

Vector fitting implementation for use of modeling of reduced self-capacitance inductor

Abstract. This paper presents a method for modeling of reduced self-capacitance inductor. Based on the frequency characteristics of the experimental model of inductor, the presented method of identification and approximation have been verified. The vector fitting method has a very good ability to match the curves and determines the parameters of the curve approximated by a rational function $h(s)$. At the stage of testing the vector fitting method ensures correct fit but wrong identification.

Streszczenie. W niniejszym artykule przedstawiono metodę modelowania dławika charakteryzującego się zredukowaną wartością pojemności pasywności uzwojeń. Na podstawie wyznaczonych charakterystyk częstotliwościowych modelu eksperymentalnego dławika przedstawiono i zweryfikowano metodę identyfikacji i aproksymacji otrzymanych charakterystyk. Metoda relokacji biegunów (ang. vector fitting) wykazuje bardzo dobrą możliwość dopasowania krzywych oraz wyznacza parametry aproksymowanej krzywej za pomocą funkcji wymiernej typu $h(s)$. Na etapie prób okazało się że metoda relokacji biegunów zapewnia poprawne dopasowanie ale złą identyfikację. (Metoda modelowania dławika charakteryzującego się zredukowaną wartością pojemności pasywności uzwojeń)

Keywords: modeling, frequency characteristics, approximation, transformer sheet, vector fitting

Słowa kluczowe: modelowanie, charakterystyka częstotliwościowa, aproksymacja, vector fitting

Introduction

Linear systems with concentrated parameters can be modeled and identified on the basis of frequency response. In the frequency identification of the measured characteristics $h(j\omega)$ usually approximates by the rational function:

$$(1) \quad h(s) \approx \frac{a_0 + a_1s + a_2s^2 + \dots + a_ns^n}{b_0 + b_1s + b_2s^2 + \dots + b_ms^m}$$

The above function is a nonlinear equation of the model parameters. Attempt to create from (1) a linear equation type $Ax=b$ by multiplying both sides by denominator leads to many errors. They result from bad conditioning of the matrix of least square method and the bias of obtained estimates. This represents a significant reduction of such approximation for higher order functions.

There are many different methods of curve fitting, each depending on the specific problems and expected results. In this case vector fitting method was chosen as more reliable and accurate to model the reduced self-capacitance inductor.

The measurements with wide frequency range can help obtaining the capacitance of examined inductor. Also, with use of a proper approximating method, the model can be found with use of impedance characteristics. This then can be applied to simulate the behavior of inductor in fast switching transients and associated with them fast currents and voltages. The parasitic parameters of components included in the main circuit converter have particular influence on this behavior, especially on capacitance of magnetic elements. Energy dissipated in these elements is dependent on fast transients of current and voltage.

Vector fitting method [1] has been found reliable and accurate for approximation of frequency characteristics different devices. In nearly every publication the cases were finished after obtaining good estimate of course and not many authors tried to find if created model was correct.

In this publication the vector fitting method was applied to create a model of equivalent electrical circuit of reduced self-capacitance inductor. The method was tested [2] for modeling of transformer sheets in wide frequency range and was found reliable and accurate for that case. Unfortunately for more complex characteristics the method identified equivalent circuit elements wrong.

Inductor design

Nowadays, a lot of new switching power devices have very good dynamic properties and can operate with high switching frequency having low values of switching losses (e.g. Silicon Carbide devices). As very fast switching transitions can be achieved with these devices, parasitic elements like the self-capacitance may have substantial impact on the switching behaviour of the devices. Recharging of the inductor parasitic capacitance during the switching process has a negative influence on the shape of the current waveforms and power losses of the converters. According to this disadvantages of typical made inductors, the special design of reduced self-capacitance inductor have been proposed in [5],[6]. The inductor has been designed using the ferrite core (U193, N87) and the number of turns ($N=40$, Litz wire 120x0.1mm) calculated from

$$(2) \quad N = \frac{L \cdot I_{L,pp}}{B_{pp} \cdot A_e}$$

where $I_{L,pp}$ and B_{pp} denote the peak-to-peak values of the inductor current and induction respectively, and A_e is the effective cross sectional area of the core, given in core's datasheet (840 mm² in this case).

