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Decision-Making under uncertainty using Info-Gap Theory and a new multi-dimensional RDM interval arithmetic

***Abstract**. Decision-making is an integral part of technical problem-solving. In this study, two decision theories, which are capable to deal with uncertain information – Info-Gap Theory and RDM (Relative-Distance-Measure) interval arithmetic, are presented. The paper shows some aspects of the robustness function and uses each approach to evaluate the example of charging car battery. The comparison of Info-Gap Theory and RDM-arithmetic not only improves understanding of these methods, it also suggests some broader insights into robustness understanding.*

***Streszczenie.** Podejmowanie decyzji jest nieodłączną częścią rozwiązywania technicznych problemów. W artykule zaprezentowano dwie metody, które rozwiązują problemy z niepewnymi danymi – Teoria Luk Informacyjnych i arytmetyka interwałowa RDM (Relative-Distance-Measure). Przedstawiono działanie każdej z metod na przykładzie ładowania akumulatora samochodowego. Porównanie Teorii Luk Informacyjnych i arytmetyki RDM nie tylko umożliwia głębsze poznanie tych metod, ale również sugeruje pewne szersze spojrzenie na rozwiązywanie problemów w warunkach niepewności. (**Podejmowanie decyzji w warunkach niepewności z wykorzystaniem Teorii Luk Informacyjnych i nowej wielowymiarowej arytmetyki interwałowej RDM**).*

Keywords: Info-Gap Theory, uncertainty, RDM-arithmetic, robustness function.
Słowa kluczowe: Teoria Luk Informacyjnych, niepewność, arytmetyka RDM, funkcja odporności.

Introduction

Decision-making under uncertainty is the process of drawing conclusions from limited information or conjecture. Uncertainty problem considered in the paper is of non-probabilistic nature. It means, there is no information on probabilities and there are not any probability distributions. Along with the development of science and technology, our understanding of uncertainty has been gradually deepened and the research concerning it has reached at a new height. Of course it is still extremely difficult to predict something precisely, but available methods and tools try to give us alternative as good as possible. Among these methods and theories we can mention: fuzzy mathematics by L.A. Zadeh (1960s) [1], interval arithmetic by R.E. Moore (1960s) [2], grey systems theory by J. Deng (1980s) [3,4], rough set theory by Z. Pawlak (1980s) [5], uncertainty mathematics by H. Bandemer (2005) [6], etc. All these works represent some of the most important efforts in the research of dealing with uncertainty. However, many scientists still try to improve the results therefore they introduce and suggest new methods, and two of them are presented in this paper. This study compares possibilities of Info-gap Theory developed by Y. Ben-Haim (2001) [7,8] and RDM interval arithmetic developed by A. Piegat (2012) [9,10]. (Do not confuse RDM-arithmetic with other RDM method – Robust Decision Making by Lempert, Popper, Bankes (2003) [11]). The two offer an interesting comparison because both provide quantitative decision analytic models designed to evaluate robust strategies using imprecise and potentially contentious information. In Poland Info-Gap Theory is investigated by scientific team on Westpomeranian University of Technology in Szczecin [12,13,14]. This study applies both Info-gap Theory and RDM-arithmetic to one test case, using the same models and data, and then compares and contrasts the results.

Description of Info-Gap Theory

Info-Gap Decision Theory is used for supporting model-based decisions with a lack of information. Info-gaps are non-probabilistic and cannot be insured against or modeled probabilistically. Examples of common info-gaps include uncertainty regarding the shape of a probability distribution, the functional form of a relationship between entities, or the values of some key parameters. The most important part of this methodology are info-gap models of uncertainty. An info-gap model is an unbounded family of nested sets that share a common structure. A frequently encountered example is a family of nested ellipsoids that have the same shape [7]. The structure of the sets in an info-gap model depends on information about the uncertainty. In general terms, the structure of an info-gap model of uncertainty is chosen to define the smallest or strictest family of sets whose elements are consistent with the prior information.
A common example of an info-gap model, which can be characterized as follows [7,8]:

$$U(\alpha,\tilde{u}) = \{u : | u - \tilde{u} | \le \alpha\}, \alpha \ge 0 \tag{1}$$

