau} e^{-A^T(T-s)} C^T C e^{A(s-T+t-\tau)} ds Bu(\tau) \right] d\tau,$$

which can be generalised to the formula (10):

$$(10) \quad x(t) = \int_{t-T}^t G_1(T-t+\tau)y(\tau)d\tau + \int_{t-T}^t G_2(T-t+\tau)u(\tau)d\tau,$$

where $G_1(t)$ and $G_2(t)$ are special matrices of observation functions. In fact, there can be found an infinite number of pairs $G_1(t)$ and $G_2(t)$ which guarantee the exactness of integral state observer [13]. In the case of perfect input and out-

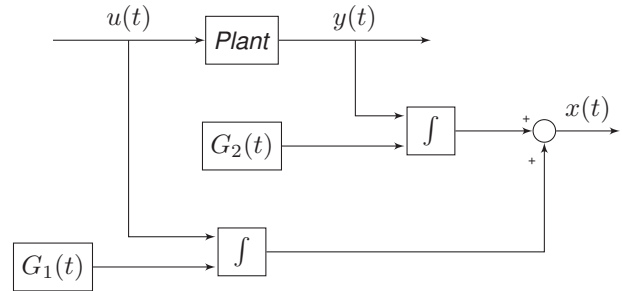


Fig. 1. Block structure of the integral observer.

put measurement, without noise or disturbances the equation (10) gives the exact reconstruction of the state vector. Nevertheless, in the real-world case, the reconstructed state vector is error-prone due to noisy measurements[13]. Let us assume that distortions in the system (1) are bounded and normalized, hence the real state estimate $\tilde{x}(t)$ contains an estimation error $\varepsilon(t)$ (12):

$$(11) \quad \tilde{x}(t) = \int_{t-T}^t G_1(T-t+\tau)[y(\tau) + v(\tau)]d\tau + \int_{t-T}^t G_2(T-t+\tau)[u(\tau) + w(\tau)]d\tau$$

$$(12) \quad \tilde{x}(t) = x(t) + \varepsilon(t)$$

where

$$(13) \quad \varepsilon(t) = \int_{t-T}^t [G_1(T-t+\tau)v(\tau) + G_2(T-t+\tau)w(\tau)]d\tau$$

Fortunately, integral operations on a finite range have a very valuable property of averaging errors, in contrast to the differential operators, which are sensitive to the noise presence in measurement data [3]. Furthermore, the length of the window T can be chosen arbitrarily, however the wider the window is the better error averaging effects can be obtained [3].

3 Power System Description

EPSs are complex dynamic systems of high order. The system dynamics is introduced by its generators, motors, dynamic loads and the transmission network. Lets consider a typical EPS described by a set of non-linear differential and algebraic equations given by (14)

$$(14) \quad \begin{cases} \dot{x}(t) = f[x(t), y(t), u(t)] \\ 0 = g[x(t), y(t)] \end{cases},$$

where $x(t) \in \mathbb{R}^n$ is a state vector, $y(t) \in \mathbb{R}^m$ is a vector of algebraic variables and $u(t) \in \mathbb{R}^p$ is a vector of system control variables. Both analyzed dynamic observer algorithms require a linear model of a measured system. To linearise the system (14) let (x_0, y_0, u_0) be an equilibrium point of (14) i.e. a solution of the power flow problem with e.g. Newton-Raphson method ($f(x_0, y_0, u_0) = 0$, $g(x_0, y_0, u_0) = 0$). The linearised model is as follows:

$$(15) \quad \begin{cases} \Delta \dot{x} = A_f \Delta x + C_f \Delta y + B_f \Delta u \\ A_g \Delta x + C_g \Delta y = 0, \end{cases}$$

where

$$(16) \quad \begin{aligned} A_f &= \left. \frac{\partial f}{\partial x} \right|_{x_0, y_0, u_0}, A_g = \left. \frac{\partial g}{\partial x} \right|_{x_0, y_0, u_0} \\ B_f &= \left. \frac{\partial f}{\partial u} \right|_{x_0, y_0, u_0} \\ C_f &= \left. \frac{\partial f}{\partial y} \right|_{x_0, y_0, u_0}, C_g = \left. \frac{\partial g}{\partial y} \right|_{x_0, y_0, u_0} \end{aligned}$$

and $\Delta x = x - x_0$, $\Delta y = y - y_0$, $\Delta u = u - u_0$. After the elimination of algebraic variables from the first equation of (15) the state-space representation of a power system for small-signal stability analysis can be obtained :

$$(17) \quad \begin{cases} \Delta \dot{x} = A \Delta x + B \Delta u \\ \Delta y = C \Delta x, \end{cases}$$

where obviously

$$(18) \quad \begin{aligned} A &= A_f - C_f C_g^{-1} A_g, & B &= B_f \\ C &= C_g^{-1} A_g \end{aligned}$$

