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Energy harvesting of a pendulum vibration absorber

Abstract. *This paper presents the numerical results of a pendulum vibration absorber. The system composed of the pendulum attached to the Duffing's oscillator. The harvester is mounted in the pendulum pivot structure and consists of a cylindrical permanent magnet mounted on rotor and of four windings fixed to the housing as a stator. The obtained results show the influence of system and electrical parameters on value of induced voltage for vibration absorption conditions.*

Streszczenie. *W pracy przedstawiono badania numeryczne wahadłowego tłumika drgań. Układ składa się z wahadła zamocowanego na oscylatorze Duffinga. Urządzenie indukujące prąd zamocowane jest w zawieszeniu wahadła i składa się z cylindrycznego magnesu zamontowanego na rotorze i czterech uzwojeń zamontowanych na stojanie. Otrzymane wyniki pokazują indukowane napięcie w warunkach dynamicznej eliminacji drgań.* ***(Odzyskiwanie energii w wahadłowym tłumiku drgań)****.*

Keywords: wahadło, odzyskiwanie energii, rezonans, indukcja, voltage
Słowa kluczowe: pendulum, energy harvester, resonance, induction, napięcie.

Introduction

Reduction of the vibration is a serious concern for many practical applications. For example, tall buildings and long-span bridges are subjected to huge dynamic loadings from the winds, earthquakes, water waves, traffics and human motions. The large vibration amplitude can damage the structures or the secondary components, or cause discomfort to its human occupants [1]. Among of many methods to reduction of vibration, the pendulum mass damper (PMD) has been proved to be a very simple and effective vibration suppression device, with many practical implementations on tall buildings, such as Taipei 101 in Taipei, Citi Group in New York, and many others [2].

Among the nonlinear systems, we can mention a special class of models which consists of at least two subsystems. If the subsystems are coupled by inertial terms, then periodic vibrations generated by one subsystem become an excitation source for the other. This phenomenon performs the so called autoparametric vibrations. Recent developments of absorption devices include the autoparametric vibration absorbers. The idea lies in attaching the absorber to the primary system in such a manner that it experiences a parametric base excitation, and therefore, the absorber frequency is tuned around one-half of the troublesome frequency value [3]. The governing system of equations of motion has quadratic nonlinearities, whose influence is activated by this tuning, enabling a parametric resonance to be excited and the absorber to respond in the desired frequency range. This device provides vibration suppression of the primary system response.

We propose to apply harvesting energy device in the autoparametric pendulum vibration absorber (APVA), this solution allows simultaneous generate electrical energy and maintain the vibration absorption effect. Autoparametric vibration systems have an interesting dynamic that results from at least two non-linear subsystems coupled to interact in a way where one of them transfers the exogenous perturbation energy to the other (vibration absorption effect). Thus, the primary system can be externally excited by some harmonic force and, when it is connected to the secondary system (absorber), it can be verified the so called parametric excitation, that is, a mechanism that transfers the exogenous energy to its absorber. The autoparametric interaction has been analyzed in the literature: (Haxton and Barr [4], Cartmell and Lawson [5], Korenev and Reznikov [6], Nayfeh and Mook [7], Warminski and Kecik [8]). There are many vibrating structures and machinery, where the pendulum type absorbers are used. Energy harvesting has been an active research area and systems refer to devices that capture and transform energy into electricity. Usually, the energy could be the kinetic energy of moving or vibrating structures. Vibration-based energy harvesters can be categorized into three types, namely electromagnetic, piezoelectric, and electrostatic, depending on the medium of the transducer. The transducers require input vibration with high frequency to generate electricity. Civil structures exhibit usually low frequency structural vibrations, which make it difficult for an energy harvester to extract energy. Therefore, tuned in such systems is very important. Vibrational energy harvesters achieve their highest output power when they are working nearly resonance regions (because of the highest amplitude of vibration).

Research in the area of piezoelectric energy harvesting involves understanding the mechanics of vibrating structures, the fundamental electrical circuit theory and the constitutive behavior of piezoelectric materials. The modeling problem of vibration energy harvesting using piezoelectric transduction is to estimate the voltage output across the resistive load in terms of the base motion input. The voltage output can then be used to calculate the power delivered to the given electrical load.

This paper shows the numerical study of an autoparametric pendulum absorber with mounted harvesting system. The study is done nearly region, where the dynamic vibration absorption effect is satisfied. Moreover, the effective electricity regarding different system and electrical parameter is discussed.