Here, $\tilde{u}$ denotes the best estimate of an uncertain function u, while the fractional error from this estimate, α, is unknown. Info-Gap Theory assumes that $\tilde{u}$ represents a poor guess at the true values of the parameters. At any level of uncertainty α, the set $U(\alpha,\tilde{u})$ contains all functions u whose fractional deviation from $\tilde{u}$ is no greater than α [15]. Uncertain variations may be either adverse or favorable. Adversity entails the possibility of failure. A robustness function expresses the greatest level of uncertainty at which failure cannot occur. More precisely, the robustness function can be expressed as the maximum value of the uncertainty parameter α of an info-gap model [7]:

$$\hat{\alpha}(q) = \max\{\alpha : \text{min requirements are always satisfied}\} \tag{2}$$

$$\hat{\alpha}(q) = \max_{\alpha \ge 0}\{\alpha : \forall u \in U(\alpha,\tilde{u})\}$$

Here, q denotes a vector of decision variables such as time of initiation, choice of a model or its parameters, or operational options. Equation expresses that robustness of q, $\hat{\alpha}(q)$, is the greatest level of uncertainty α, or the greatest possible variation, for which specified minimal requirements are always satisfied. $\hat{\alpha}(q)$ expresses robustness — the degree of immunity against errors or deviations from ones' assumptions — so a large value of $\hat{\alpha}(q)$ is desirable. The robustness function involves maximization of the uncertainty, or the range of variation in a variable, parameter or model, at which decision q would satisfy the performance at a tolerable level [7]. The robustness function specifies the trade-offs associated with a policy that one faces in a given situation.

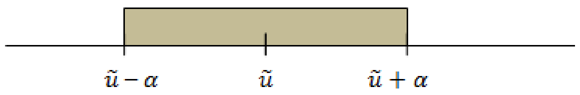

Fig. 1. The space of possible values of uncertain variables

The robustness function reflects the pernicious effects of uncertainty. To help inform decision makers, Info-gap Theory presents visualizations showing robustness for each strategy as a function. Typically, uncertainty is plotted on the y-axis and target performance values on the x-axis and robustness describes the maximum level of uncertainty that can be borne while ensuring a given "critical" outcome. Info-gap theory has been studied or applied in a range of applications including engineering, biological conservation, theoretical biology, homeland security, economics, project management and statistics. In this study it is presented in the numerical example of mechanical problem.

Description of RDM-arithmetic

RDM-arithmetic is a new approach to an interval arithmetic. Typical interval arithmetic has many faults that rather are known [16], for example *the excess width effect problem* or *dependency problem*. RDM-arithmetic was found to eliminate all these defects. The abbreviation RDM means Relative Distance Measure. An information piece in this method is given as a variable x, which has a value contained in interval $x \in [\underline{x}, \overline{x}]$, where $\underline{x}$ is the lower limit and $\overline{x}$ is the upper limit of the interval.

Thus variable x can be described with formula [9]:

(2) $$x \in [\underline{x}, \overline{x}] : x = \underline{x} + \alpha_x(\overline{x} - \underline{x}), \alpha_x \in [0,1]$$

Variable α_x can be interpreted as measure of relative distance and is illustrated in Figure 2.

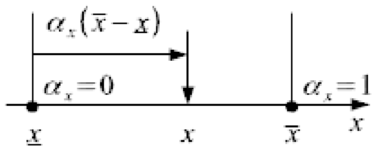

Fig. 2. Illustration of notion Relative Distance Measure

Let us consider addition of two intervals.

(3) $$[\underline{a}, \overline{a}] + [\underline{b}, \overline{b}] = [\underline{x}, \overline{x}]$$

Using RDM variables equation (4) can be transformed in (5).

(5) $$\underline{a}, +\alpha_a(\overline{a} - \underline{a}) + \underline{b}, +\alpha_b(\overline{b} - \underline{b}) = x,$$
$$\alpha_a \in [0,1], \alpha_b \in [0,1]$$

Depending on values of variables α_a and α_b the resulting variable x assumes various values. It should be noted that this sum is 3-dimensional: it depends on 2 variables α_a and α_b. Table 1 shows values of x for border values of RDM-variables α_a and α_b.

Table 1. Values of the sum for various border values of RDM-variables

α_a α_b	0 0	0 1	1 0	1 1
x	$(\underline{a}+\underline{b})$	$(\underline{a}+\overline{b})$	$(\overline{a}+\underline{b})$	$(\overline{a}+\overline{b})$

After projecting the input granule on the functional addition surface a 3D-solution granule is achieved. RDM-arithmetic has the same operations as interval arithmetic, but this method is free of interval arithmetic's defects. More details on RDM interval arithmetic can be found in [9,10] and on Andrzej Piegat's web page.

