The FEM analysis of electromagnetic torque of hybrid stepper motor with different load

**Abstract.** This paper presents the method of modeling and numerical simulation of a hybrid stepper motor. The problem is solved by the electromagnetic field modeling, using the time-stepping finite element method. Circuit equations are coupled with field equations creating one global system of non-symmetric equations. The mechanical motion of the rotor is determined by solving the second order differential motion equation. The magnetic torque is calculated by the Maxwell stress tensor. The paper shows how motor load modifications influence the response dynamic and torque ripples in the electromagnetic torque characteristic. It is shown that the FEM stepper model including the analysis of inertia controlled system can be used as support for improving tool and technics designed as predicting and controlling the electromagnetic torque ripples for motor drive applications. The simulations’ results of electromagnetic torque are verified and compared with the results obtained from laboratory kit using FAST (ang. Force Angle Speed Torque) technology sensor. **Analiza FEM momentu elektromagnetycznego** **hybrydowego silnika krokowego z** **różnym obciążeniem.**

**Słowa kluczowe:** hybrydowy silnik krokowy, metoda elementów skończonych, moment elektromagnetyczny

**Keywords:** hybrid stepper motor, Maxwell stress tensor, electromagnetic torque, finite element method (FEM)

**Introduction.** Along with dynamic development new industrial technology appears. Mechanics are looking for efficient, reliable, not demanding specialist service drives and control systems. Above-mentioned requirements should be performed in stepping motors systems, hence recently it is possible to notice significant growth of applications these solutions [6].

Stepper motors are electromagnetic incremental-motion devices which convert digital pulse inputs to the analog output motion. They can be successfully used in applications where a control of rotation angle, speed, position and synchronism is needed. Because of the inherent advantages listed previously, stepper motors have found their place in many different applications. Some of these include printers, plotters, office equipment, hard disk drivers, medical equipment, fax machines, automotive and many other [2,3,6].

In this paper a modeling method and a numerical simulation of the two-phased hybrid steppe motor is presented (Figure 1). The motor consists of slotted stator equipped with two bipolar coils and a rotor with no windings.

![Rotor laminations](image)

Fig. 1. A view of hybrid stepper motor rotor

Time stepping finite element approximation is applied to calculate the coupled field-circuit problem. High performance of motor drive applications require smooth torque with minimum torque ripples. The authors examined a few types of load which change dynamics of the motor, and the most interested of them are presented in this paper.

**Abstract.** Article presents numerical method of modeling and simulation of hybrid stepper motor. Pole electromagnetic models were done on model elements sketched which were solved with equations determined by solving the second order differential motion equation using the Maxwell stress tensor. The FEM stepper model including the analysis of inertia controlled system can be used as support for improving tool and technics designed as predicting and controlling the electromagnetic torque ripples for motor drive applications. The simulations’ results are verified and compared with the results obtained from laboratory kit using FAST (ang. Force Angle Speed Torque) technology sensor. **Analiza FEM momentu elektromagnetycznego hybrydowego silnika krokowego z** **różnym obciążeniem.**

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Obtained responses of examined models are compared with real hybrid stepper motor 57BYGH804 Wobit Company [9]. This is the universal motor, which can be controlled both bipolar and unipolar modes.

**Table 1. The hybrid stepper motor parameters**

<table>
<thead>
<tr>
<th><strong>Type of control</strong></th>
<th><strong>Unipolar/Bipolar</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal step</strong></td>
<td>1.8 [angle]</td>
</tr>
<tr>
<td><strong>Voltage supply</strong></td>
<td>3.3 [V]</td>
</tr>
<tr>
<td><strong>Current</strong></td>
<td>3 [A]</td>
</tr>
<tr>
<td><strong>Resistance</strong></td>
<td>1.1 [Ω]</td>
</tr>
<tr>
<td><strong>Inductance</strong></td>
<td>1.4 [mH]</td>
</tr>
<tr>
<td><strong>Torque</strong></td>
<td>1.2 [Nm]</td>
</tr>
<tr>
<td><strong>Inertia</strong></td>
<td>440 [gcm²]</td>
</tr>
<tr>
<td><strong>Damping</strong></td>
<td>0.0001 [Nms]</td>
</tr>
<tr>
<td><strong>Weight</strong></td>
<td>1.1 [kg]</td>
</tr>
</tbody>
</table>

