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Diffused mode of vacuum arc in high current interrupters

***Abstract**. Using stationary solutions for electron plasma column in axial external magnetic field the analysis of spreading of initially high concentrated local currents is done. Early stage homogenization of local discharge is considered. It is shown that fast relaxation of electron density profile towards equilibrium is achieved with decreasing of maximum density more than 1000 times.*

***Streszczenie.** Wykorzystując stacjonarne rozwiązania dla plazmy elektronowej w zewnętrznym, osiowym polu magnetycznym przeprowadzono analizę rozmywania silnie skoncentrowanych lokalnych wyładowań . Skupiono się na wczesnym stadium procesu homogenizacji. Wykazano, że zachodzi szybka relaksacja do rozkładu równowagowego z równomiernym rozkładem gęstości prądu i maksymalną wartością gęstości zmniejszoną ponad 1000 razy (**Jednorodne wyładowanie w próżniowych wyłącznikach wysoko prądowych**).*

Keywords: high-current interrupters, electrodes erosion, stationary solution for electron plasma in axial magnetic field,.
Słowa kluczowe: wyłączniki wysoko prądowe, erozja elektrod, stacjonarne rozwiązania dla plazmy elektronowej w osiowym polu magnetycznym.

Introduction

The main problem in constructing high current interrupters is high erosion of electrodes due to vacuum arc which is unavoidable especially during disconnecting. Particularly harmful is multiple arcs electric discharge with very non-uniform distribution of current on the surface of electrodes. To avoid such possible local high concentration of current an external axial magnetic field is applied in vacuum interrupters [1, 2, 3, 4]. Because of shielding effect in plasma (Yukawa-type electric potential $(1/r)\exp(-\lambda_D/r)$, λ_D - Debye's radius) magnetic forces can dominate over electric ones leading to pinch effect (parallel currents attract). The strong external axial magnetic field dominates over the self magnetic fields and can radially confine plasma in so called *diffused mode* with uniform distribution of current.

We find stationary solutions of Vlasov-Maxwell equations describing radially symmetric electron plasma column in macroscopic approximation. Next we obtain also the thermal equilibrium solutions. We use these solutions to analyze mechanism of smearing of local high density currents.

We propose the configuration of electrodes presented in Fig.3 self-producing high external axial magnetic field.

Stationary solutions

In short time scale which depend on density of particles binary collisions can be neglected and plasma is described by Vlasov-Maxwell equations [6, 7, 8]:

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla_{\mathbf{x}}+ q_j\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times(\mathbf{B}+\mathbf{B}_{ext})\right)\right)\cdot\nabla_{\mathbf{p}} f_j(t,\mathbf{x},\mathbf{p})=0$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}$$

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\sum_j q_j\int d^3p\ \mathbf{v}\ f_j(t,\mathbf{x},\mathbf{p})+\frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}$$

$$\nabla\cdot\mathbf{E}=4\pi\sum_j q_j\int d^3p\ f_j(t,\mathbf{x},\mathbf{p})$$

$$\nabla\cdot\mathbf{B}=0$$

where: $\mathbf{E},\mathbf{B}$ – electric and magnetic fields, $\mathbf{B}_{ext}=B_0\mathbf{e}_z$, $B_0 = const$, c – light velocity, q_j – charge of j-component of plasma, f_j – one-particle distribution function of j-component of plasma. Thus, $f_j(t,\mathbf{x},\mathbf{p})d^3x d^3p$ – number of particles of j-component located in volume $d^3x d^3p$ at the phase space point $(\mathbf{x},\mathbf{p})$ at time t, $n_j(t,\mathbf{x}) = \int d^3p\ f_j(t,\mathbf{x},\mathbf{p})$ – particle density of j-component, $\rho(t,\mathbf{x}) = \Sigma_j q_j n_j(t,\mathbf{x})$ – charge density, $\mathbf{J}(t,\mathbf{x}) = \Sigma_j q_j \int d^3p\ \mathbf{v}\ f_j(t,\mathbf{x},\mathbf{p})$ – current density.

For long time binary collisions lead to thermal equilibrium (Maxwell distribution) which is the solution of Boltzmann equation (Vlasov equation with collision term on the right-hand side).

