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A Hybrid Constrained State Estimator with Pseudo Injection Measurements

***Abstract**. The proposed hybrid state estimator calculates pseudo-injections from the voltage and current phasors collected by Phasor Measurement Units (PMUs). The pseudo-injections are combined with the voltage phasors and conventional measurements as inputs for the iterative algorithm with addition of equality constraints for zero-injections. The methodology is tested on the IEEE test systems with 14, 30, 57 and 118 buses as well as on the Croatian transmission power system and compared to the classical solution that uses conventional measurements only.*

***Streszczenie.** Zaproponowano hybrydowy estymator stanu obliczający pseudo-injekcję z napięcia i prądu fazora. Metode przetestowano w systemie IEEE z różną liczbą szyn a także w chorwackim systemie energetycznym. (**Hybrydowy wymuszony estymator stanu wz pomiarem pseudo-injekcji**)*

Keywords: Hybrid Power System State Estimator, Phasor Measurement Unit (PMU), Synchronized Measurement Technology, Smart Grid
Słowa kluczowe: esrtymator stanu, pseudo-injekcja, pomiar fazora.

Introduction

Operation of the power systems became a challenging task for system operators. Due to restrictions for building transmission corridors and increased outputs from modernized and more efficient power plants, the power flows became higher. Large scale integration of the distributed renewable energy sources, which are intermittent by their nature, and introduce uncertainties in the power system operation, pushes the power systems closer to their operating limits and eventually contingencies and blackouts occur [1]. The aging infrastructure urges for solutions that would support continuous rise in energy needs and trends in global energy markets.

The synchronized measurement technology (SMT) offers numerous benefits in comparison with the conventional measurements obtained from the traditional Supervisory Control And Data Acquisition (SCADA) system, and it is applied in the development of Wide-Area Monitoring, Protection and Control (WAMPAC) systems [2]-[4]. The synchronized phasor measurements (synchrophasors) are collected by Phasor Measurement Units (PMUs) that measure geographically dislocated voltage and current phasors. Utilities all over the world are collaborating with experts from both industry and academia to develop technologies and implement solutions that would transform the transmission power system into a Smart Transmission Grid. Applications based on the SMT provide power system operators with a better overview of real-time conditions, which is valuable information for operating the power system in more economical and secure manner.

The core application of the Energy Management System (EMS) is the state estimator, since control of the power system demands the information about the power system state. Input for the power system state estimator is a set of measurements which include noise due to measurement and communication equipment imperfections. Also, a part of the measurements can be temporarily unavailable. Thus, the observability analysis is needed to determine whether the power system is observable from the given set of measurements, i.e. whether the power system state can be uniquely determined. To achieve a certain degree of measurement redundancy, the standard practice assumes that the number of measurements is larger than the number of the power system state variables. As the power system state estimation is an overdetermined problem, the state estimator is needed to determine the power system state that is closest to the true power system state, and therefore it is optimal state when speaking in terms of the state estimation problem. The power system state estimator also has to detect, identify and process gross measurement errors that can significantly deteriorate the final solution [5]. The output of the power system state estimator is then used by other applications in the EMS.

Since the state estimator offers voltage phasors at all the buses in the observed power system, it presents a logical application of the SMT. If there is a sufficient number of PMUs which are optimally located in the power system to provide complete observability [6], the problem of estimating the power system state becomes linear [7]. Considering the number of PMUs needed for larger power systems and availability of the conventional measurements that contribute to the power system observability and redundancy of the measurements, the more realistic option are hybrid models that combine the conventional measurements together with the synchrophasors. A topic widely investigated is an optimal inclusion of the synchrophasors in the set of measurements in order to enhance the power system estimator performance and consequently to enable the power system operator a real-time monitoring of the power system conditions. Reference [8] relates the current phasors with the bus voltages through a set of constraints, while in [9] the pseudo-voltages on the buses adjacent to the PMU buses are calculated. The phasors of current can also be transformed from polar to rectangular coordinates and included in the set of measurements, as proposed in [10]. Operation of the power system state estimator in the polar and the rectangular coordinates is assessed in [11].

