Outage probability analysis of system with dual selection combining over correlated Weibull fading channel in the presence of $\alpha$-$\mu$ co-channel interference

Abstract. In this paper, the performance of the system with dual selection combining over correlated Weibull fading channel in the presence of $\alpha$-$\mu$ distributed co-channel interference is studied. The closed-form expressions for probability density function and cumulative distribution function of the signal-to-interference ratio at the output of the selection combining receiver are presented. The effect of correlation on the system performances is investigated and numerical results are shown.

Keywords: $\alpha$-$\mu$ fading, co-channel interference, correlated fading, selection combining, Weibull fading.

Introduction

In wireless communication systems, phenomenon of the variance of momentary value of the received signal, called fading, is one of the main causes of degradation of system performances. There are various statistical models that explain the nature of fading and several distributions which describe fading statistics including Rayleigh, Rice, Nakagami-$m$ and Hoyt. Nakagami-$m$ distribution is preferred because wide range of its applicability and mathematical tractability. Also, it can be reduced to Rayleigh distribution for appropriate value of parameter $m$. The main problem is that there is no such distribution which adequately fit to measured data. One of the distributions that show good fit with experimental data is Weibull distribution [1]–[4].

To describe the fading models, besides the Weibull, there is a more generalized $\alpha$-$\mu$ distribution which is valid for the non-linearity of the propagation medium as well as for the multipath clustering. The $\alpha$-$\mu$ distribution is a general fading distribution that can be used to represent the small-scale variation of the fading signal in a non-line-of-sight fading condition. This fading model has mathematical precedence and huge flexibility, providing a very good fit to measured data over a wide range of fading conditions. This distribution has two physical parameters, $\alpha$ and $\mu$. The parameter $\alpha$ is related to the non-linearity of the environment, whereas the parameter $\mu$ is associated to the number of multipath clusters [5]–[7].

Diversity technique is one of the most used methods for minimizing fading effect and increasing the communication reliability without enlarging either transmitting power or channel’s bandwidth. Diversity techniques combine the multiple received signals in reception device, on different ways. There are several types of diversity combining techniques. Maximum ratio combining (MRC) and equal gain combining (EGC) techniques require more information about channel: fading amplitude, phase and delay [8]. The implementation of these diversity techniques is quite complex and expensive since they require a separate receiver for each branch. On the other hand, selection combining (SC) diversity technique is simpler for implementation because the SC systems process only one of the diversity branches. If the noise power is equally distributed over branches, SC receiver selects the branch with the highest signal-to-noise ratio (SNR) and that is the branch with the strongest signal. In fading environments, the level of noise can be sufficiently low compared with the level of co-channel interference (CCI). In this case, SC combiner processes the branch with the highest signal-to-interference ratio (SIR-based selection diversity) [1]–[3], [9]–[12].

In communication systems where antennas are sufficiently apart, it is considered there is no correlation between transmitted signals, as well as between interferences at the reception. Diversity systems in the presence of uncorrelated fading and interference were observed in [13, 14]. However, it cannot be always done in practice because there is insufficient antenna spacing when diversity is applied in small devices. The SC systems performances with a correlation between transmitted signals and between interferers are considered in [1]–[4].

Performance evaluation of SIR-based dual selection diversity over channels in the presence of correlated Nakagami-$m$ fading is presented in [10]. Performance analysis of the system with dual SC over channels in the presence of correlated Weibull fading and co-channel interference is observed in [1] and [4]. The similar SC system, but triple, is analyzed in [2], and SC combiner with L-branches in [3].

In this paper, we study dual selection diversity system, where the desired signal suffers correlated Weibull fading and the interfering signal endures correlated $\alpha$-$\mu$ fading. The case that fading in the interference channel can be different from fading in the desired transmitted channel is scrutinized. This assumption is justified because we consider that interference comes from adjacent cells in which the propagation conditions may be different from those in the observed cell. The closed form expressions of probability density function (PDF) and cumulative distribution function (CDF) are derived when there is the correlation of desired signals and between interferers.

Analytical expression for outage probability is determined and numerical results are graphically shown.

Statistics of output SIR

The $\alpha$-$\mu$ distribution is a general fading distribution that can be used to represent the small-scale variation of fading signal. The probability density function $f_d(R)$ is $\alpha$-$\mu$ distributed and can be written as [5]:

$$f_d(R) = \frac{\alpha \mu^\alpha R^{\alpha-1}}{\Gamma(\mu)} \exp\left(-\frac{R}{\mu}\right)$$

The effect of correlation on the system performances is analyzed by focusing on the performance of selection combining over correlated Weibull fading channels. The closed-form expressions for the probability density function and cumulative distribution function of the signal-to-interference ratio at the output of the selection combining receiver are presented. The effect of correlation on the system performances is investigated and numerical results are shown.
with parameter $\mu$, an arbitrary parameter $\alpha>0$, and $\alpha$-root mean value $\bar{R} = \sqrt[\alpha]{R}$. If $\Omega_s = E[R^2]$ is the mean power of the signal, then it is:

$$\Omega_s = E[R^2] = \frac{\Gamma(\mu+\frac{2}{\alpha})}{\Gamma(\mu)\alpha^\frac{2}{\alpha}} \bar{R}^2. \tag{2}$$