Model of an autoparametric pendulum harvester

The main idea is to generate the electricity by convert the kinetic energy (the rotation or swings of the pendulum) into the voltage. The model of an autoparametric pendulum harvester in *Fig. 1* is shown. This model is composed of the harmonic excited oscillator (the mass m_1) and attached pendulum (the masses m_2 and m_3). The stiffness of springs are denote: k and k_2 - linear characteristics, k_1 – nonlinear characteristic. In the pendulum pivot, harvesting device is mounted. This device consist of a rotor a cylindrical permanent magnet and a stator. The device is connected directly to the electrical load, L_d. The scheme of harvesting device mounted in the pendulum pivot in *Fig. 2* is shown.

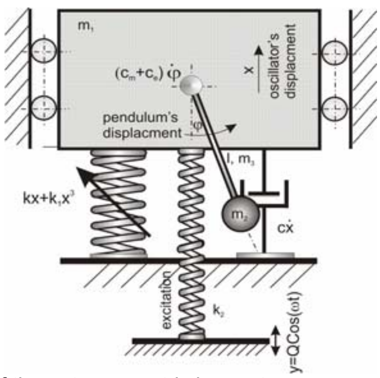

Fig.1.Model of the autoparametric harvester.

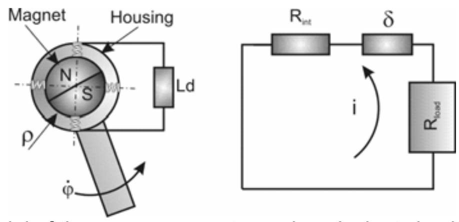

Fig.2. Model of the energy converter and equivalent circuit.

The equations of motion are derived by Lagrande's equations of the second kind. The detailed derivation of dimensional equations can be found in [9]. They are written as:

$$(m_1+m_2+m_3)\ddot{x}+c\dot{x}+(k+k_2)x+k_1x^3 \tag{1}$$
$$+\left(m_2+\frac{m_3}{2}\right)l\left(\ddot{\varphi}\sin\varphi+\dot{\varphi}^2\cos\varphi\right)=k_2Q\cos\omega t,$$

$$\left(m_2+\frac{m_3}{3}\right)l^2\ddot{\varphi}+\left(c_e+c_m\right)\dot{\varphi}+\left(m_2+\frac{m_3}{2}\right)l \tag{2}$$
$$\left(\ddot{x}+g\right)\sin\varphi=0.$$

The first, Eq. (1) represents the oscillator's motion while the second Eq. (2) denotes motion of the pendulum. If the pendulum does not move, it becomes a part of the oscillator mass, only. The c_m is the damping coefficient used to describe the mechanical damping, c_e denotes electrical damping in the electrical devices mounted in the pendulum pivot and c is a viscous damping coefficient in damper.

To calculate the electrical damping coefficient, the equation for the electrical circuit is used by applying Kirchhoff's law to the electrical circuit

$$i\left(R_L+R_I\right)-\delta\dot{\varphi}(\tau)\rho=0, \tag{3}$$

where: δ is the transduction factor, ρ is the radius of housing
on which the coils are mounted, R_L is the resistance of the external load, R_I is the internal resonance. The δ equals the magnetic flux gradient. This term is commonly depends on: number of coil turns, average magnetic field strength, and the coil length. The voltage can be estimated based on Ohm's law

$$U=iR_L=\delta\frac{R_L\dot{\varphi}(t)\rho}{R_L+R_I}. \tag{4}$$

Then the electrical damping coefficient defined as

$$c_e=\frac{\delta^2\rho}{R_L+R_I}. \tag{5}$$

The value of this coefficient depends on electrical properties and construction of the harvester device.

Introducing dimensionless time $\tau=\omega_0 t$ where ω_0 is the natural frequency of the linear oscillator (the nonlinear part comes from the nonlinear spring is neglected), and then the dimensionless coordinates $X = x/x_{st}$ and $\varphi\equiv\varphi$ where x_{st} is the static displacement of the linear oscillator we obtain the equations (1) and (2) in dimensionless form:

$$\ddot{X}+\alpha_1\dot{X}+X+\gamma X^3+\mu\lambda\left(\ddot{\varphi}\sin\varphi+\dot{\varphi}^2\cos\varphi\right)=q\cos\vartheta\tau, \tag{6}$$
$$\ddot{\varphi}+\alpha_2\dot{\varphi}+\beta\dot{\varphi}+\lambda\left(\ddot{X}+1\right)\sin\varphi=0.$$