In Republic of Croatia there is a significant operational experience with the SMT, as the Croatian Transmission System Operator (TSO) developed and deployed its own monitoring system with an originally developed Phasor Data Concentrator (PDC) [12]. The WAMPAC applications are currently being developed. The hybrid state estimator proposed in the paper uses the pseudo-injections calculated from the voltage and current phasors. The pseudo-injections are combined together with the voltage phasors and the conventional measurements as inputs for the iterative Weighted Least Squares (WLS) method with equality constraints for zero-injections. The hybrid state estimator is tested for several IEEE test systems and for the Croatian transmission power system model and compared with the classical state estimator that uses only SCADA measurements.

Classical Power System State Estimation

The majority of the state estimators used in the power system control centres are based on the WLS method, which starts from relationship between the measurement vector $\mathbf{z}$ and the state vector $\mathbf{x}$ [13]:

(1) $$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e}$$

where $\mathbf{h(x)}$ is the non-linear functions vector and $\mathbf{e}$ is the vector of measurement errors. Minimization of the weighted sum of squares of residuals, known as the objective function $J(\mathbf{x})$, yields the state estimation optimization problem:

(2) $$\min \quad J(\mathbf{x}) = \frac{1}{2}[\mathbf{z} - \mathbf{h}(\mathbf{x})]^T \mathbf{R}^{-1}[\mathbf{z} - \mathbf{h}(\mathbf{x})]$$

where $\mathbf{R}$ is the measurement covariance matrix, while $\mathbf{W} = \mathbf{R}^{-1}$ is the matrix of measurement weights, which differentiate contribution of each measurement according to its accuracy. The more accurate measurements have larger weight factors, and contribute more to the final solution. The objective function minimum is obtained from fulfilling the following condition:

(3) $$\mathbf{g}(\mathbf{x}) = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} = -\mathbf{H}^T(\mathbf{x})\mathbf{R}^{-1}[\mathbf{z} - \mathbf{h}(\mathbf{x})] = \mathbf{0}$$

where $\mathbf{H(x)}$ is the Jacobian matrix of partial derivatives of $\mathbf{h(x)}$ with respect to the state variables. The solution is found with an iterative procedure that gives the change of the state vector in the k-th iteration:

(4) $$\mathbf{G}(\mathbf{x}^k)\Delta\mathbf{x}^k = \mathbf{H}^T(\mathbf{x}^k) \cdot \mathbf{R}^{-1} \cdot [\mathbf{z} - \mathbf{h}(\mathbf{x}^k)]$$

(5) $$\Delta\mathbf{x}^k = \mathbf{x}^{k+1} - \mathbf{x}^k$$

where $\mathbf{G}(\mathbf{x}^k)$ is the gain matrix given as:

(6) $$\mathbf{G}(\mathbf{x}^k) = \frac{\partial \mathbf{g}(\mathbf{x}^k)}{\partial \mathbf{x}} = \mathbf{H}^T(\mathbf{x}^k) \cdot \mathbf{R}^{-1} \cdot \mathbf{H}(\mathbf{x}^k)$$

The iterative procedure stops when a maximal change in the state vector gets smaller than a previously chosen tolerance:

(7) $$\max\left|\Delta\mathbf{x}^k\right| < \varepsilon$$

Hybrid State Estimator with Pseudo Injection Measurements

Fig. 1 gives a pi-model of the network branch, with a PMU deployed at one end of the branch [14].

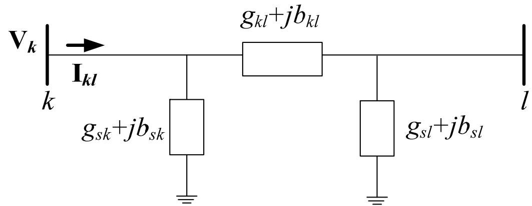

Fig. 1. A pi-model of the network branch

If we assume that the PMU is accommodated at the bus k, it measures the voltage V_k and the current I_{kl} magnitudes as well as the voltage θ_{Vk} and the current θ_{Ikl} angles. The magnitudes and angles form the voltage $\mathbf{V}_k$ and the current $\mathbf{I}_{kl}$ phasors:

(8) $$\mathbf{V}_k = V_k \angle \theta_{Vk}$$

(9) $$\mathbf{I}_{kl} = I_{kl} \angle \theta_{Ikl}$$

The active and reactive power injections at the bus k are calculated as:

(10) $$P_k = V_k \cdot I_k \cdot \cos(\theta_{Vk} - \theta_{Ik})$$

(11) $$Q_k = V_k \cdot I_k \cdot \sin(\theta_{Vk} - \theta_{Ik})$$

where I_k and θ_{Ik} are the current magnitude and the current angle obtained after summing all the currents measured by the PMU at the bus k:

(12) $$\mathbf{I}_k = I_k \angle \theta_{Ik} = \sum_{i=1}^{n} \mathbf{I}_{kn}$$

where n is the number of branches emanating from the PMU bus. The presumption needed for the power injection calculation is that PMUs measure currents on all the branches incident to their buses.