The probability density function $f_r(r)$ is given with Weibull distribution:

$$f_r(r) = \frac{\alpha r^{\alpha-1}}{\bar{R}^{\alpha}} \exp\left(-\frac{r^{\alpha}}{\bar{R}^{\alpha}}\right) \tag{3}$$

where $\bar{R} = \sqrt[\alpha]{E[R^{\alpha}]}$ is a $\alpha$-root mean value of the parameter $\alpha$. The mean power of the signal is now

$$\Omega_s = E[r^2] = \Gamma\left(1 + \frac{2}{\alpha}\right) \bar{R}^2 \tag{4}$$

Now, the case with dual selection combined diversity system operating in environment where the desired signal suffers correlated Weibull fading and the interfering signal endures correlated $\alpha$-$\mu$ fading due to insufficient spacing between antennas is examined.

The joint probability density function of the interfering signal envelopes $r_1$ and $r_2$ is [5]:

$$f_{r_1, r_2}(R_1, R_2) = \frac{\alpha \alpha_1 \mu^{\mu+1} R_1^{\frac{\alpha_1}{\mu} (\mu+1)-1} R_2^{\frac{\alpha_2}{\mu} (\mu+1)-1}}{R_1^{\frac{\alpha_1}{\mu} (\mu+1)} R_2^{\frac{\alpha_2}{\mu} (\mu+1)}} \Gamma(\mu) (1-\delta) \delta^\frac{1}{\alpha} \times \exp\left(-\mu \left(\frac{R_1^{\frac{\alpha_1}{\mu}}}{R_1} + \frac{R_2^{\frac{\alpha_2}{\mu}}}{R_2}\right)\right) \times I_0\left(2\mu \delta R_1^{\frac{\alpha_1}{\mu}} R_2^{\frac{\alpha_2}{\mu}} \left(1 - \delta\right) \right) \tag{5}$$

where $\delta$ is the correlation coefficient, $\Omega_1$ and $\Omega_2$ are the average signal interference powers at the first and the second branches, respectively. $I_0(.)$ is the modified Bessel function of the first kind and zero order. Using development of $I_0(.)$ into the order [15, eq. 03.02.02.0001.01], it is obtained:

$$f_{r_1, r_2}(r_1, r_2) = \frac{\alpha \alpha_1 \mu^{\mu+1} r_1^{\frac{\alpha_1}{\mu} (\mu+1)-1} r_2^{\frac{\alpha_2}{\mu} (\mu+1)-1}}{r_1^{\frac{\alpha_1}{\mu} (\mu+1)} r_2^{\frac{\alpha_2}{\mu} (\mu+1)}} \Gamma(\mu) (1-\delta) \delta^\frac{1}{\alpha} \times \exp\left(-\frac{1}{1-\delta} \left(\frac{r_1^{\frac{\alpha_1}{\mu}}}{r_1} + \frac{r_2^{\frac{\alpha_2}{\mu}}}{r_2}\right)\right) \times I_0\left(2\delta^\frac{1}{\alpha} r_1^{\frac{\alpha_1}{\mu}} r_2^{\frac{\alpha_2}{\mu}} \left(1 - \delta\right) \right) \tag{8}$$

The instantaneous values of SIR at the diversity branches input can be defined as $\xi_1 = r_1/R_1$ and $\xi_2 = r_2/R_2$. The selection combiner chooses and outputs the branch with the largest SIR: $\xi = \max(\xi_1, \xi_2)$.

The average SIR’s at two input branches of the selection combiner are $S_1 = \Omega_1/\Omega_1$ and $S_2 = \Omega_2/\Omega_2$, which can be determined using (2) and (4) as

$$S_i = \frac{\xi_i \Gamma(\mu) \mu^\frac{1}{\alpha} \bar{R}}{\eta_i \bar{R}^2}, \quad i = 1, 2, \tag{9}$$

where

$$\xi_i = \Gamma\left(1 + \frac{2}{\alpha}\right), \quad i = 1, 2, \tag{10}$$

$$\eta_i = \Gamma\left(\mu + \frac{2}{\alpha}\right), \quad i = 1, 2. \tag{11}$$

The joint probability density function of instantaneous values of SIR at two output branches $\xi_1$ and $\xi_2$ is [1]

$$P_{\xi_1, \xi_2}(t_1, t_2) = \frac{1}{4t_1 t_2} \int \int f_{\xi_1, \xi_2}(x_1, x_2, t_1, t_2) \times f_{\xi_1, \xi_2}(x_1, x_2) dx_1 dx_2 \tag{12}$$