The dimensionless parameters take the following definitions:

$$\alpha_1=\frac{c}{\left(m_1+m_2+m_3\right)\omega_0},\ \alpha_2=\frac{c_m}{\left(m_2+\frac{m_3}{3}\right)l\omega_0},$$

$$\beta=\frac{\delta^2\rho}{\left(R_L+R_I\right)\left(m_2+\frac{m_3}{3}\right)l\omega_0},\ q=\frac{k_2Q}{(k+k_2)x_{st}},$$

$$\omega_0=\sqrt{\frac{k+k_2}{(m_1+m_2+m_3)}},\ x_{st}=\frac{(m_1+m_2+m_3)g}{(k+k_2)},\ \gamma=\frac{k_1}{k+k_2}x_{st}^2, \tag{7}$$

$$\mu=\frac{\left(m_2+\frac{1}{2}m_p\right)l^2}{\left(m_1+m_2+m_3\right)x_{st}^2},\ \lambda=\frac{(m_2+\frac{1}{2}m_3)x_{st}}{(m_2+\frac{1}{3}m_3)l},\ \vartheta=\frac{\omega}{\omega_0}.$$

The motion of the dynamic pendulum absorber is described by two generalized coordinates, namely the displacement of the oscillator in the vertical direction X, and the angle of the pendulum rotation φ.

Harvesting energy and absorption effect

The autoparametric vibration pendulum absorber (AVPA) is designed to absorb energy from the primary system (main mass). This absorption effect is efficient only in the limited band of vibration frequencies of the main system. Therefore we analyze dynamics both systems i.e. the pendulum and the oscillator motion. The system parameters are: α_1=0.1, α_2=0.002, μ=6, λ=0.3, q=0.2, γ=0. The electrical parameters are: R_I=188Ω, R_L=$10^6\Omega$, δ=7.752Vs/m, ρ=0.005m. The simulation time was taken for two thousand periods where first thousands of them were cut as a transient time. The calculations have been done in Matlab environment by means of Runge–Kutta method using solver ode45. The results of this work based of the voltage root mean square (quadratic mean) which derivatives $RMS_V=\langle V\rangle/\sqrt{2}$. This is a statistical measure of the magnitude of a varying quantity. It is especially useful when variates are positive and negative.

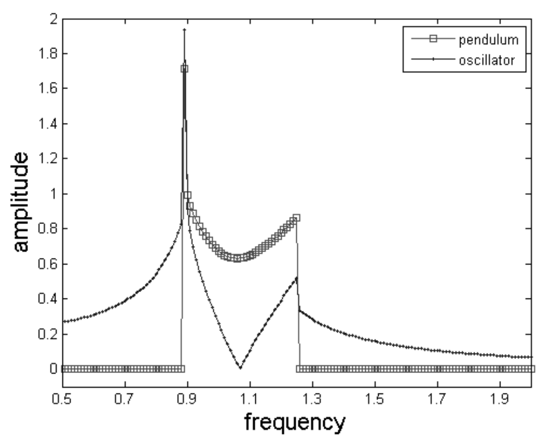

Fig.3. The resonance curves for the pendulum (blue) and the oscillator (red).

The resonance curves in *Fig. 3* are presented. The line (·) and (▫) denote the pendulum and the oscillator resonances, respectively. Near the frequency ϑ=1.1, the dynamical vibration absorption effect is clearly visible, the amplitude of oscillator is reduced to zero, the full energy takes the pendulum.

The amplitude of excitation (*q*) is a crucial parameter which influence on the dynamics of an autoparametric system. The generating electricity depends on this parameter, also. This situation in *Fig. 4* is presented. Increase amplitude of excitation causes, that resonance region is expanded and induced electricity growing. Unfortunately, this parameter may cause reduction in efficiently of absorption effect (the chaotic motions or the rotations of the pendulum is possible [9]). The maximal value of root mean square of voltage equals about *1.5*.

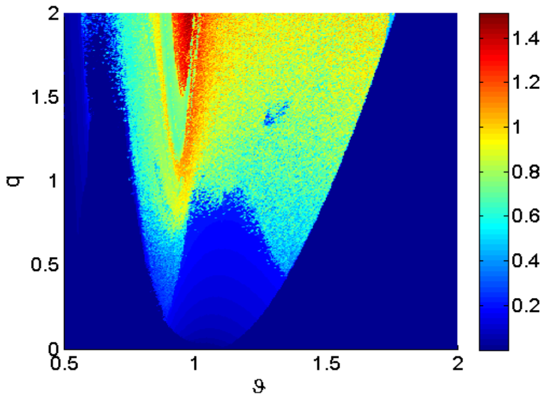

Fig.4.Influence of amplitude excitation (q) versus frequency of excitation (ϑ) on the RMS_v voltage near the resonance region.