The power injections that are obtained from the voltage and current phasors are referred as the pseudo-injections. The measurement vector is then formed as:

(13) $$\mathbf{z} = \begin{bmatrix} \mathbf{z}_c & \mathbf{z}_s \end{bmatrix}^T$$

The vector of conventional measurements $\mathbf{z}_c$ is given as:

(14) $$\mathbf{z}_c = \begin{bmatrix} \mathbf{V} & \mathbf{P}_{flow} & \mathbf{Q}_{flow} & \mathbf{P}_{inj} & \mathbf{Q}_{inj} \end{bmatrix}^T$$

where $\mathbf{V}$ is the vector of voltage magnitudes, $\mathbf{P}_{flow}$ and $\mathbf{Q}_{flow}$ are the vectors of power flows, while $\mathbf{P}_{inj}$ and $\mathbf{Q}_{inj}$ are the vectors of power injections collected by the SCADA system. The vector $\mathbf{z}_s$ combines the vectors of voltage magnitudes $\mathbf{V}_{PMU}$ and voltage angles $\boldsymbol{\theta}_{PMU}$ that are directly measured by the PMUs, with the vectors of pseudo-injections $\mathbf{P}_{injPMU}$ and $\mathbf{Q}_{injPMU}$ that are obtained from expressions (10) and (11):

(15) $$\mathbf{z}_s = \begin{bmatrix} \mathbf{V}_{PMU} & \boldsymbol{\theta}_{PMU} & \mathbf{P}_{injPMU} & \mathbf{Q}_{injPMU} \end{bmatrix}^T$$

Relating the given measurement vector $\mathbf{z}$ to the state vector $\mathbf{x}$ which comprises voltage magnitudes and angles at all the buses in the power system, the Jacobian matrix structure becomes:

(16) $$\mathbf{H(x)} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \mathbf{H}_{13} & \mathbf{H}_{14} & \mathbf{H}_{15} & \mathbf{H}_{16} & \mathbf{H}_{17} & \mathbf{H}_{18} & \mathbf{H}_{19} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \mathbf{H}_{23} & \mathbf{H}_{24} & \mathbf{H}_{25} & \mathbf{H}_{26} & \mathbf{H}_{27} & \mathbf{H}_{28} & \mathbf{H}_{29} \end{bmatrix}^T$$

Table 1. Elements of the Jacobian Matrix

Element		Element	
$\mathbf{H}_{11}$	$\partial\mathbf{V} / \partial\boldsymbol{\theta}$	$\mathbf{H}_{21}$	$\partial\mathbf{V} / \partial\mathbf{V}$
$\mathbf{H}_{12}$	$\partial\mathbf{P}_{flow} / \partial\boldsymbol{\theta}$	$\mathbf{H}_{22}$	$\partial\mathbf{P}_{flow} / \partial\mathbf{V}$
$\mathbf{H}_{13}$	$\partial\mathbf{Q}_{flow} / \partial\boldsymbol{\theta}$	$\mathbf{H}_{23}$	$\partial\mathbf{Q}_{flow} / \partial\mathbf{V}$
$\mathbf{H}_{14}$	$\partial\mathbf{P}_{inj} / \partial\boldsymbol{\theta}$	$\mathbf{H}_{24}$	$\partial\mathbf{P}_{inj} / \partial\mathbf{V}$
$\mathbf{H}_{15}$	$\partial\mathbf{Q}_{inj} / \partial\boldsymbol{\theta}$	$\mathbf{H}_{25}$	$\partial\mathbf{Q}_{inj} / \partial\mathbf{V}$
$\mathbf{H}_{16}$	$\partial\mathbf{V}_{PMU} / \partial\boldsymbol{\theta}$	$\mathbf{H}_{26}$	$\partial\mathbf{V}_{PMU} / \partial\mathbf{V}$
$\mathbf{H}_{17}$	$\partial\boldsymbol{\theta}_{PMU} / \partial\boldsymbol{\theta}$	$\mathbf{H}_{27}$	$\partial\boldsymbol{\theta}_{PMU} / \partial\mathbf{V}$
$\mathbf{H}_{18}$	$\partial\mathbf{P}_{injPMU} / \partial\boldsymbol{\theta}$	$\mathbf{H}_{28}$	$\partial\mathbf{P}_{injPMU} / \partial\mathbf{V}$
$\mathbf{H}_{19}$	$\partial\mathbf{Q}_{injPMU} / \partial\boldsymbol{\theta}$	$\mathbf{H}_{29}$	$\partial\mathbf{Q}_{injPMU} / \partial\mathbf{V}$