**Field-motion coupled model.**

Using the magnetic vector potential A and electric scalar potential V as electromagnetic field variables, the magnetic flux density B in conducting and non-conducting region $(\Omega_C \cup \Omega_N)$ is defined as [10-13]:

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \text{in} \quad \Omega_C \cup \Omega_N$$

In this case, the boundary value problem in terms of potentials is expressed as follows:

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A}(t) \right) = \mathbf{j}(t) + \mathbf{j}_m(t) \quad \text{in} \quad \Omega_C \cup \Omega_N$$

$$\mathbf{j}_m(t) = \nabla \times \mathbf{M}$$

where $\mu$ is a permeability, $(t)$ current density of the thin coil and $\mathbf{M}$ is magnetization vector. If voltage excitation is given, first the equations (1) expressed in term of magnetic vector potential must be considered as [1, 5]:

$$\frac{d}{dt} \int \mathbf{A}(t) dl + R_1 \mathbf{i}_1(t) = u_1(t)$$

$$\frac{d}{dt} \int \mathbf{A}(t) dl + R_2 \mathbf{i}_2(t) = u_2(t)$$
where $R$ is the winding resistance of a one phase, $i$ is the phase current and $u$ is the supply voltage. For two-phase voltage forced system above equations are expressed by matrix of dynamic impedances. Discrete form of (4) can be defined as bellow:

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
=
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\]

where:

\[
Z_{ij} = R_i + \frac{1}{T_p} \int I_{i_j} \, dI_j \\
Z_{jk} = Z_{kj}
\]

Global impedance matrix includes own impedances $Z_{ii}$ and mutual impedance $Z_{ij} = Z_{ji}$. For $I_j = 1 \, A (j=1,2)$ and for $I_{k\neq j} = 0 \, A (k=1,2)$ are performed calculations for matrix of impedance $Z$, next the current vector $I$ in both coils are calculated from formulation:

\[
I = Z^{-1}U
\]

when a voltage vector $U$ is known. Calculation of the force is performed using the Maxwell’s stress tensor method. The force density is given by following formula:

\[
f = \nabla \cdot T
\]

where $T$ denotes modified Maxwell’s stress tensor proposed as follows [2, 4]:

\[
T = \begin{bmatrix}
T_{11} & T_{12} & T_{13} \\
T_{21} & T_{22} & T_{23} \\
T_{31} & T_{32} & T_{33}
\end{bmatrix}
\]

where:

\[
T_{11} = \frac{1}{2\mu_0} B_x^2 - \frac{1}{2} \mu_0 (H_y^2 + H_z^2), \quad T_{12} = H_y B_x, \\
T_{13} = H_x B_x, \quad T_{21} = H_x B_y, \quad T_{23} = H_z B_y, \\
T_{22} = \frac{1}{2\mu_0} B_y^2 - \frac{1}{2} \mu_0 (H_x^2 + H_z^2), \quad T_{32} = H_y B_z, \\
T_{33} = \frac{1}{2\mu_0} B_z^2 - \frac{1}{2} \mu_0 (H_x^2 + H_y^2), \quad T_{31} = H_x B_z
\]

The rotor displacement is evaluated by solution of the mechanical motion equation:

\[
f \frac{d^2 \Theta}{dt^2} + b \frac{d \Theta}{dt} = T
\]

where $\Theta$ is the rotor displacement, $J$ is the rotor inertia, $b$ is the damping coefficient, $T = T_E - T_L$ is the difference between the electromagnetic and the load torque.

The force is evaluated along a surface in the airgap around the rotor. The torque component $T$ is obtained from the relationship [7]:

\[
T = \int_S [r \times P] \, dS
\]

where $r$ is the position vector of the integration contour, $P$ denotes the stress component defined around the rotor, $dS$ is a surface segment. Only z-component of the torque $T = [0 \quad 0 \quad T]$ is taken into consideration.
It is especially useful wherever there is a need to perform complex, repetitive sequence of many parameters (both position and velocity) [9].

The control signals are realized by miniature controller SMC64v2 designed to work with two-phase stepper motor with bipolar (8 or 4 wires) or unipolar (6 wires) windings [9]. It allows full step control or broken step (2/4/5/8/10 or 16 parts) by forcing constant value of current in the motor regardless of the value of voltage supply.