The nonlinearity of a supporting spring (γ) slightly influences location of the resonance and absorption regions. The positive characteristic of spring shifted the resonance region in right, the negative characteristic in left side (*Fig. 5*). From, the practical point of view, very important is that nonlinearity of the spring practically doesn't decrees the electricity generated, but the dangerous region can be move away.

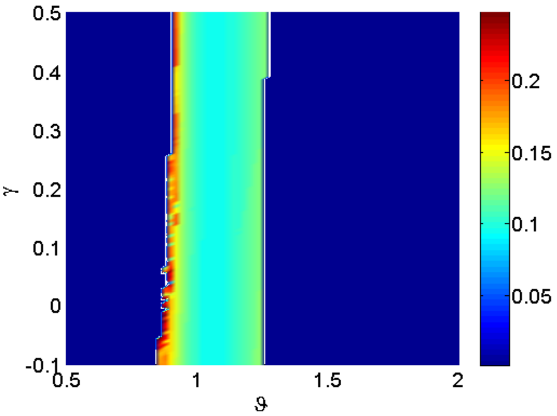

Fig.5. Influence of spring's nonlinearity (γ) versus frequency of excitation (ϑ) on the RMS_v voltage near the resonance region.

The dynamics of the system (and of course induced electricity) strongly depend on the values of the nonlinear terms which couple the main structure (the oscillator) and absorber (the pendulum). The response of the system is very sensitive to changes to the parameters λ and μ. Parameter λ ad μ couples the oscillator and pendulum motion. Parameter μ only appears in the oscillator equation of motion and plays the role of a gain in the product $\mu\lambda$ in the oscillator equation (first in Eq. 6). In a real system, the length of the pendulum and its mass moment of inertia are responsible for these parameters values (see eq. (7)).The influence of both parameters λ and μ on the value of RMS_V voltage in *Fig. 6* is presented.By combination of these parameters, for the analyzed range, we can increase several times in the RMS_V voltage. Additionally, the region where the electricity is induced is clearly visible (the pendulum executes motion). For, λ<0.15, the pendulum stopped and the electricity is not generated.

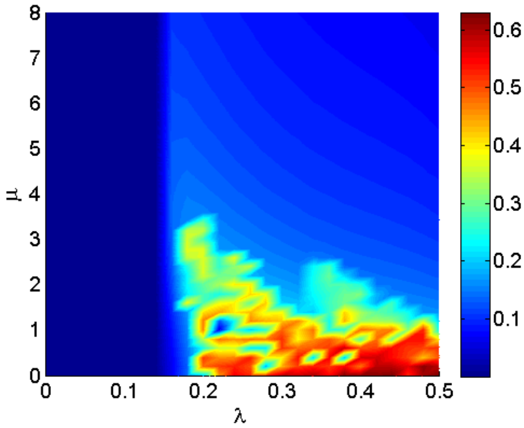

Fig.6. Influence of pendulum's parameters (μ versus λ) on the RMS_v voltage, for ϑ=1.05.

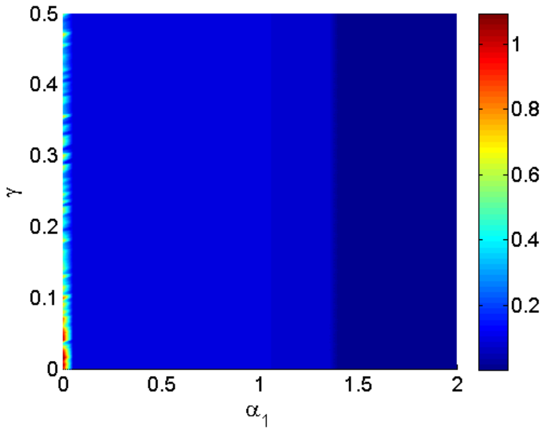

Fig.7. Influence of suspension's parameters (γ versus α_1) on the RMS_v voltage, for ϑ=1.05.

The suspension of an autoparametic vibration absorber can be used for a control of dynamics of absprber in difficullt situations. Interesingly, that induced electricity practically is independent on the spring stifness (γ), especially for larger oscillator damping. This results is similar with *Fig 5*, where nonlinearity of spring sligtly shiffted the resonance frequency. Using damping in the suspension of an autoparametric vibration system we can adjust the value of electricity induced. In, *Fig. 7*, we can see the influence of oscillator damping versus nonlinearity of spring. Analyzing, the results, we can conclude, that contol of electricity is easy by coefficient α_1 (for example: changing damping in viscous damper by hydraulic valve or current in magnetorheological damper). These region is a wide, therefore easily adjust the system parameters to the required voltage.