Full understanding of Info-Gap Theory and RDM-arithmetic requires training on problem examples. Therefore, in the next chapter an interesting problem is presented. Charging car battery is an appropriate problem to be solved using methods under uncertainty, not only Info-Gap Theory but RDM-arithmetic as well.

Numerical example

The automotive service has to charge discharged batteries 90 [Ah]. According to the employees' opinion a battery charge status is between 10 [Ah] and 20 [Ah] and it is an uncertain information. The discharged battery draws a charge current of typically 9 [A] to 11 [A]. Simple chargers do not regulate the charge current, and the user needs to stop the process or lower the charge current to prevent excessive gassing of the battery.

The formula of charging car battery is described:

(6) $$q(t) = q_0 + i \cdot t$$

where: $\tilde{q}_0$ = 15 [Ah] the nominal value of q_0 provided by an expert; α_{q_0} = 5 [Ah] the average deviation of batteries' charge; $\tilde{i}$ = 10 [A] the nominal value of i provided by an expert; α_i = 1 [A] the average deviation of the current level.

The question is how long the batteries should be charging to the final state q ranged [85, 90] [Ah]. An important clue is the fact that overcharging batteries is more dangerous than undercharging.

This information allows to create uncertainty model of initial charge q_0 and charging current i, presented below and based on (1):

(7) $$U(\alpha) = \{(q_0, i) : q_0 + i \cdot t \in [85, 90],$$
$$\frac{|q_0 - 15|}{5} < \alpha, \frac{|i - 10|}{1} < \alpha\}, \alpha \geq 0$$

where α is the horizon of uncertainty.

The exact values of initial charge q_0 and charging current i are unknown, so there is no possibility to determine time t. However, Info-Gap Theory allows to determine it so accurately, safely and sensibly as possible within incomplete of knowledge.

The robustness function

The robustness function assesses the greatest tolerable horizon of uncertainty. The robustness function is based on a satisficing performance requirement. The info-gap robustness model in this case is as follows:

(8) $$\hat{\alpha}(t) = \max_{\alpha \geq 0}\{\alpha : q_0 + i \cdot t \in [85, 90],$$
$$\forall (q_0, i) \in U(\alpha, \tilde{u}), \tilde{u} = (\tilde{q}_0, \tilde{i})\}$$

This function is given an uncertainty model and a minimum level of desired outcome. The minimum level of final charge q is 85 [Ah].

(9) $$q = q_0 + i \cdot t \quad \rightarrow \quad q = (15 - 5 \cdot \alpha) + (10 - \alpha) \cdot t$$

$q = q_{\min} = 85 = (15 - 5\cdot\alpha) + (10 - \alpha)\cdot t$, $\hat{\alpha} = \frac{10\cdot t - 70}{t+5}$

The horizon of uncertainty $\hat{\alpha}$ informs us how much may fall q_0 and i below their nominal value to secure the critical minimal charge of battery. The maximum level of final charge q is 91 [Ah].

$q = q_{\max} = 91 = (15 + 5\cdot\alpha) + (10 + \alpha)\cdot t$, $\hat{\alpha} = \frac{76 - 10\cdot t}{t+5}$