It is convenient to describe plasma in terms of dimensionless parameters: $\omega_{pj}=\sqrt{4\pi n_j q_j^2/m_j}$ - *plasma frequency* of j-component (m_j – mass of j-component), $\omega_{cj}=|q_j\mathbf{B}/m_j c|$ - *cyclotron frequency* of j-component.

We consider nonneutral electron ($q_j = -e$, $m_j = m_e$) plasma column of infinite axial extent confined radially by external uniform axial magnetic field as it is shown in Fig.1. We use the cylindrical coordinates (r, φ, z) where: $x = r\cos\varphi$ and $y = r\sin\varphi$.

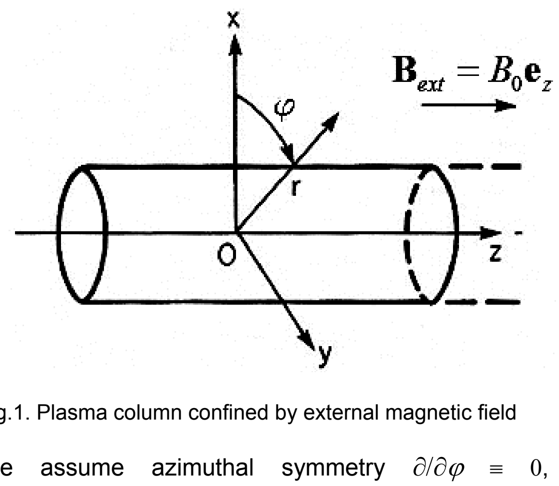

Fig.1. Plasma column confined by external magnetic field

We assume azimuthal symmetry $\partial/\partial\varphi \equiv 0$, and independence on z-coordinate $\partial/\partial z \equiv 0$. Infinite extension of plasma column means that processes in neighbourhood of electrodes are neglected. We simply assume that charges produced at one electrode are annihilated at the other one. Independence on z-coordinate corresponds to our goal of finding distribution in x-y plane of current lines (current density $\mathbf{J}(t, \mathbf{x})$ integrated with respect to z).

As first step we check if there exist the stationary solution corresponding to ideally diffused mode described by the rectangular density profile

(1) $$n(r) = \begin{cases} n = \text{const}, & 0 \le r \le r_b \\ 0, & r > r_b \end{cases}$$

In this case charge density $\rho(r)$ is constant ρ for $0 \le r \le r_b$ and equal to zero for $r > r_b$. So the steady-state solution ($\partial/\partial t \equiv 0$) for electric field $E_r(r)$ is well known solution of electrostatic Poisson's equation (Gauss law and symmetry consideration)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rE_r(r)\right) = -4\pi\rho(r)$$

in the form

$$E_r(r) = \begin{cases} 2\pi\rho r, & 0 \le r \le r_b \\ 2\pi\rho \dfrac{r_b^2}{r}, & r > r_b \end{cases}$$

For electron plasma this solution takes form

(2) $$E_r(r) = \begin{cases} -\dfrac{m_e}{2e}\omega_{pe}^2 r, & 0 \le r \le r_b \\ -\dfrac{m_e}{2e}\omega_{pe}^2 \dfrac{r_b^2}{r}, & r > r_b \end{cases}$$

where $\omega_{pe} = \sqrt{4\pi n e^2 / m_e}$ - electron plasma frequency.

Motion in the plane perpendicular to z-axis in the plasma column follows from radial forces balance between outward centrifugal and electrical forces and inward magnetic force:

$$-m_e\Omega^2(r)r = -eE_r(r) - \frac{e}{c}\Omega(r)rB_0$$

where: $\Omega(r)$ – angular velocity. Substituting the solution (2) and using the definition of electron cyclotron frequency $\omega_{ce} = eB_0 / m_e c$ we obtain the following algebraic equation for angular velocity:

(3) $$-\Omega^2 + \omega_{ce}\Omega - \frac{1}{2}\omega_{pe}^2 = 0\ , \quad \text{for } 0 \le r \le r_b$$

Solutions of the equation (3)

(4) $$\Omega = \omega^{\pm} \equiv \frac{\omega_{ce}}{2}\left(1 \pm \sqrt{1 - \frac{2\omega_{pe}^2}{\omega_{ce}^2}}\right)$$

do not depend on r so plasma treated as charge fluid behaves as *rigid body* (rigid rotator) in the case of rectangular density profile. Solutions $\omega^{\pm}$ as functions of the parameter $s_e = 2\omega_{pe}^2 / \omega_{ce}^2$ are shown in the picture Fig.2. Equilibrium is possible only for $s_e \le 1$ and this condition gives us estimation for minimum value of applied external magnetic field

$$B_0 \ge \sqrt{8\pi n e m_e c^2}$$

in the case of electron plasma.