Table 1 provides elements of the Jacobian Matrix, while exact expressions for partial derivatives of each type of measurement in respect to the state variables are available in reference [13].

The matrices $\mathbf{H}_{11}$, $\mathbf{H}_{16}$ and $\mathbf{H}_{27}$ are zero matrices. Elements of the matrices $\mathbf{H}_{17}$, $\mathbf{H}_{21}$ and $\mathbf{H}_{26}$ are equal to zero or one, depending on the measurements taken in the vectors $\boldsymbol{\theta}_{PMU}$, $\mathbf{V}$ and $\mathbf{V}_{PMU}$, respectively. As for the remaining types of measurements, the relationship between the measurements and the state vector elements is nonlinear; the state estimate is determined by using the iterative WLS method. Additionally, the zero-injections are introduced through equality constraints as proposed in [15], [16], which equals to the approach applied for the classical state estimator deployed in the Croatian power system control centre.

Propagation of Measurement Uncertainties

The transformation of measurements is needed to obtain the pseudo-injections. First the current phasors are transformed from polar to rectangular coordinates. The sum of the current phasors at each PMU bus is then returned to polar coordinates and used together with the voltage phasor to calculate the pseudo-injections. The classical measurement uncertainty propagation theory is used to assign proper weight factors to the pseudo-injections [17]:

$$\sigma_{P_{injPMU}} = \sqrt{\sum_{n=1}^{4}\left[\partial P_{injPMU} / \partial \mathbf{f}(n)\right]^2 \cdot \sigma_{\mathbf{f}(n)}^2} \tag{17}$$

$$\sigma_{Q_{injPMU}} = \sqrt{\sum_{n=1}^{4}\left[\partial Q_{injPMU} / \partial \mathbf{f}(n)\right]^2 \cdot \sigma_{\mathbf{f}(n)}^2} \tag{18}$$

where $\sigma_{\mathbf{PinjPMU}}$ and $\sigma_{\mathbf{QinjPMU}}$ are uncertainties of the pseudo-injections while $\sigma_{\mathbf{f}(n)}$ are uncertainties of elements in the vector $\mathbf{f}(n) = [V_k, \theta_{Vk}, I_{kl}, \theta_{Ikl}]$.

Case Studies

The developed hybrid state estimator was tested on several power system models in order to investigate its operation for systems of different sizes, topologies and configuration of measurements. Standard power systems used for testing were the IEEE test systems [18]. Table 2 – Table 5 give locations of the measurements for the IEEE tests systems with 14, 30, 57 and 118 buses.

Table 2. Locations of the measurements for the IEEE 14 system

Type	Locations
PMU (# bus)	1 and 6
Voltage magnitude (#bus)	2, 3, 8, 10, 12
Power flow (#from - #to)	1-2, 4-7, 4-9, 5-6, 6-12, 6-13, 7-9, 13-14
Power injection (#bus)	1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 13, 14

Table 3. Locations of the measurements for the IEEE 30 system

Type	Locations
PMU (# bus)	1, 5, 12, 15, 27
Voltage magnitude (#bus)	2, 3, 6, 9, 11, 14, 16, 17, 25, 30
Power flow (#from - #to)	1-3, 2-4, 2-6, 4-6, 5-7, 6-8, 6-9, 6-10, 12-13, 12-15, 14-15, 16-17, 15-18, 10-20, 10-17, 15-23, 25-26, 25-27, 28-27, 29-30, 6-28
Power injection (#bus)	1, 2, 4, 6, 10, 11, 12, 15, 18, 19, 24, 25, 27, 30

As an example of the real power system, the model of the Croatian TPS was used. The Croatian TPS model includes 110, 220 and 400 kV voltage levels, with 200 buses and 287 branches. During the last decade, the Croatian Transmission System Operated (TSO) conducted research projects and studies that aimed to investigate the benefits of applications based on the SMT. The deployment of the PMUs was divided in several phases; resulting in 14 PMUs at all the 400 kV and a part of the 220 kV buses. Locations of the PMUs in the Croatian TPS are given in [3].