4 Case study and experiments

As a case study we have chosen a 9-bus, 3 machine power system given in [4], page 70 and depicted in a single-line form in Fig.2. In addition to 3 synchronous generators Gen.1, Gen.2 and Gen.3, the system comprises of 3 induction machines at buses 5, 7, 9. System parameters such as synchronous generators, line parameters and induction motors are collected in Tables 1, 2, 3, respectively. Each synchronous generator is modelled as a second-order system with the angle δ and the speed ω as state variables. Induction motors are modelled as third-order single-cage systems, where the slip σ and the voltage behind the stator resistance both on real e'_r and imaginary e'_m axis are state variables. Thus, the system is of 15th order and the state vector is defined as follows:

$$(19) \quad x = [\delta_1 \omega_1 \delta_2 \omega_2 \delta_3 \omega_3 e'_{r1} e'_{m1} \sigma_1 e'_{r2} e'_{m2} \sigma_2 e'_{r3} e'_{m3} \sigma_3]^T$$

The experiments were done in Matlab/Simulink environment. The non-linear models of the generators, machines and grid are a part of the the Power System Toolbox which was also used to calculate the state-space matrices.

Table 1. Synchronous generators data.

machine	bus	base mva	x'_d [pu]	H [s]	d_o [pu]
1	1	100	0.0608	23.64	13.64
2	2	100	0.1198	6.4	6.4
3	3	100	0.1813	3.01	3.01

4.1 Evaluation and comparison of observers performance

For the first experiment purposes the simulated EPS was excited by the introduction of a pulse signal of 5 seconds width to one of the state variables - slip σ_1 . Additionally, the input and the output of the plant was distorted using Gaussian noise generator: SNR= 46dB. It was done in order to

Table 2. Line parameters.

line	from	to	r [pu]	x [pu]	charging/2 [pu]
1	1	4	0.	0.0576	0.
2	4	5	0.017	0.092	0.079
3	5	6	0.039	0.17	0.179
4	3	6	0.	0.0586	0.
5	6	7	0.0119	0.1008	0.1045
6	7	8	0.0085	0.072	0.0745
7	8	2	0.	0.0625	0.
8	8	9	0.032	0.161	0.153
9	9	4	0.01	0.085	0.088

simulate the plant under circumstances similar to those in the real world and to examine noise vulnerability of the observers described in section 2.