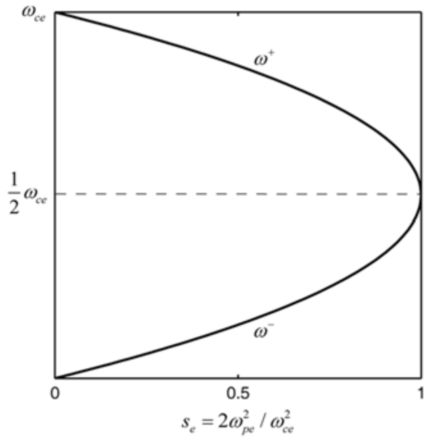

Fig.2. Rigid rotor solutions for angular velocity

Thermal equilibrium

The above simple model of ideally diffused mode with rectangular density profile corresponds to the so called *cold plasma* when binary collisions between particles are neglected. More realistic model is described by Boltzmann-Maxwell equations. It is very difficult problem to find solutions of these equations, even approximate ones, for finite time. However for infinite time binary collisions lead to thermal equilibrium described by Maxwell distribution. In practice, for typical time of disconnecting an interrupter, plasma in vacuum arc is in the state close to thermal equilibrium. Thus the thermal equilibrium solutions give us useful information about evolution of vacuum arc.

So we postulate the specific form of one-particle distribution function corresponding to Maxwell distribution in thermal equilibrium. Such a distribution function is used in Poisson equation.

It is well known fact that one-particle distribution function depending on $\mathbf{x}$ and $\mathbf{p}$ only through constants of motion of a single particle in self-consistent equilibrium electromagnetic field solves exactly the steady-state ($\partial / \partial t \equiv 0$) Vlasov equation [5,6,7]. This could be verified by direct substitution of such a distribution function into Vlasov equation. In our rigid-rotor equilibrium there are three following constants of motion:

1. Energy of a particle in electrostatic potential ϕ ($E_r(r) = -\partial\phi / \partial r$):

$$H = \frac{1}{2m_e}(p_r^2 + p_\varphi^2 + p_z^2) - e\phi$$

2. Canonical angular momentum:

$$P_\varphi = r(p_\varphi - m_e r\omega_{ce} / 2)$$

3. Axial momentum: p_z

In our case of mean azimuthal motion being rigid rotation with angular velocity Ω = const the effective energy is equal to $H - \Omega P_\varphi$ and the general steady-state solution has the form:

$$f(r, \mathbf{p}) = f(H - \Omega P_\varphi, p_z)$$

In thermal equilibrium of electron plasma the distribution function takes Maxwell distribution form with respect to effective energy

(5) $$f_e(r, \mathbf{p}) = \frac{n}{(2\pi m_e k_B T_e)^{3/2}} \exp\left(-\frac{H - \Omega P_\varphi}{k_B T_e}\right)$$

The density profile corresponding to Maxwell distribution function (5) is equal

$$n_e(r) = \int d^3p\, f_e(r, \mathbf{p}) =$$

(6)
$$n \exp\left(-\frac{m_e}{2k_B T_e} \left[r^2(\Omega\omega_{ce} - \Omega^2) - \frac{2e}{m_e}\phi(r) \right] \right)$$

We have to substitute the density profile (6) into Poisson equation

(7)
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \phi(r)}{\partial r} \right) = -4\pi\rho(r) = 4\pi e n_e(r)$$

Because this equation is highly nonlinear we can not find analytical solutions of it. Numerical example solution for electron plasma with parameters $s_e = 2\omega_{pe}^2 / \omega_{ce}^2 = 0.5$ and $\Omega = 1.0001\omega^-$ is illustrated in picture Fig.3.