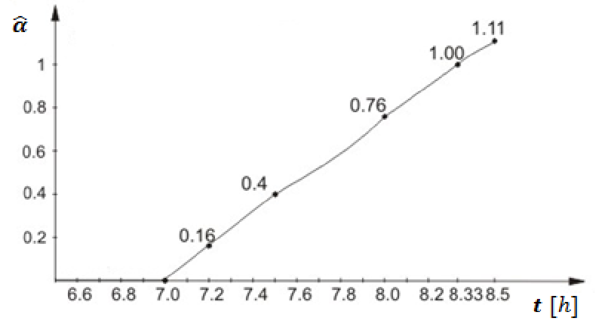

Fig. 3a. The robustness function of time t in a case of undercharging

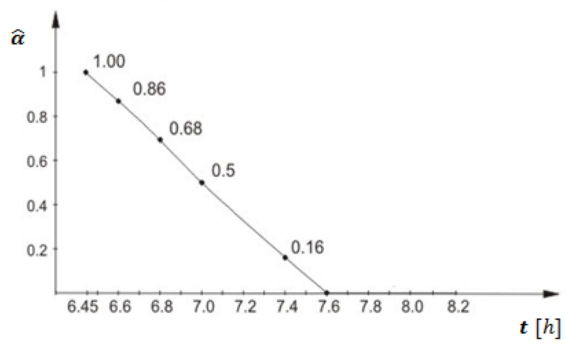

Fig. 3b. The robustness function of time t in a case of overcharging

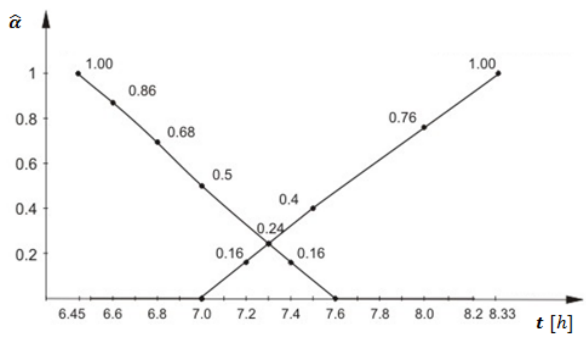

Fig. 4. The final robustness function of time t and i

(10) $\frac{10\cdot t - 70}{t+5} = \frac{76 - 10\cdot t}{t+5} \quad \rightarrow \quad t = 7,3$

However, overcharging batteries is more risky than undercharging because it can damage car battery and this information should be taken into account. The final decision is implemented in Figure 5.

This 2-dimensional graph shows the sense of the dangers present in the charging process. If at the same time both the state q_0< 14,2 [Ah] and i < 9,84 [A], it is certain that battery can be undercharged and if q_0> 16,5 [Ah] and i > 10,33 [A] than it can be overcharged.

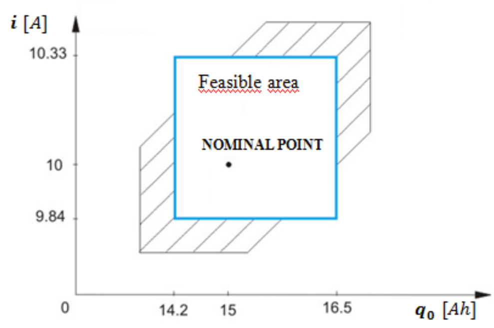

Figure 5. The full space of uncertain variables q_0 resolving a problem

The same problem can be solved using RDM method. Maintaining all the signs associated with this method, the formula of charging car battery takes form (11).

Initial charge - $q_0 \in [10;20]$ [Ah]

Charging current - $i \in [9;11]$ [A]

(11) $$[10;20] + [9;11]\cdot t = [85;91]$$

Upper $\overline{t}$ and lower $\underline{t}$ time of charging should be found. The first step is an inaccurate conventional solution delivered by Moore's arithmetic [2]:

$$[10;20] + [9;11]\cdot[\underline{t};\overline{t}] = [85;91]$$

$$10 + 9\cdot\underline{t} = 85 \quad \rightarrow \quad \underline{t} = \frac{75}{9} = 8,333$$

$$20 + 11\cdot\overline{t} = 91 \quad \rightarrow \quad \overline{t} = \frac{71}{11} = 6,455$$

$$t \in [8,333;6,455]$$

The solution is absurd because $\overline{t} < \underline{t}$. Moore's interval arithmetic is not able to solve this problem, therefore RDM-arithmetic [9] will be applied.

In equation (11) particular intervals are models of approximately known values of q_0 , i , t and q . The model of interval type $[\underline{q}_0, \overline{q}_0]$ used by Moore's arithmetic defines only the outer limits of the interval without any information about interior. RDM interval arithmetic introduces internal variables α , $\alpha \in [0,1]$ which concern also interior.

Formulas (12) show the interval models in terms of RDM arithmetic.

$$[\underline{q}_0, \overline{q}_0]: \ q_0 = \underline{q}_0 + \alpha_{q_0}(\overline{q}_0 - \underline{q}_0), \alpha_{q_0} \in [0,1]$$

(12) $$[10;20]: \ q_0 = 10 + 10\cdot\alpha_{q_0}, \alpha_{q_0} \in [0,1]$$

$$[\underline{i}, \overline{i}]: \ i = \underline{i} + \alpha_i(\overline{i} - \underline{i}), \alpha_i \in [0,1]$$

$$[9;11]: \ i = 9 + 2\cdot\alpha_i, \alpha_i \in [0,1]$$

The new interval models express the fact that in the case of a particular car battery it occurs only one value of q_0 , i , q , not the set of values of these variables.

