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A randomization of parameters in Wireless Sensor Networks

***Abstract**. In this paper we present a model of single-hop type wireless sensor networks with random access and one-way transmission. We replace deterministic network parameters by random variables. This applies to parameters such as: the total number of nodes, the number of groups, the nodes percentage in each group, the average time between transmissions of each node in subsequent groups. We obtain formulas for the collision probability for different cases. The purpose of this approach is to match better the network model with random access to exact applications.*

***Streszczenie.** W pracy przedstawiono model bezprzewodowej sieci czujników z losowym dostępem typu single-hop z jednokierunkową transmisją. Zastąpiono deterministyczne parametry sieci zmiennymi losowymi. Dotyczy to takich parametrów jak: całkowita liczba węzłów, liczba grup węzłów, udział procentowy węzłów w poszczególnych grupach, średnie czasy pomiędzy transmisjami węzłów w grupach. Otrzymano wzory na prawdopodobieństwo kolizji dla różnych przypadków. Celem jest lepsze dopasowanie modelu sieci z losowym dostępem do konkretnych aplikacji.* ***Randomizacja parametrów w bezprzewodowej sieci czujników**.*

Keywords: Wireless Sensor Network (WSN), Poisson Arrivals See Time Averages (PASTA), probability of collision, random access.
Słowa kluczowe: Bezprzewodowe sieci czujników, strumień Poissona, prawdopodobieństwo kolizji, dostęp losowy.

Introduction

The development of radio communication and of information systems has opened a range of possibilities to create and use a wireless network also in areas related with acquiring of information and its processing [1].

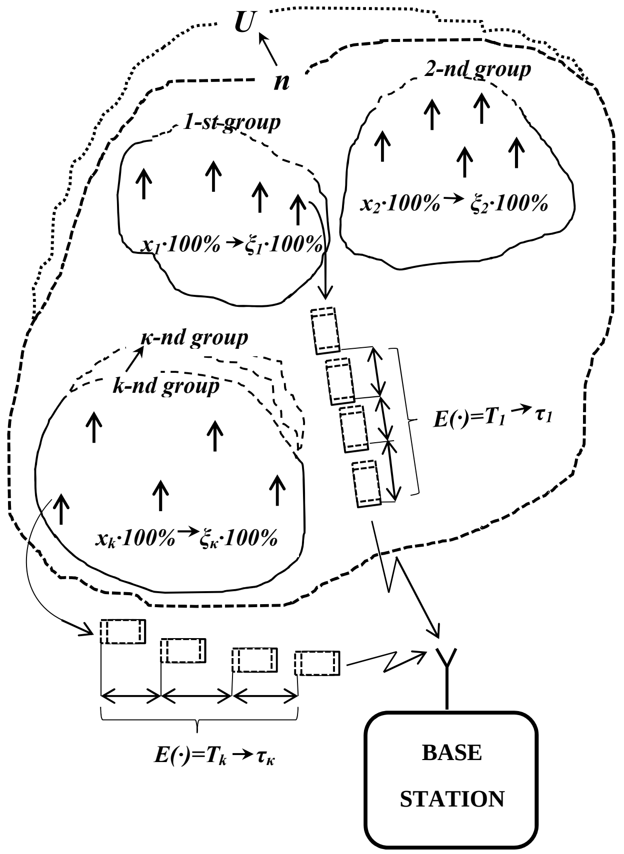

Fig. 1. Diagram of a wireless network with random parameters

Wireless Networks (WSN-Wireless Sensor Network) are a specific area used for obtaining of measurement data [2]. This is one of the most dynamically developing areas using wireless communication with a large number of applications [3, 4, 5, 6, 7]. The need to share a common transmission medium (radio space) by many users in these networks is a basic issue An important property of WSN is that often many users briefly uses the transmission medium. Establish then a permanent connection between them would be a very inefficient solution. A very good solution then is a random access. The issue of random access to the transmission medium is the subject of many works and many studies as well as network solutions [8, 9, 10]. The innovative solution was launching the ALOHA random access in 1971 in Hawaii. Further research developed these methods and they were widely used especially in local area networks known as CSMA access. Random access WSN networks seems to be very useful due to the short time required for the protocol transmission and a large number of transmitters (nodes) in the network.