In order to reduce the eddy currents and winding losses at the operating frequency of 100 kHz, a Litz copper wire, consisting of 120 strands with 0.1 mm diameter each, has been used. The skin depth (δ) can be calculated from

$$(3) \quad \delta = \sqrt{\frac{2\rho_c}{\mu\omega}}$$

where ρ_c is the electrical resistivity of the conducting material (for copper it is $23 \cdot 10^{-9}$ Ωm at 100°C), μ is the permeability of the material (for copper $\mu \approx \mu_0$), and $\omega = 2\pi f$, is the angular frequency. With these values the penetration depth is approximately 0.241 mm.

Turns have been wound in two layers on a single coil former. In order to reduce the inter-layer capacitance, the inductor makes use of a specially developed spacer between the 1st and the 2nd layer as shown in Figure 1. This spacers increases the distance between the layers, and thus, reduce the layer-to-layer parasitic capacitance. The spacers, made of TECAVINYL PVC material, have low relative permittivity $\epsilon_r = 3$, and maximum operating temperature of 100 °C. They provide a distance of $d = 3$ mm

between the winding layers. Therefore, the insulation material is mostly air with a relative permittivity close to unity what decreasing value of parasitic capacitance of inductor windings.

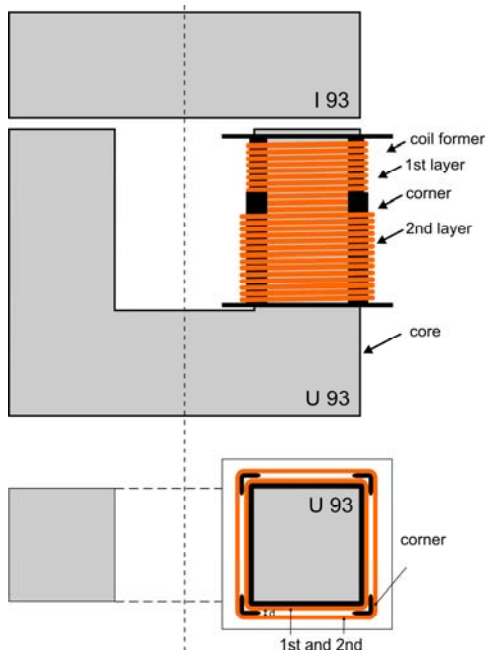


Fig. 1. Layout of the optimized, low self-capacitance inductor



Fig. 2. Photo of reduced self-capacitance inductor

The parasitic capacitance of the inductor was measured according to the four measuring methods described in [5]. The parasitic capacitance of the inductor ranges from 10.88 pF to 20.45 pF. Self-capacitance of presented inductor design was also determined by simple analytical calculations. The model of a plate capacitor was employed for the inter-winding capacitance calculation. The result of this calculation gives value of parasitic capacitance equal to 16.45pF. This noticeable values of the parasitic capacitance shows that the layer-to-layer capacitance is the most significant part of the total self-capacitance. The turn-to-turn capacitances in a given winding layer are actually in series and contribute slightly to the total self-capacitance. On the other hand, the parasitic capacitances between different layers are in parallel and add to each other. Therefore, the most effective way to reduce the total parasitic capacitance is to minimize the layer-to-layer capacitance, which is exactly what the proposed winding method does.

Measurement technique

The frequency characteristics of reduced self-capacitance inductor were measured by impedance analyzer HP4192A in a setup from Fig. 3.