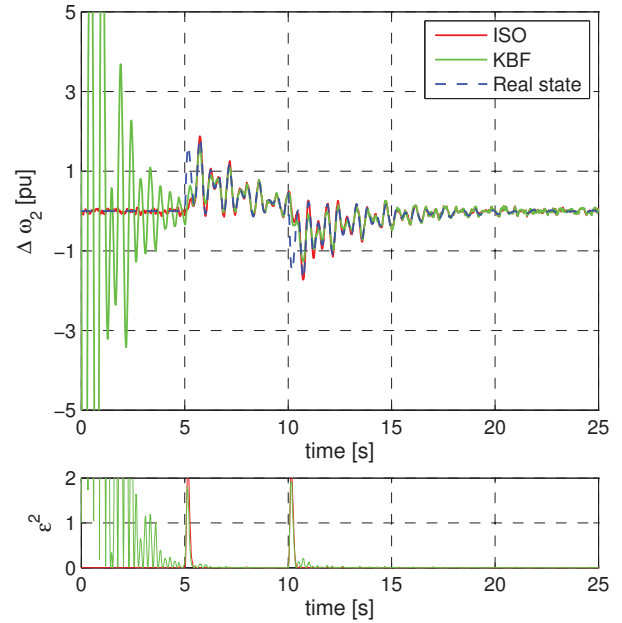


Fig. 3. Evolution of change in speed ω_2 and square of estimation error ε^2

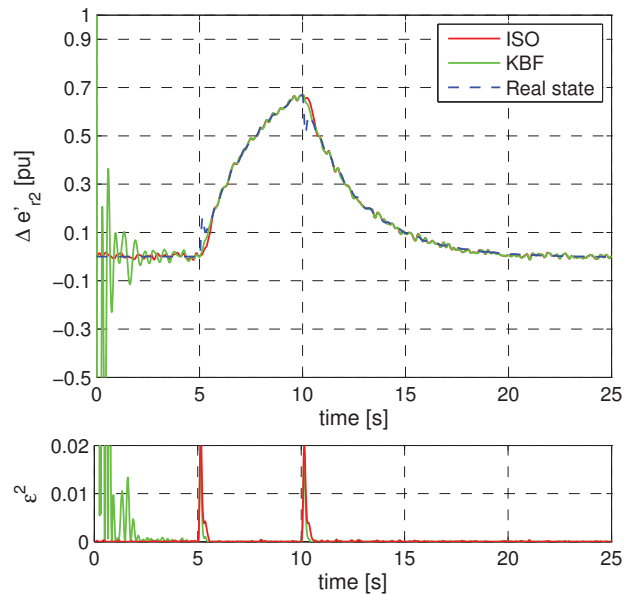


Fig. 4. Evolution of change in d-axis stator voltage of motor 2: $\Delta e'_{r2}$ and square of estimation error ε^2

As a result, in Fig.3 and Fig.4 one can see the transients of the two arbitrary-chosen state variables: speed ω_2 and d-

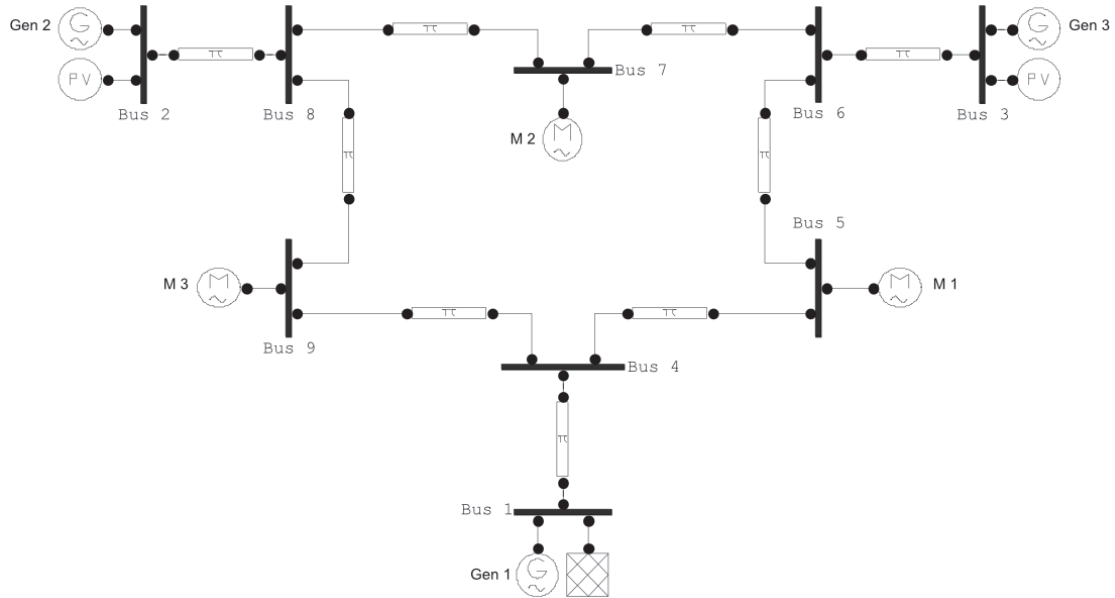


Fig. 2. A nine bus, three machine system [4].

Table 3. Induction motor data.

motor	bus	MVA base	r_s [pu]	x_s [pu]	X_m [pu]	r_r [pu]	x_r [pu]	H [s]	r_{r1} [pu]	x_{r1} [pu]	dbf	i_{sat} [pu]
1	5	28.6	0.03274	0.08516	3.7788	0.06164	0.06005	1.0	0.01354	0.07517	0.0	3
2	7	120.0	0.0229	0.1153	3.469	0.0102	0.0877	1.0	0.0	0.0	7.469	3
3	9	120.0	0.03274	0.08516	3.7788	0.06164	0.06005	1.0	0.01354	0.07517	0.0	3

axis stator voltage of motor 2 e'_{r2} , respectively. Additionally, in each figure KBF and ISO state estimates and square of estimation error are given. It can be noticed that in both cases KBF needs some non-negligible time (app. 5 and 2.5 seconds), which depends on the difference between the initial states of the EPS system and KBF, in order to converge to the reliable estimate. On the contrary, the ISO performance does not depend on the system initial state which results in the convergence to the observed state after the time equal to the size of the moving window which is too small to be indicated in the figure. For each state and for both KBF and ISO there are mean square errors (MSE) and standard deviations of square errors (SDSE) collected in Tab.4. The ISO's performance is noticeably better than KBF's due to rapid convergence to every observed state.