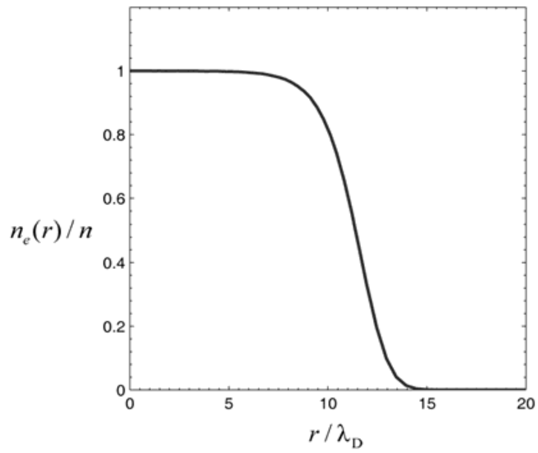

Fig.3. Example of thermal equilibrium density profile for electron plasma

The diameter of the plasma column is approximately given by $2r_b \approx 15\lambda_D$ where λ_D is Debye's radius.

Diffusing of local high currents

Now we perform asymptotic analysis of solution (6) and equation (7) based on the numerical solution presented in Fig.3. For typical situation shown in this figure the density profile is almost constant in the region $0 \le r \le r_b$ not too different from the rectangular profile (1). So in this region electric field is approximately given by (2). Electrostatic potential corresponding to this field is of the form

(8)
$$\phi(r) \approx (m_e / 4e)\omega_{pe}^2 r^2$$

For $0 \le r \ll r_b$ this approximation is very good. For $r \gg r_b$ asymptotic form of the potential corresponding to (2) is of the form $\phi(r) \approx (m_e / 4e)\omega_{pe}^2 r_b^2 + (m_e / 2e)\omega_{pe}^2 r_b^2 \ln(r / r_b)$ (continuity condition at $r = r_b$ was used).

With the asymptotic approximation (8) the density profile (6) is equal to

(9)
$$n_e(r) = n \exp\left(-\frac{m_e}{2k_B T_e} r^2 W(\Omega) \right)$$

where the polynomial $W(\Omega)$ is given by

(10)
$$W(\Omega) = -\Omega^2 + \omega_{ce}\Omega - \omega_{pe}^2 / 2$$

From the form of exponential damping factor in (9) and asymptotic form of the electrostatic potential in whole region it is easily seen that plasma column is radially confined ($n_e(r \to \infty) = 0$) provided $W(\Omega) > 0$. Comparing (10) and (3), (4) we see that this condition is equivalent to

(11)
$$\omega^- < \Omega < \omega^+$$

for a given parameter s_e, that is $W(\Omega) > 0$ between roots of the polynomial W. Thus all values of the angular velocity of rigid rotation laying inside the parabola shown in Fig.2 are now possible.

In general the density profile $n_e(r)$ is bell-shaped as is illustrated in Fig.3. However for values of Ω near to ω^+ or ω^- the profile $n_e(r)$ falls more abruptly tending to the rectangular profile (1). If no special means are applied then plasma starts to rotate with the smallest possible angular velocity. So $\Omega \approx \omega^-$ as in numerical solution presented in Fig.3.

Now we are ready to consider the problem of diffusing of local high current discharge. Suppose that in a some small region of the diameter ~2mm the big number N_e of electrons was initially injected from one of the electrodes. Without external magnetic field this concentrated current could lead to local erosion of the electrodes. Due to the pinch-effect concentration of current can even rise. With axial external magnetic field applied this concentration is smeared over bigger region and the density profile is given by (6). Now we use the numerical solution shown in Fig.3 because this solution corresponds to typical experimental situation. For comparison we choose such a number of electrons N_e that corresponds to the axial density n of the numerical solution for (6). We assume that the initial local current is diffused over the whole surface of an electrode. Typical radius of electrodes is equal to 3-5cm. So from Fig.3 we have reasonable estimation $10\lambda_D \approx$ 4cm. From the thermal electron Debye's radius

(12)
$$\lambda_D = \sqrt{\frac{k_B T_e}{4\pi n e^2}} \approx 0.4\mathrm{cm}$$

where T_e ~ 1eV is the electron temperature (work function) we find the axial electron density

$$n \approx 2.3 \cdot 10^6 \mathrm{cm}^{-3}$$

Total number of electrons in column of length ~ 1cm is obtained by integration of the density profile of the type presented in Fig.3. It is enough to approximate this profile by rectangular profile ending at $10\lambda_D \approx$ 4cm so the value of this integral can be estimated as follow

(13)
$$N_e \approx n \int_0^{2\pi} d\varphi \int_0^4 r dr = 32\pi n$$

If such a number of electron is concentrated in the region of diameter ~2mm, that is with radius 40 times smaller, then concentration of electrons grows 40^2 = 1600 times to the value

(14)
$$n_c \approx 3.7 \cdot 10^9 \mathrm{cm}^{-3}$$

In the case without external magnetic field such a high local concentration (14) of electrons can further grow as the collisions of these electrons with electrodes produce new electrons and positive ions. This can lead to local high concentrated currents that are particularly harmful. They are main cause of erosion of electrodes.