Thus, equation (9) $q = q_0 + i\cdot t$ can be formulated in this form (13):

(13) $$(10+10\cdot\alpha_{q_0})+(9+2\alpha_i)\cdot t=q,$$
$$\alpha_{q_0}\in[0,1],\alpha_i\in[0,1]$$

It should be pointed that in equation (9) q (final state) and t (charging time) were variables and q_0 and i were constants. It was linear 2-dimensional model. Using RDM arithmetic nonlinear 4-dimensional model (13) is received, where α_{q_0}, α_i, t and q are variables. A significant fact is that each inconstant parameter increases dimensionality of model by 1 and increases the degree of non-linearity. Thus solving problems with uncertainty is more difficult than solving problems without uncertain parameters. Charging time t has to be chosen fulfilling minimum (14) and maximum (15) inequalities.

(14) $$(10+10\cdot\alpha_{q_0})+(9+2\alpha_i)\cdot t\geq 85,$$

(15) $$(10+10\cdot\alpha_{q_0})+(9+2\alpha_i)\cdot t\leq 91,$$
$$\alpha_{q_0}\in[0,1],\alpha_i\in[0,1]$$

Taking into account (14) and (15) minimum time $t_{\min}$ and maximum time $t_{\max}$ of charging car battery can be calculated.

(16) $$t_{\min}=\frac{75-10\cdot\alpha_{q_0}}{9+2\cdot\alpha_i},$$

(17) $$t_{\max}=\frac{81-10\cdot\alpha_{q_0}}{9+2\cdot\alpha_i},$$
$$\alpha_{q_0}\in[0,1],\alpha_i\in[0,1]$$

As shown, $t_{\min}$ and $t_{\max}$ are functions of two variables: $t_{\min}=\underline{f}(\alpha_{q_0},\alpha_i)$ and $t_{\max}=\overline{f}(\alpha_{q_0},\alpha_i)$.

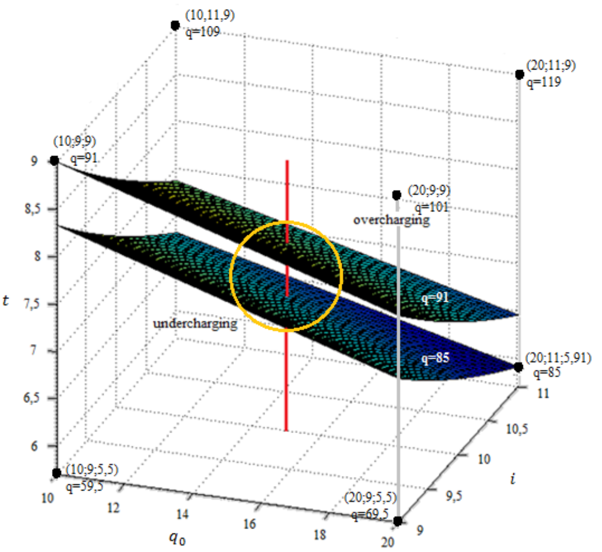

Figure 6. The location of undercharging bound $\underline{t}=\underline{f}(\alpha_{q_0},\alpha_i)$ and overcharging bound $\overline{t}=\overline{f}(\alpha_{q_0},\alpha_i)$ and the nominal charging line in the car battery's state space

The bounds of $t_{\min}$ and $t_{\max}$ are defined by equations (16) and (17) and they determine information granule, which is a space of acceptable status for charging car battery. The states which are under or over this information granule lead to undercharging or overcharging car battery. It is shown on Figure 6. A red vertical line is called nominal charging line and it applies to the most common charging car battery status defined by two nominal values $\tilde{q}_0$ and $\tilde{i}$ (18).