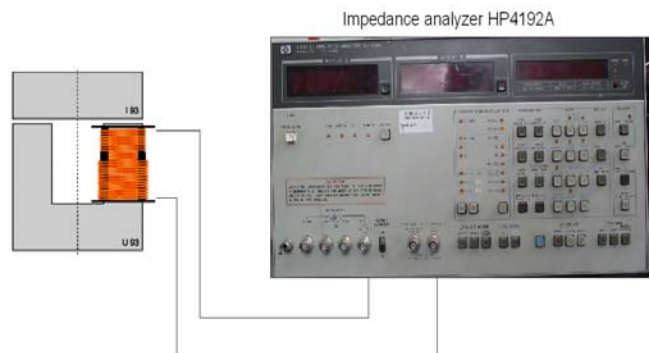


Fig. 3. Measurement setup

Impedance analyzer HP4192A measured characteristics as a function of frequency in range 10Hz – 5MHz:

$$(4) \quad A = 20 \log_{10} \left(\frac{Z}{R} \right)$$

where: Z – impedance of inductor, R – resistance of current shunt additional in HP4192A.

Vector fitting modeling

The main principle of vector fitting method (VF) [1] is to find rational approximation of frequency characteristic of reduced self-capacitance inductor with use of rational function (2):

$$(5) \quad h(s) \approx \sum_{j=1}^N \frac{c_j}{s - a_j} + d + sh$$

where: c_j – zeros of function $h(s)$; a_j – poles of function $h(s)$; d, h – constant values.

Poles and zeros can be both real or complex, constants d and h can be only real. The equation (1) is nonlinear, because zeros and poles are unknown and also constant values are unknown. VF method solves this equation in 2 steps creating dividing it into linear problem.

In first step starting poles \bar{a}_j are assumed and then function $h(s)$ has to be multiplied by unknown function $\sigma(s)$:

$$(6) \quad \sigma(s) \approx \sum_{j=1}^N \frac{\tilde{c}_j}{s - \bar{a}_j} + 1$$

where: \tilde{c}_j – zeros of unknown function $\sigma(s)$, \bar{a}_j – poles of unknown function $\sigma(s)$.

After multiplication (5) with function (6) may be obtained:

$$(7) \quad \sum_{j=1}^N \frac{c_j}{s - \bar{a}_j} + d + sh \approx \left(\sum_{j=1}^N \frac{\tilde{c}_j}{s - \bar{a}_j} + 1 \right) h(s)$$

Equation above (3) is linear to the searched parameters $x = [c_1 \dots c_N \tilde{c}_1 \dots \tilde{c}_N]^T$ and can be written also like:

$$(8) \quad Ax = b$$

This over-defined equation (5) may be solved with use of least-square method by calculating the residues of function under identification.

In the second step VF method calculates zeros of function $h(s)$ [1].

In the VF method, particular attention should be paid to the initial guess (starting poles) and the order of approximation. Improperly formed initial poles cause significant errors in the fitting function. Authors B. Gustavsen and A. Semlyen pay particular attention to two cases resulting in significant errors [1]:

- The nature of the initial poles - real or complex values,
- A significant gap between starting and real values of the poles.

From the results can be seen effectively match the new function to the data measured in the full frequency range. In fact, those good results were obtained after many attempts, during which different initial values of the poles and different levels of fit (N) were tested. It should be noted that for proper fit, must first be used a method that could with a high degree of accuracy find the poles, then you can use the VF method. Some of improvement in that field was shown in [3].

VF method was used in [2] to approximate frequency characteristics of transformer sheets and was found reliable and fine method. The mechanism of forming initial poles was also improved.

Results

VF method was applied to approximate frequency characteristics of reduced self-capacitance inductor (Fig. 4). Characteristics was taken by use of impedance analyzer HP4192A in logarithm frequency range 10Hz – 5MHz.

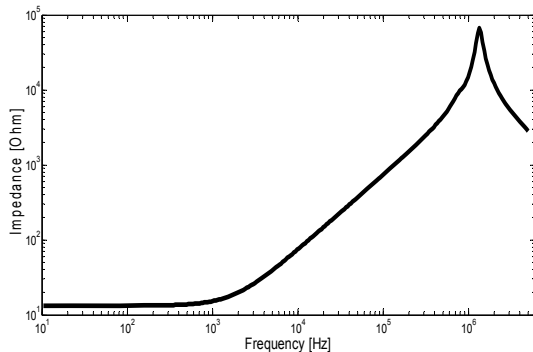


Fig. 4. Impedance of reduced self-capacitance inductor in wide frequency range.