WSN networks are characterized by specific architectural and communication properties arising from many conditions [2]. Here you can include features such as transmitting nodes mobility, communication of "many to one" type, the problem is to use the radio space as a communication medium with limited frequency bands with radio traffic organization, changing measuring environmental conditions, network configuration changes, complex algorithms of operations (single-hop, multi-hop, etc.).

In this context, a random access in WSN has an especial importance related with the joint use of the transmission medium for many users and network work organization. In the papers [11, 12, 13] were analyzed random access network, when the nodes transmit the information independently of each other. The network model was built based on the PASTA system (Poisson Arrivals See Time Averages). In the paper [13] nodes were divided into groups with the same average time between transmissions, which improved the working conditions of the network. In the analyzed cases, were imposed additional restrictions: one-way transmission (simplex), one carrier frequency (the lack of two-way transmission between the node and the base station as well as synchronization).

The imposition of such restrictions gives a lot of benefits because it allows for a significant simplification of the network architecture but mostly the network system. The nodes may exist in the network on the plug and play principle.There are no restrictions to the occurrence neither nor difficulties related to the procedures associated with the transmission (the lack of: monitoring, return channels, validation, synchronization, etc.). Fully random procedures in the radio communications environment, like WSN, in the case of a certain class of network applications with random access, give very big benefits, with significant savings related to expenses for the network implementation and its exploitation.

A deeper analysis of these issues and expectations in terms of applications leads to conclusions which shows that actually each of the parameters characterizing WSN networks in concrete application conditions can be variable. How to design networks, therefore, on the assumption that actually all characterized networks parameters can be variable? Conducted research and applications of WSN networks [11, 12, 13, 14] for single-hop type networks with random access control, shows a need to introduce characterizing the network parameters as random variables. This paper concerns the issue of analysis of the network work quality of single-hop type random access networks for a generalized model, in which we consider the random variables rather than deterministic previously used parameters characterizing network.

Problem formulation

The paper concerns a generalization of WSN single-hop type network model with controlled random access, which has been studied in [13,14], by randomization of characterizing network parameters.

The network nodes are equipped with radio transmitters which transmit informations from a connected sensors, which measure the physical parameters required. We assume that each node has the same frequency channel and transmits information to the base station randomly according to a Poisson distribution. The transmission is one-way from the nodes to the base station, which gives a lot of exploitation benefits. The main problem with this type of network is to ensure a minimum collision of probability of radio signals from different nodes. The development of this network concept, mainly due to the practical conditions of network applications, requires an examination transmission quality in situations, when consideration of deterministic parameters which characterize the network becomes insufficient. This leads to the conclusion that instead of deterministic parameters which characterize this type of network, it is better to consider the random variables. We assume, therefore, that so far deterministic parameters as: the number of nodes, the number of nodes groups due to different average time between transmissions, the average time between nodes transmissions in each group as well as nodes percentage, in each group, are random variables. The solution of such task allows WSN network design to be more adapted to the actual application needs with full quality control associated with the radio collision probability.

Network model

The network consists of n sensors which are able to send information about the measured physical magnitude on one selected radio frequency to the receiving base, quite independently of each other. Duration of communication protocol is t_p, the nodes send the information to the receiving point in randomly selected moments, every T s. at an average.

Beginning and cessation of the transmission of a particular node takes place in random moments but these moments are relatively rare. It is a one-way transmission, i.e. from nodes to the receiving base. The nodes are completely independent of one another and their on or off state is of no influence on the operation of the network. All the nodes (sensor-senders) or a part of them may be mobile assuming that their senders have been left within the radio range of the receiving base. If one or more nodes start sending while protocol transmission of t_p time is going on from another nodes then such a situation is called collision. The collision excludes the possibility of the correct receiving of information by the receiving base. Such a disturbed signal is ignored. The receiving base rejects the erroneous message and waits for a retransmission to be made after the average time T. We must accept a certain loss of information in exchange for simplicity in respect of both system and equipment.