In order to properly model a drive system both friction load and load inertia must be considered. Many simulations indicate that this is a crucial part of small drive system modeling. Gear drive system is one of the most popular way of connecting hybrid stepper motor.

![Fig. 4. The laboratory kit with FAST technology torque sensor DFM22-2.5](image)

Fig. 4. The laboratory kit with FAST technology torque sensor DFM22-2.5: where: 1 – unipolar/bipolar control mode switch, 2 – torque gauge MD100M, 3 – gear drive system, 4 – DFM 22-2.5 torque sensor , 5 – clutch, 6 – stepper motor, 7 – motor controller, 8, 10 – voltage power supply, 9 – modul that defines the motor trajectory (PC connected)

When gears are used to drive a load, the inertia reflected to the motor is expressed by the following equation [9]:

\[
J = \left( \frac{\alpha_1}{\alpha_2} \right)^2 \left( J_2 + J_3 \right) + J_1
\]

Fig. 5. Gear drive system model

where: \( J_1, J_2 \) and \( J_3 \) are inertias, \( \alpha_1 \) is number of load gear teeth, \( \alpha_2 \) is number of motor gear teeth. In the experiment shown in Figure 9 and Figure 10, the gears has been used with the following parameters:

\[
J_1 = 1.12 \, D^{-3} \, \text{kg} \cdot \text{m}^2, \quad J_2 = 1.34 \, D^{-3} \, \text{kg} \cdot \text{m}^2, \\
J_3 = 0.29 \, D^{-3} \, \text{kg} \cdot \text{m}^2, \quad \alpha_2 = 44 \quad \text{and} \quad \alpha_1 = 18.
\]

The motor was controlled in bipolar full step mode with 3,3 [V] voltage excitation (Figure 6 and Table 2). During control without external load only half the bandwidth of the motor winding was used. In the second control mode (gear drive system) full bandwidth of the motor winding was supplied.

![Fig. 6. Bipolar control mode of stepper motor](image)

Table 2. Bipolar commutation of stepper motor

<table>
<thead>
<tr>
<th>STEP</th>
<th>T_{1,4}</th>
<th>T_{2,3}</th>
<th>T_{5,8}</th>
<th>T_{6,7}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

The analysis was focused on electromagnetic torque characteristics. The figures presented below compare data from a measurement of the stepper motor system equipped with a FAST torque sensor and FEM simulation of distributed parameters model.

First part of the research concerns one of the least favorable mode of stepping motor work - its revolutions without load. The torque characteristics (simulated and measured) are presented in Figure 7 and Figure 8. In this control mode appears unfavorable resonance frequency. The resonance is a result of no continue revolution’s angular stator magnetic field. Consequently, deliver energy to rotor is pulsating and torque characteristic is oscillating.

![Fig. 7. Torque characteristic of stepper motor measured using FAST technique without external motor load](image)

This can create problems of increased audible noise, fatigue of the shaft, and possible speed and displacement oscillations. The torque ripples could be significant reduced by appropriate designing the gear drive system.
Including additional load inertia of controlled object reduces threshold of resonance frequency. In many cases, gear drive system is used as a method which enables to increase and stabilize torque dynamics [9]. Figure 9 and Figure 10 shows the results of electromagnetic torque obtained by considering additional model inertia shown in Equation 12. This research demonstrates the correctness of the implemented drive system. With accurate model of the motor with distributed parameters, the designer can adjust the optimal gear parameters to the control strategy.

**REFERENCES**


**Autozy:** dr inż. Jakub Bernat, Poznan University of Technology, Chair of Computer Engineering, ul. Piotrowo 3a, 60-965 Poznan, E-mail: Jakub.Bernat@put.poznan.pl; dr inż. Jakub Kolota, Poznan University of Technology, Chair of Computer Engineering, ul. Piotrowo 3a, 60-965 Poznan, E-mail: Jakub.Kolota@put.poznan.pl; dr inż. Slawomir Stepien, Poznan University of Technology, Chair of Computer Engineering, ul. Piotrowo 3a, 60-965 Poznan, E-mail: Slawomir.Stepien@put.poznan.pl.