Table 4. Locations of the measurements for the IEEE 57 system

Type	Locations
PMU (# bus)	1, 6, 9, 18, 19, 30, 55
Voltage magnitude (#bus)	1, 3, 4, 5, 7, 8, 11, 15, 17, 22, 27, 31, 37, 44, 52, 54
Power flow (#from - #to)	1-15, 1-17, 2-3, 3-4, 4-5, 4-18, 6-7, 7-8, 7-29, 8-9, 9-10, 9-11, 9-12, 9-13, 12-16, 12-17, 13-15, 14-15, 14-46, 18-19, 22-23, 22-38, 24-25, 28-29, 24-26, 26-27, 32-33, 35-36, 38-48, 46-47, 52-29, 52-53
Power injection (#bus)	1, 2, 5, 6, 10, 12, 13, 15, 18, 19, 25, 27, 30, 32, 35, 41, 43, 44, 47, 49, 51, 53, 54, 55, 57

Table 5. Locations of the measurements for the IEEE 118 system

Type	Locations
PMU (# bus)	24, 40, 59, 75, 80, 100, 103, 113, 114
Voltage magnitude (#bus)	4, 10, 12, 18, 25, 27, 36, 40, 59, 73, 76, 82, 86, 92, 107, 111, 112, 117
Power flow (#from - #to)	1-2, 2-12, 3-5, 5-6, 6-7, 9-10, 4-11, 5-11, 7-12, 12-14, 14-15, 16-17, 17-18, 21-22, 23-24, 28-29, 30-17, 17-31, 23-32, 34-36, 37-40, 39-40, 40-41, 43-44, 34-43, 46-48, 45-49, 51-52, 52-53, 54-55, 56-57, 50-57, 51-58, 59-60, 60-62, 64-65, 62-67, 68-65, 47-69, 71-72, 71-73, 69-75, 74-75, 76-77, 78-79, 81-80, 77-82, 84-85, 86-87, 85-88, 91-92, 92-93, 93-94, 94-95, 82-96, 92-100, 95-96, 98-100, 99-100, 100-101, 101-102, 100-106, 105-107, 105-108, 108-109, 103-110, 109-110, 110-112, 17-113, 27-115, 114-115, 75-118, 76-118
Power injection (#bus)	3, 4, 8, 12, 13, 16, 15, 19, 20, 22, 24, 25, 27, 31, 33, 35, 36, 42, 44, 46, 47, 49, 52, 53, 54, 55, 61, 66, 70, 72, 74, 77, 79, 83, 85, 86, 89, 90, 92, 94, 96, 97, 98, 99, 102, 104, 105, 110, 111, 112, 116, 117, 118

The simulation of the proposed hybrid state estimator was performed by using the power flow solution as a true state, to which a random Gaussian noise was added to generate the noisy measurements. Table 6 provides standard deviations for the conventional measurements and synchrophasors, expressed in percentage of the actual values. These standard deviations were used to generate the random Gaussian noise with a zero mean and given uncertainties. The hybrid model was compared with the classical state estimator that is currently deployed in the Croatian power system control centre.

Table 6. Standard deviations of measurements

Conventional measurements		
Power flow	Power injection	Voltage magnitude
2%	2%	0.2%
Synchrophasors		
Current magnitude	Voltage magnitude	Voltage and current phase angle
0.03%	0.02%	0.01°

Evaluation Criteria for the State Estimator

The first criterion used was the variance of the estimated states that shows the state estimator accuracy:

$$\sigma_{\Sigma}^2 = \sum_{i=1}^{2N}\left(\hat{x}_i - x_i\right)^2 \tag{19}$$

where N is the number of buses in the system, while $\hat{x}$ and x are estimated and true state values, respectively.

Filtering of the measurement errors is defined as the ratio:

$$\xi\left(\hat{x}\right)=\frac{\sum_{i=1}^{m}\left(\hat{z}_i-z^t{}_i\right)^2}{\sum_{i=1}^{m}\left(z_i-z^t{}_i\right)^2} \tag{20}$$

where $\hat{z}$, z^t and z are the estimated, true, and noisy measurements, respectively.