(18) $\tilde{q}_0=15$ [Ah], $\tilde{\alpha}_{q_0}=0,5$ $\tilde{i}=10$ [A], $\tilde{\alpha}_i=0,5$

The aim of this research is to determine nominal charging time $\tilde{t}$, which will be well matched to other nominal variables $\tilde{q}_0$ and $\tilde{i}$ (18). Generally speaking, time t for any charging status (q_0,i) should satisfy the following condition:

(19) $$t\in[t_{\min},t_{\max}]=[\underline{t},\overline{t}]=\left[\frac{75-10\cdot\alpha_{q_0}}{9+2\cdot\alpha_i},\frac{81-10\cdot\alpha_{q_0}}{9+2\cdot\alpha_i}\right],$$
$$\alpha_{q_0}\in[0,1],\alpha_i\in[0,1]$$

The condition (19) is written in a conventional interval arithmetic way. Using RDM arithmetic, the value of charging time can be noted with variable α_t in the following form:

(20) $$t=\underline{t}+\alpha_t(\overline{t}-\underline{t})=\frac{75-10\cdot\alpha_{q_0}}{9+2\cdot\alpha_i}+\alpha_t\left(\frac{81-10\cdot\alpha_{q_0}}{9+2\cdot\alpha_i}-\frac{75-10\cdot\alpha_{q_0}}{9+2\cdot\alpha_i}\right)=\frac{75+6\cdot\alpha_t-10\cdot\alpha_{q_0}}{9+2\cdot\alpha_i}$$
$$\alpha_{q_0}\in[0,1],\alpha_i\in[0,1],\alpha_t\in[0,1]$$

This interval model of charging time is not defined by 2 boundary values like in Moore's arithmetic but by 2 boundary functions $\underline{t}=\underline{f}(\alpha_{q_0},\alpha_i)$ and $\overline{t}=\overline{f}(\alpha_{q_0},\alpha_i)$.

Inserting values ($\tilde{\alpha}_{q_0}$ = 0,5, $\tilde{\alpha}_i$ = 0,5) for nominal charging status to equations (16) and (17), the minimum and maximum time can be calculated. Thus, the nominal time $\tilde{t}$ which is matched to $\tilde{q}_0$ = 15 [Ah] and $\tilde{i}$ = 10 [A] should satisfy the following condition:

(21) $$\tilde{t}\in[\underline{t},\overline{t}]=[7,0;7,6]$$

In RDM arithmetic an interval (21) takes the form:

(22) $$\tilde{t}=\underline{t}+\alpha_t(\overline{t}-\underline{t})=7,0+0,6\cdot\alpha_t,\ \alpha_t\in[0,1]$$

The value α_t means a distance of nominal charging state from the undercharging boundary.

If it assumes that $\alpha_t=\frac{1}{6}$, then t = 7,1 [h] and a nominal point will be too close to undercharging boundary, which is shown on Figure 7.

In the case that a real charging state is totally different from the nominal state ($\tilde{q}_0$ = 15, $\tilde{i}$ = 10) there would be considerable risk of undercharging this car battery.

Assuming a nominal charging time $\tilde{t}$ = 7,3 h, which is a mean of acceptable range $\tilde{t}\in[7,0;7,6]$, where α_t = 0,5 thus the distance of nominal charging state ($\tilde{q}_0$ = 15, $\tilde{i}$ = 10) from the undercharging boundary ($\underline{q}=85$ [Ah]) and overcharging boundary ($\overline{q}=91$ [Ah]) is the same (see Fig. 8.).

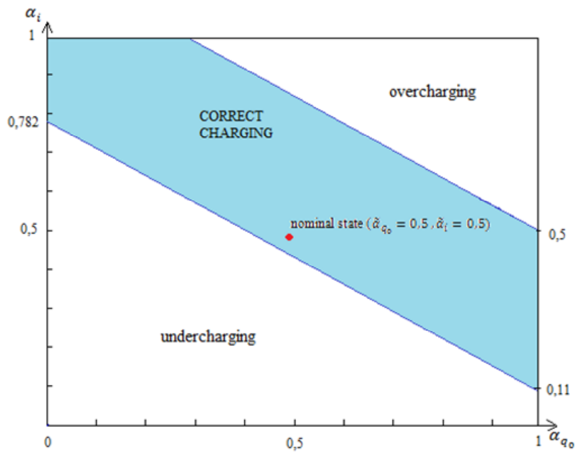

Figure 7. The space of correct and incorrect states of the car battery charging process for charging time t = 7,1 h.