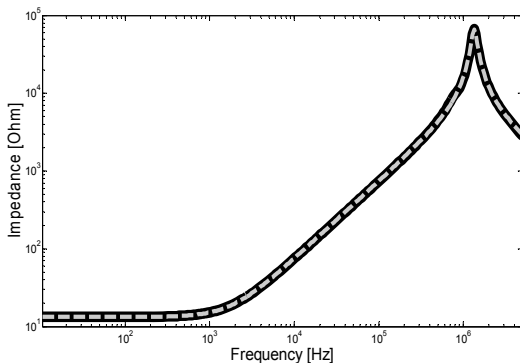


Fig. 5. Impedance characteristic and its approximation in wide frequency range

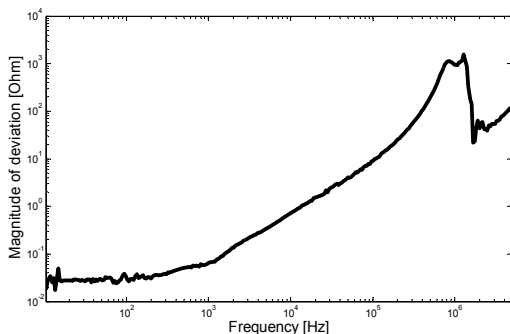


Fig. 6. The deviation of approximated course

Starting poles were chosen as below (Table 1). The Marti's method [4] to improve the forming of initial poles [2] cannot be applied in this problem. The transformer core was modeling in wide frequency range [2]. The impedance of transformer sheets was modeled as two-terminal networks RL connected in series. The reduced self-capacitance inductor shows more complex equivalent circuit, which can be found after obtaining proper model i.e. described by rational function $h(s)$.

Table 1. Starting poles for approximation with vf method

Number of a pole	Value of a pole
1	$-0.1256 \cdot 10^7$
2	$(-0.0600 + 2.6766i) \cdot 10^7$
3	$(-0.0600 - 2.6766i) \cdot 10^7$

The results of approximation with use of VF method was shown on Fig. 5. The approximated course was shown with gray dashed line. On Fig. 6 the deviation from impedance characteristic and approximated course was shown

VF method calculated the poles and zeros of new function (Table 2).

Table 2. Calculated poles and zeros

No.	Value of a pole	Value of a zero
1	$-5.5126 \cdot 10^7$	$0.5758 \cdot 10^{10}$
2	$(-0.0638 + 0.8409i) \cdot 10^7$	$(4.2148 + 0.3585i) \cdot 10^{10}$
3	$(-0.0638 - 0.8409i) \cdot 10^7$	$(4.2148 - 0.3585i) \cdot 10^{10}$

With calculated poles and zeros the rational impedance function $Z(s)$ can now be obtained:

$$(9) \quad Z_{eq}(s) = \frac{9.003 \cdot 10^{10} s^2 + 4.648 \cdot 10^{18} s + 4.911 \cdot 10^{22}}{s^3 + 5.64 \cdot 10^7 s^2 + 1.415 \cdot 10^{14} s + 3.92 \cdot 10^{21}}$$

This can be expressed in more relevant form:

$$(10) \quad Z_{eq}(s) = C \cdot \frac{(s - z_1)(s - z_2)}{(s - p_1)(s - p_2)(s - p_3)}$$

where z_1, z_2 –zeros from function roots of numerator from (9); p_1, p_2, p_3 –poles from function roots of denominator from (9), C – scale factor

Next step was to create equivalent electrical circuit with elements like resistance R, inductance L and capacitance C. To simplify calculations now impedance was changed to admittance. Equation (9) now have to be modified:

$$(11) \quad Y_{eq}(s) = As + B + \frac{Cs + D}{9.003 \cdot 10^{10} s^2 + 4.648 \cdot 10^{18} s + 4.911 \cdot 10^{22}} = \frac{1}{Z_{eq}(s)}$$

where A, B, C, D are distribution coefficients of denominator of rational impedance function $Z_{eq}(s)$. After calculations these coefficients are equal to:

$$(12) \quad A = 1.1107 \cdot 10^{-11}, B = 5.3013 \cdot 10^{-5}, \\ C = -1.0545 \cdot 10^{14}, D = 3.9174 \cdot 10^{21}$$