Finally, convergence capabilities of the state estimator are tested by observing the number of iterations necessary to reach the desired tolerance of 10^{-6}.

Results

The average values of 100 Monte Carlo (MC) simulations were taken to obtain results presented in this section. In every trial, a different set of random errors was taken when generating the noisy measurements.

The results for the IEEE test systems and the Croatian TPS are presented in Table 7. When comparing the proposed hybrid model with the classical state estimator that uses only the conventional measurements, it can be concluded that inclusion of the synchrophasors into the set of measurements enhances the estimation practice for all the test systems. The hybrid model offers the more accurate state estimation visible as lower values of the variance of the estimated states, which means that the estimated states are closer to their true values. The highly accurate voltage and current synchrophasors used in the hybrid state estimator to derive pseudo-injections at the PMU buses also bring improvement of the filtering capabilities for the random measurement errors. The hybrid state estimator converged for every simulation and the number of the iterations needed to reach the tolerance is similar to the classical solution.

Table 7. Results for the test systems – average values (100 MC)

System	State Estimator	σ_{Σ}^2	$\xi\left(\hat{x}\right)$	*Iter*
IEEE 14	Classical	7.9950×10^{-5}	0.4502	4.96
	Hybrid	3.2020×10^{-6}	0.0744	5.00
IEEE 30	Classical	2.5582×10^{-5}	0.3760	4.58
	Hybrid	1.1239×10^{-6}	0.0418	4.00
IEEE 57	Classical	1.3508×10^{-3}	0.3080	5.00
	Hybrid	8.6752×10^{-4}	0.0533	5.00
IEEE 118	Classical	5.7932×10^{-4}	0.6809	4.96
	Hybrid	7.0112×10^{-5}	0.5307	4.45
Croatian TPS	Classical	1.4086×10^{-5}	0.2692	4.00
	Hybrid	8.0159×10^{-6}	0.1295	4.00

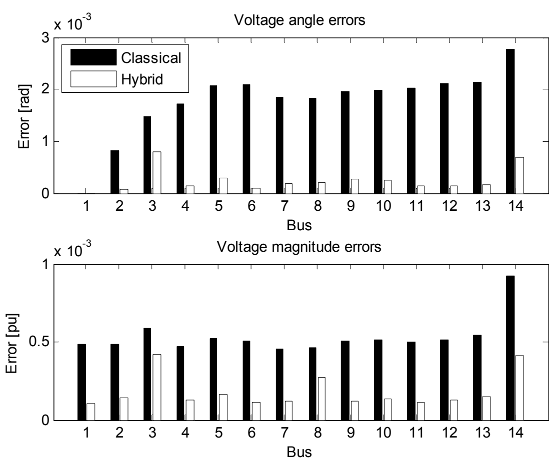

Fig. 2. Voltage angle and magnitude errors for the IEEE 14 - average values of 100 Monte Carlo simulations

The voltage angle and magnitude errors for all the buses in the IEEE 14 buses test system are given in Fig. 2, while Table 8 gives the true states and the average values of the estimated states obtained by using the classical and the proposed hybrid state estimator.

Table 8. States of the IEEE 14 system – average values (100 MC)

Bus nr.	True		Classical		Hybrid	
	Mag [p.u.]	Ang [°]	Mag [p.u.]	Ang [°]	Mag [p.u.]	Ang [°]
1	1.060	0.000	1.059	0.000	1.060	0.000
2	1.045	-7.531	1.043	-7.561	1.045	-7.529
3	1.010	-17.033	1.008	-17.125	1.010	-17.032
4	0.988	-15.554	0.986	-15.615	0.988	-15.556
5	0.993	-13.710	0.992	-13.778	0.993	-13.723
6	1.070	-25.954	1.069	-26.029	1.070	-25.958
7	1.016	-22.748	1.015	-22.793	1.016	-22.737
8	1.090	-22.748	1.089	-22.796	1.090	-22.741
9	0.980	-26.640	0.979	-26.675	0.980	-26.620
10	0.988	-26.792	0.986	-26.831	0.988	-26.773
11	1.024	-26.463	1.023	-26.521	1.024	-26.457
12	1.027	-27.774	1.025	-27.847	1.027	-27.780
13	0.988	-28.216	0.986	-28.279	0.988	-28.220
14	0.782	-35.983	0.779	-36.017	0.782	-35.986

The active and reactive power injection errors for all the buses are given in Fig. 3, while the active and reactive power flow errors on all the branches in the IEEE 14 buses test system are given in Fig. 4.