Substituting (6) and (8) in (12), we have

$$P_{\xi_1, \xi_2}(t_1, t_2) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{\alpha_1 \alpha_2 \delta^{\mu+k} (1 - \delta)^{\mu+k}}{4m^\mu k^k \Gamma(k + \mu)} \Gamma\left(1 + m + k + \mu\right) \frac{t_1^{\alpha_1 (m+1)-1}}{t_1^{\alpha_1 (m+1)}} \frac{t_2^{\alpha_2 (m+1)-1}}{t_2^{\alpha_2 (m+1)}} \Gamma(\mu) \frac{\alpha^{2}\delta^{\mu+k} (1 - \delta)^{\mu+k}}{2m^\mu k^k \Gamma(k + \mu)} \sum_{i=1}^{2} \frac{\alpha_i^{2}(m+1)-1}{\eta_i^{2}} \times \left( \frac{t_i \Gamma(\mu) \xi_i}{t_i^{\alpha_i (m+1)}} \right) \left( S_i \xi_i \right)^{\alpha_i (m+1)-1} \tag{13}$$
The joint cumulative distribution function can be found as:

\[(14) \quad F_{S_1S_2}(t_1,t_2) = \int_0^1 p_{S_1S_2}(x_1,x_2)dx_1dx_2\]

Substituting (13) in (14), the joint cumulative distribution function is now:

\[(15) \quad F_{S_1S_2}(t_1,t_2) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{\delta^{m+k}}{m!k!} \left(1 - \delta\right)^{m+k} \Gamma \left(1 + m + k + \mu\right) \cdot \frac{\alpha^m}{(m+k+\mu)^{\alpha}} \cdot \frac{\mu^k}{k!} \Gamma \left(\mu + k\right) \Gamma \left(\mu\right) \cdot \left(1 + \frac{t_1 S_1}{S_2}\right) \cdot \left(1 + \frac{t_2 S_2}{S_1}\right) \cdot \left(t_1 t_2\right)^{\alpha \gamma} \cdot \left(m+k+\mu\right) \cdot \left(1 + \frac{t_1 S_1}{S_2}\right) \cdot \left(1 + \frac{t_2 S_2}{S_1}\right) \cdot \left(t_1 t_2\right)^{\alpha \gamma} \cdot \left(m+k+\mu\right)
\]

The cumulative distribution function of the output SIR could be derived from (15) by equating two arguments \(t_1 = t_2 = t\):

\[(17) \quad F_t(t) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{\delta^{m+k}}{m!k!} \left(1 - \delta\right)^{m+k} \Gamma \left(1 + m + k + \mu\right) \cdot \frac{\alpha^m}{(m+k+\mu)^{\alpha}} \cdot \frac{\mu^k}{k!} \Gamma \left(\mu + k\right) \Gamma \left(\mu\right) \cdot \left(1 + \frac{t S}{S}\right) \cdot \left(t S\right)^{\alpha \gamma} \cdot \left(m+k+\mu\right)
\]

The probability density function of the output SIR is obtained from previous expression in the next form:

\[(18) \quad p_t(t) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{\delta^{m+k}}{m!k!} \left(1 - \delta\right)^{m+k} \Gamma \left(1 + m + k + \mu\right) \cdot \frac{\alpha^m}{(m+k+\mu)^{\alpha}} \cdot \frac{\mu^k}{k!} \Gamma \left(\mu + k\right) \Gamma \left(\mu\right) \cdot \left(1 + \frac{t S}{S}\right) \cdot \left(t S\right)^{\alpha \gamma} \cdot \left(m+k+\mu\right)
\]

The outage probability \(P_{out}\) is usual system performance measure of diversity system operating over channels in the presence of fading. In the channels with interference, the \(P_{out}\) is defined as the probability that the output SIR falls below a specified threshold \(g\):

\[(19) \quad P_{out} = P_R(\xi, g) = \int_0^1 p_{t}(t)dt = F_t(g)\]
The specified threshold $g$ is called a protection ratio and it depends on modulation technique and expected quality of service (QoS).

Fig. 2 gives the outage probability dependence versus values of threshold, for different values of correlation coefficient $\delta$, for balanced and unbalanced ratio of SIR.

The dependence of the outage probability versus the normalized average SIR at the input of the first branch of the selection combiner, $S_1$, for different values of correlation coefficient, $\delta$, and for balanced and unbalanced ratio of SIR, is presented in Fig. 3.

With increasing of correlation, the system has worse performance. The outage probability decreases when the average SIR on one of the branches increases, as was expected.

**Conclusion**

In this paper, the performances of the system with dual selection combining over fading channel in the presence of interference have been studied. The fading between interferers is correlated and $\alpha$-$\mu$ distributed. The $\alpha$-$\mu$ distribution can be used to represent the small-scale variation of the fading signal under a non-line-of-sight fading condition. Weibull distribution is used as a fading model between the diversity branches. The statistical characteristics, such as probability density function, cumulative distribution function and the outage probability, for the selection combining output SIR are given in the closed form. These results are graphically presented with curves showing the influence of correlation coefficient and distribution parameters.

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**REFERENCES**