Of course, the parameters of harvester device system is also important. The dimensionleess electrical damping (β) depends the radius of housing (ρ), magnetic flux gradient (δ) and the resistance of the external and internal load (R_L and R_I). The influence of the electrical damping (β) causes reduction both to the right and the left hand sides of the resonance region, which essentially decreases the effectiveness of the pendulum as a dynamical damper (*Fig. 8*). Interestingly, that maximal value of generate electricity exists for small region, only. Similar effect appears for mechanical damping of the pendulum (α_2), because the electrical damping is a part of total damping of the pendulum ($\alpha_2+\beta$). The total damping is an important parameter, for dynamic absorption of vibrations. A slightly difference in coefficients (α_2 or β) influences effectiveness absorption effect. Increase damping in the pendulum causes decrease of efficiently of absorption effect [7]. For the damping in the harvester device about $\beta \approx 0.5$, the pendulum stopped and electricity is not induced. For our parameters the electrical damping coefficient is too small of the oreder $3 \cdot 10^{-6}$.

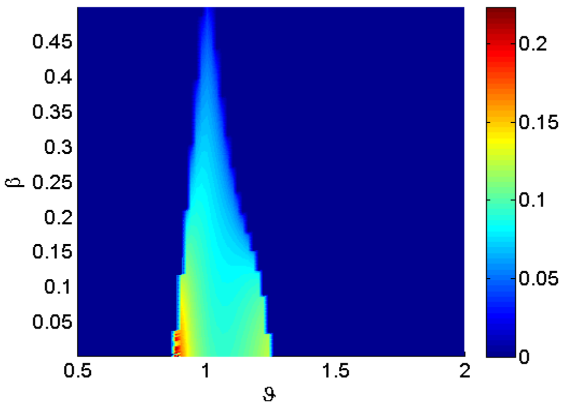

Fig.8. Influence of electrical damping coefficient (β) on the RMS_v voltage near the resonance region.

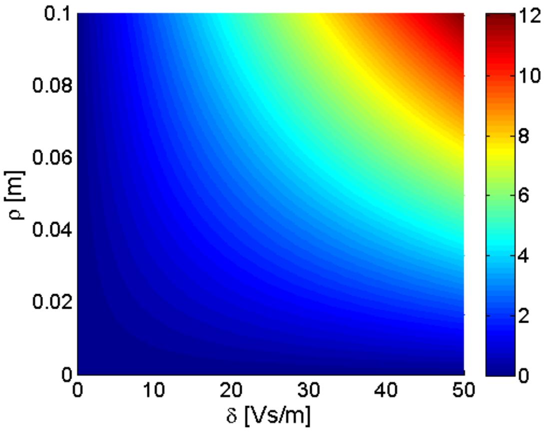

Fig.9. Influence of harvester device parameters (ρ versus δ) on the RMS_v voltage for the absorption effect, ϑ=1.05.

Radius of housing versus magnetic flux gradient, obtained for linear spring γ=0 is presented in *Fig. 9*.These parameters significantly influences electricity. For, example, for ρ=0.06m, the voltage root mean square can achieve about 7V (for ρ=50Vs/m).

It is worth noting that most of the numerical results presented in this work (without installed harvesting device) were verified analytically and experimentally on the special laboratory system [8,10].

Conclusions and final remarks

The vibration absorption systems are device dedicated to reduce dangerous vibration. This work proposes the application of energy harvesting device, mounted in the pivot of the pendulum vibration absorber. Because of the small electrical damping (see eq. 7) the effectiveness of absorption effect is practically unchanged. The excessive increase in damping of the pendulum causes decaying a big reduction of the resonant curve (Fig. 8), an unwanted effect from the vibration dynamical absorption point of view.

To control of dynamic and voltage induced can be realized by proper choice of the pendulum parameters (μ and λ), but these parameters cannot be modified online, as the pendulum operates. Therefore, another solution is proposed, based on control the damping and the stiffness of a suspension of an autoparametric vibration pendulum absorber. The amplitude of excitation (q) influences on absorption effect, therefore in this type of absorbers required to apply the control with feedback.

In future, the harvesting device will mounted in the laboratory rig, and these results will be experimentally verified.

Acknowledgments

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