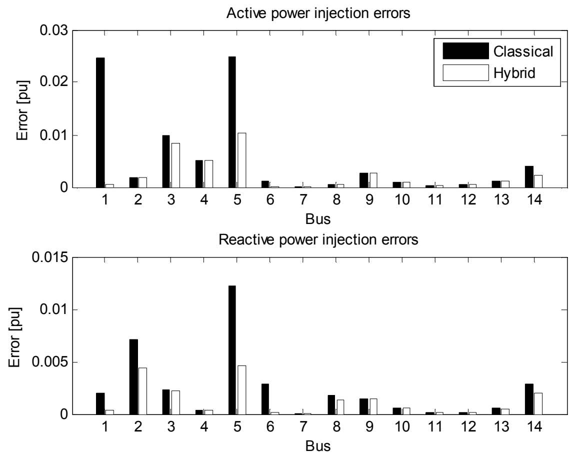

Fig. 3. Power injection errors for the IEEE 14 - average values of 100 Monte Carlo simulations

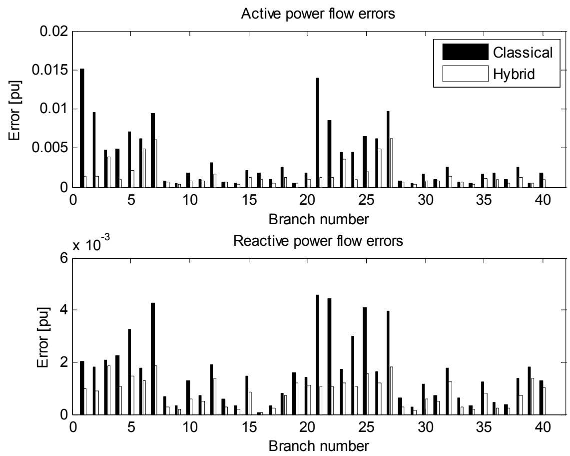

Fig. 4. Power flow errors for the IEEE 14 - average values of 100 Monte Carlo simulations

For the Croatian TPS the results are given for part of the branches observable by the PMUs and all the buses were the PMUs are deployed, because of a large number of buses and branches. In the Croatian TPS there are 14 PMUs placed at 9 buses, as they were originally deployed for applications other than the state estimation. The voltage angles and magnitudes on all the PMU buses are given in Fig. 5, while Table 9 gives the average states of the Croatian TPS. The active and reactive power injection errors for the PMU buses are presented in Fig. 6, while the active and reactive power flow errors for part of the branches are given in Fig. 7.

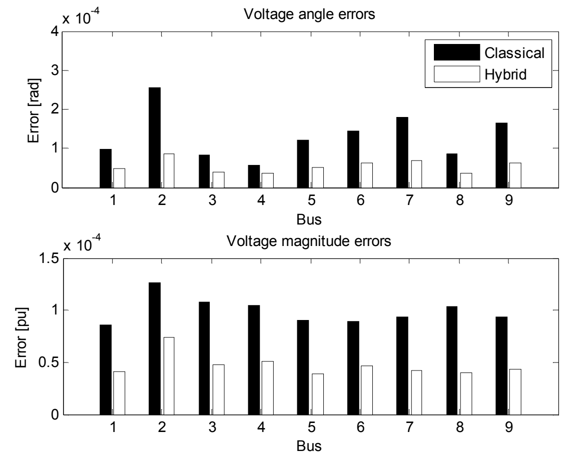

Fig. 5. Voltage angle and magnitude errors for the Croatian TPS - average values of 100 Monte Carlo simulations

Table 9. States of the Croatian TPS – average values (100 MC)

Bus nr.	True		Classical		Hybrid	
	Mag [p.u.]	Ang [°]	Mag [p.u.]	Ang [°]	Mag [p.u.]	Ang [°]
1	1.061	0.679	1.061	0.675	1.061	0.673
2	1.090	9.725	1.090	9.719	1.090	9.724
3	1.053	0.775	1.053	0.776	1.053	0.775
4	1.044	0.091	1.044	0.094	1.044	0.090
5	1.068	1.886	1.068	1.888	1.068	1.882
6	1.040	2.968	1.040	2.964	1.040	2.970
7	1.070	6.968	1.070	6.980	1.070	6.971
8	1.064	0.622	1.064	0.623	1.064	0.621
9	1.085	8.018	1.085	8.025	1.085	8.024