4.2 Influence of sliding window size on the quality of observation

According to the results given in the section 2.2, the size of the sliding window should not affect the ISO estimate error provided that the input and output measurements are ideal. In order to check this statement we have extended the observation window by 0.02 second for every consecutive simulation experiment done on non-noisy plant from 0.2 to 0.9 seconds. As a result, the relation between the sliding window width and ISO mean square error for six state variables, for which errors are most apparent, is depicted in Fig. 5. Surprisingly, the observation error is variable though all the characteristics should be flat and constant. The reason for that can be numerical precision of the integral methods used in the simulation. In most cases however, the wider the observation window T is the less sensitive to disturbances the observer is.

5 Conclusions

In this paper a successful attempt of the application of integral state estimation algorithms in the EPS field was pre-

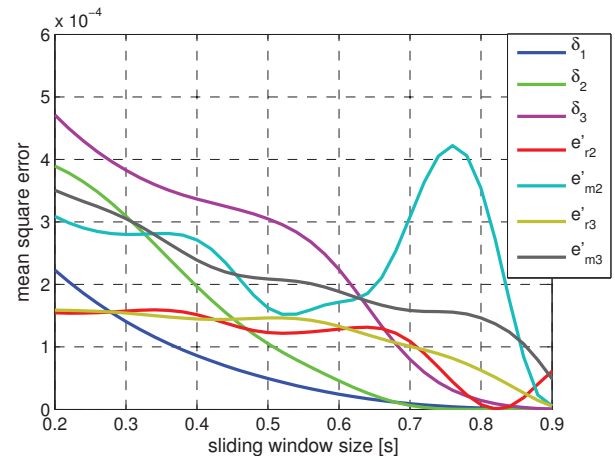


Fig. 5. Effect of window size (a fixed period of observation) on the quality of the state observation (without disturbances) of selected state variables.

sented. It was shown that the primary advantage of the integral state observers in comparison to the classical approach is the accurate state reconstruction invulnerable to the initial system conditions. This resulted in better performance of ISO with regards to the estimation mean square error. Furthermore, it was shown that numerical methods applied to the simulation influence the ISO estimate error but they can be averaged by the appropriate choice of the moving window width. In the future work we are planning to implement the optimal version of the ISO in the hardware and test it for the real-world plant. Additionally, we would like to compare ISE with the approaches presented in [1] and [10].

Acknowledgements

This work has been funded by KIC InnoEnergy Innovation projects: Active SubStation (ASS) and Controllable and Intelligent Power Components (CIPOWER).

Table 4. Mean square error (*MSE*) and standard deviation of square error (*SDSE*) of Kalman-Bucy Filter state estimation (*KBF*) and Integral Observer state estimation (*ISO*).

no.	sym.	state vector component	MSE KBF	SDSE KBF	MSE ISO	SDSE ISO
1	δ_1	Rotor angle of gen. 1	0.0175	0.1667	0.0002	0.0013
2	ω_1	Speed of gen. 1	2.2251	26.9610	0.0135	0.1059
3	δ_2	Rotor angle of gen. 2	0.0707	0.8726	0.0004	0.0029
4	ω_2	Speed of gen. 2	7.5437	108.6743	0.0181	0.1637
5	δ_3	Rotor angle of gen. 3	0.1083	1.4482	0.0005	0.0043
6	ω_3	Speed of gen. 3	15.0267	180.7549	0.0543	0.4574
7	e'_{r1}	Principal d-axis stator voltage of motor 1	0.0996	1.5366	0.0201	0.3139
8	e'_{m1}	Principal q-axis stator voltage of motor 1	0.1121	1.8200	0.0221	0.3803
9	σ_1	Slip of motor 1	0.0260	0.0796	0.0249	0.0749
10	e'_{r2}	Principal d-axis stator voltage of motor 2	0.0882	1.6061	0.0002	0.0014
11	e'_{m2}	Principal q-axis stator voltage of motor 2	0.2532	4.7794	0.0004	0.0025
12	σ_2	Slip of motor 2	0.0013	0.0298	0.0000	0.0000
13	e'_{r3}	Principal d-axis stator voltage of motor 3	0.0494	0.9478	0.0002	0.0017
14	e'_{m3}	Principal q-axis stator voltage of motor 3	0.1695	3.3544	0.0004	0.0029
15	σ_3	Slip of motor 3	0.0011	0.0282	0.0000	0.0000

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