The rational admittance function can be now described as below:

$$(13) \quad Y_{eq}(s) = 1.1107 \cdot 10^{-11} s + 5.3013 \cdot 10^{-5} + \frac{-1.0545 \cdot 10^{14} s + 3.9174 \cdot 10^{21}}{9.003 \cdot 10^{10} s^2 + 4.648 \cdot 10^{18} s + 4.911 \cdot 10^{22}}$$

The part $As+B$ describing the parallel RC circuit, where capacitance C is equal to coefficient A and resistance R is equal to coefficient B .

The problem occurred on this step of modeling. The coefficient C cannot have negative value. It has no physical equivalent element from electrical circuit. The possible explanation is that the identification made by VF was resulting in artificial outcomes, even if the approximation seems to be good. VF found first zero - $5758 \cdot 10^6$ but the frequency range of measurement was $10\text{Hz} - 5\text{MHz}$. The method is rather comparing the results of deviation from both measurement course and approximated one, than exactly identify the elements of a model. The vector fitting method needs improvement or there should be created another proper method.

For clarifying influence of those artificial pole and zero, attempts were made to cancel (a_2, b_1) from (10), and the equivalent impedance can be now described as below:

$$(14) \quad Z_{eq}(s) = C \cdot \frac{s + 1.14 \cdot 10^4}{s^2 + 1.276 \cdot 10^6 s + 7.112 \cdot 10^{13}} = \frac{8.447 \cdot 10^{10} s + 9.638 \cdot 10^{14}}{s^2 + 1.276 \cdot 10^6 s + 7.112 \cdot 10^{13}}$$

Value $1.14 \cdot 10^4$ was taken by the localization method. The denominator was calculated by multiplying $(s - p_2)(s - p_3)$ with new poles $(-0.0638 + 0.8409i) \cdot 10^7$ and $(-0.0638 - 0.8409i) \cdot 10^7$. C is a scale factor. That operation gave nearly the same results. On Fig. 7 the deviation from impedance characteristic and approximated course was shown.

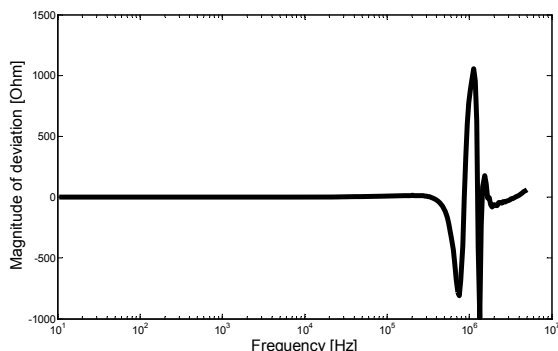


Fig. 7. The deviation of approximated course

The model now can be build as parallel connection of capacity C and in series resistance R and inductance L (Fig. 8).

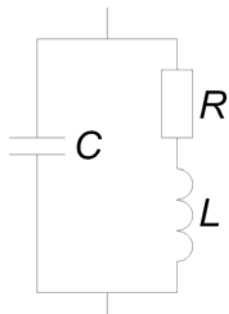


Fig. 8. The model of impedance characteristic of reduced self-capacitance inductor

The elements of a model have values:

$$(15) \quad R = 13.5517 \Omega, L = 1.0620 \cdot 10^{-5} H, \\ C = 1.3239 \cdot 10^{-9} F$$

Conclusions

In article the construction of reduced self-capacitance inductor was shown. Authors made frequency response measurements and applied vector fitting method to find an equivalent electrical circuit. The approximation seen on Fig. 5 seemed to be quite proper, but unfortunately the identification was wrong. Authors idea is to create another method for modeling and identification of frequency characteristic of inductor and find proper equivalent electrical circuit.

To improve identification new method was presented. This one is based on locating zeros with "finger" – looking at impedance course. What can be seen from Fig 8 is that normally capacity C is connected also with conductance G . Authors next step will be to improve this method.

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