When the external axial magnetic current is applied these local currents are smeared out over the whole surface of an electrode producing the so called *diffused mode of vacuum arc* with uniform distribution of a current. Formally this diffused mode is achieved in infinite time corresponding to transition to thermal equilibrium. In practice this process is very quick. The time t_d of diffusing of high local currents can be estimated as a few relaxation times t_r for electron plasma with given electron concentration. The predicted by collisional transport theory [8] value of t_r corresponding to concentration (14) is of order t_r ~10^{-5}sec. For decreasing value of n_c during diffusing process the relaxation time grows according to

$$t_r \sim \frac{1}{n_c}$$

At the end of the diffusing process the value of t_r is ~1600 times bigger

$$t_r \sim 1.6 \cdot 10^{-2}\, s$$

Thus it is reasonable to use the value of t_r equal to

$$t_r \sim 3 \cdot 10^{-4}\, s$$

and estimate the diffusing time as

$$t_d \approx 3t_r \sim 10^{-3}\, s \qquad (15)$$

The diffusing time t_d ~ 1ms much smaller than the half of cycle of current in power network and also much smaller than time of disconnecting electrodes of interrupter. The experiment [9] demonstrates relaxation time up to 5000 times smaller than predicted by collisional transport theory. This is explained by high nonlinearity of the process. However experimental parameters of plasma where quite different from our problem.

Discussion and conclusions

We presented the so called *rigid body rotator* solutions of Vlasov-Maxwell equations in macroscopic approximation for the cylindrical plasma column in the external axial magnetic field. Next these solutions where generalized to thermal equilibrium solutions which model the diffused mode of vacuum arc in high current interrupters. Because obtained equations are highly nonlinear we can not find exact analytical solutions of them. We used the typical numerical solution of obtained equations and asymptotic analysis based on the form of this solution to get insight into the process of smearing of local high concentration currents over the surface of electrodes. This analysis shows the fast relaxation of local current towards diffuse mode in thermal equilibrium with decreasing of electron concentration more than 1000 times.

As our analysis is only approximate the experimental verification is highly desired. Scaling to smaller diameter of electrodes and corresponding smaller currents will simplify the experimental setup. Fast cameras can be used to observe uniformity of the discharge during disconnecting electrodes. After appropriate number of cycles of discharge the surface of electrodes should be inspected and compared with interrupters without external magnetic field. The concept of electrodes construction self-producing high external axial magnetic field is presented in Fig.4.

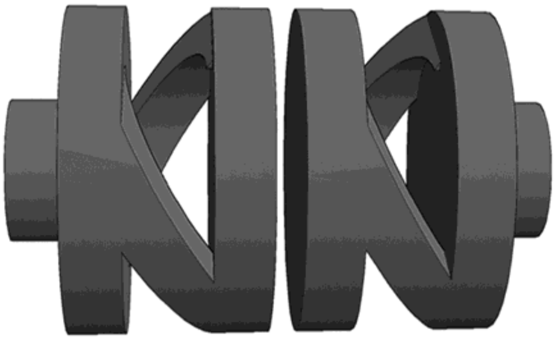

Fig.4. Electrodes self-producing external axial magnetic field

In the future we plan to check if other exact analytical solutions of Vlasov-Maxwell equations exist and looking for them. Advanced methods such as the symmetry analysis of equations (Lie group theory) seems to be useful to this end. More detailed and exact description of an arc discharge in interrupters needs the use of full kinetic theory (Boltzmann equation). The most desirable but also most difficult is to construct method of finding time depending solutions of Boltzmann-Maxwell equations. Our only hope is to find such an approximate method (asymptotic or perturbation theory) based on canonical exact solutions.

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