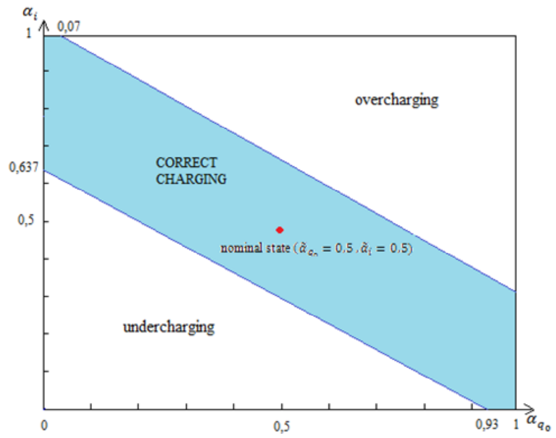

Figure 8. The space of correct and incorrect states of the car battery charging process for charging time t = 7,3 [h] and α_t = 0,5.

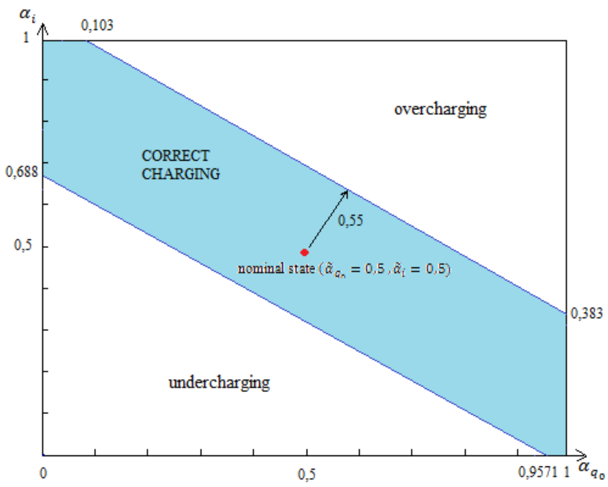

Figure 9. The position of nominal charging state for charging time t = 7,27 [h] and α_t = 0,45.

It should be noted that overcharging car battery is more dangerous than undercharging it.
Hence, the distance from the overcharging boundary $\overline{q}$ should be greater than from the undercharging boundary $\underline{q}$. Suppose that an expert defined the value of α_t, and it is 0,45. It means that the value of relative distance of nominal charging state from the undercharging boundary should be smaller by 0,1 from the overcharging boundary (1-α_t = 0,55). Figure 9. shows the position of nominal charging state to undercharging and overcharging boundaries for α_t =0,45.

The decision on the appropriate value of α_t is specific to a particular expert and depends on its risk assessment

Comparison of Info-Gap Theory and RDM interval arithmetic

Info-gap Theory in its original form presented in [7] is a very interesting and inventive method for decision under uncertainty. It does not require to establish any probability density distributions for the variables of which we have very little information and which are important for solving the problem. This theory is based on the measurement of the distance from the robustness representing the unfavorable state of the object and the distance from opportuneness representing a very favorable state of the object. However, this method examines the issues in simplified way, which is criticized by some scientists, eg [18]. Firstly, Info-gap theory analyzes the problem rather locally and around the nominal state of the problem, which is shown for charging car battery in Figure 6., where a local area was marked by a circle. Whereas RDM interval arithmetic can detect a full global space of possible solutions (granules of solutions), which may also contain other very favorable local solutions to the problem. In Figure 6. a global granule of solutions is a space named "correct charging space". Another drawback of Info-gap theory is to assign one and the same robustness variable α and opportuneness variable β for all uncertain parameters of the problem. The result is that the theory takes the feature of MAX-MIN method, where the selection of the best decision is made by measuring the distance between the worst and the best solution to the problem. However, in many decision making problems it is necessary to consider a distance from continuous boundaries rather than 2 decision points. This situation requires the improvement of Info-gap theory, which the basic idea is very reasonable. This possibility gives RDM interval arithmetic, as it is shown in the article on the example of charging car battery. Using this arithmetic is quite simple and it can take the form of algorithm given below:
Step 1: Determine mathematically the variables and constraints in the problem.
Step 2: Specify the nominal state of the object in the problem.
Step 3: Enter the nominal state of the object to the description of the problem and specify the best value or range of values of the decision variable for this state.

Conclusions

This approach allows one to manage uncertainty by choosing a policy that delivers an acceptable performance for a known range of parameter outcomes. RDM-arithmetic is not a competitor in terms of the Info-Gap Theory, but both methods can often be used in parallel. These two theories together can better support decision making than only one of them. The problem of decision making is not one-dimensional and reducing it to one dimension is generally worse. The most important thing is the fact that Info-Gap Theory utilizes effectively the most probable values, but RDM analyzes the boundary of values: pessimistic and optimistic. The best solution is to create a new decision making theory using these three assessments together.

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