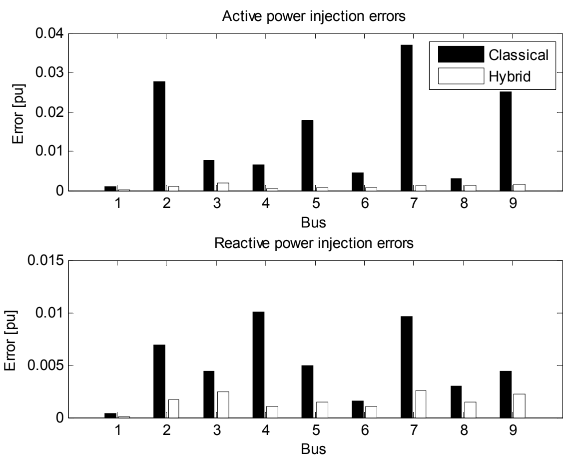

Fig. 6. Power injection errors for the Croatian TPS - average values of 100 Monte Carlo simulations

The results indicate that inclusion of the synchrophasors into the proposed hybrid state estimator brings overall improvement of the state estimator performance. When analysing the results for the IEEE 14 test system and for the Croatian TPS model given in Fig. 2 and Fig. 5 as well as in Table 8 and Table 9 it can be concluded that the states estimated by using the developed hybrid state estimator are closer to the true states in comparison with the states estimated using the classical state estimator. The more accurate estimation of the voltage magnitudes and angles is the result of using the highly accurate voltage and current synchrophasors to derive the pseudo-injections. Further, Fig. 3 and Fig. 6, which depict the power injection errors, show that the more accurate estimation of the system states results in the more accurate calculation of the power injections. Finally, Fig. 4 and Fig. 7 indicate that the errors of the calculated power flows are also lower when using the hybrid model.

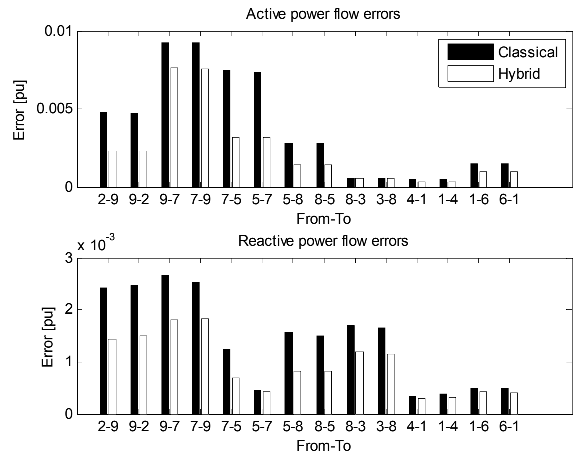

Fig. 7. Power flow errors for the Croatian TPS - average values of 100 Monte Carlo simulations

Conclusions

The SMT is already well established and recognized as one of fundamental technologies for building modern power system solutions. As the PMUs start to populate transmission power systems, the synchrophasors are measured and collected in the control centres. Applications based on the SMT would thus improve the power system operation and enable its monitoring, protection and control in real-time.

Inclusion of the highly accurate synchrophasors in the power system state estimation offers many potential benefits, as the state estimator output is used for several other EMS applications. That is why an optimal solution for incorporating the conventional SCADA measurements with the synchrophasors is needed in order to enhance the state estimator performance.

The hybrid state estimator presented in the paper uses current and voltage phasors to calculate pseudo-injections at the buses equipped with PMUs. The pseudo-injections are then combined in the measurement vector together with the voltage phasors and the conventional measurements, with appropriate weight factors assigned to the each type of measurements. The iterative procedure based on the WLS method with the equality constraints for zero-injections is applied to obtain the power system state vector.

The developed hybrid state estimator was tested on the IEEE test systems with 14, 30, 57 and 118 buses as well as on the Croatian TPS model. The comparison with the classical state estimator, which runs in the Croatian power system control centre, shows that the proposed methodology significantly improves the state estimator accuracy and filtering of the measurement errors. Obtaining the more accurate estimate of the state variables, the power system operator is provided with the more accurate calculation of the power injections and the power flows.

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