Mathematical network model

We model our wireless network using a Poisson process. Mathematically the process N is described by so called counter process N_t or $N(t)$ of rate $\lambda > 0$. The counter tells the number of events that have occurred in the interval $[0, t]$ $(t \geq 0)$. N has independent increments (the number of occurrences counted in disjoint intervals are independent from each other), such that $N(t) - N(s)$ has the Poisson $(\lambda (t-s))$ distribution (with the mean $\lambda (t-s)$), for all $t \geq s \geq 0$, $j = 0,1,2,\ldots,$

$$P\{N(t)-N(s)=j\}=e^{-\lambda(t-s)}\frac{[\lambda(t-s)]^j}{j!}.$$

Let us state our main assumptions. There are some number nodes observing a dynamical system and reporting to a central location over the wireless sensor network with one radio channel. For simplicity, we assume that our sensor network is a single-hop network with star topology. We also assume every node (sender-sensor) always has packet ready for transmission. We assume that nodes send probe packets (a communication protocol) at poissonian times. Duration of the communication protocol is t_p. We say that a collision occurs in time interval s, if there exist at least two nodes which start sending within this interval with the difference between the beginning of their sending times not exceeding the value of t_p. Let $P(A_s)$ denote the probability of collisions (or the collision probability) in the time interval s. We proved the following theorem on the probability of collisions [12,13].

Theorem 1

Let $N(t)$ be a Poisson process with the rate $\lambda>0$, representing the time counter of transmissions of nodes. Then the probability of collisions in the time interval of s length $(s > t_p)$ is given by the formula

(1) $$P(A_s)=\sum_{j=2}^{\infty}e^{-\lambda s}\frac{(\lambda s)^j}{j!}[1-(1-j\frac{t_p}{s})^j],$$

where t_p is the duration time of a protocol.

In [11, 12] we consider the case, when there are n identical nodes, with the same average time between transmissions of each node. In [11] we give some conditional probability of collisions, and in [12] unconditional probability of collisions.

Theorem 2 ([12]) Let n be the number of nodes and let T be the average time between transmissions of a node. Then the probability of collisions in the interval of s length $(s > t_p)$ is given by the formula (1) with $\lambda = n/T$.

In [13] we study the case, when the average times between transmissions of nodes are not necessarily the same. In [13] can be found the following theorem on the probability of collisions in this case.

Theorem 3 Let n be the number of nodes. Assume that all nodes are divided into k groups $(1 \leq k \leq n)$, such that $n = n_1 +$ n_k, where n_i is the number of nodes from the i–th group and T_i is the average time between transmissions of each node from the i–th group $(i = 1, 2, \ldots k)$. Then the probability of collisions in the interval of s length $(s > t_p)$ is given by (1) with λ given by the formula

(2) $$\lambda = \sum_{i=1}^{k} \frac{n_i}{T_i}.$$

Note that, by denoting

(3) $$x_i = \frac{n_i}{n} \quad (i = 1, 2, ..., k),$$

(4) $$A = A(x_1, ..., x_k; T_1, ..., T_k) = \sum_{i=1}^{k} \frac{x_i}{T_i},$$

the rate parameter λ given by (2) can be rewritten in the form

$$\lambda = nA.$$

Consequently, the probability of collisions, given in Theorem 3, can be written in the form

(5) $$P(A_s) = \sum_{j=2}^{\infty} e^{-nAs} \frac{(nAs)^j}{j!}[1-(1-j\frac{t_p}{s})^j].$$

We denote

(6) $$Q(t, n, A) = \sum_{j=2}^{\infty} e^{-nAt} \frac{(nAt)^j}{j!}[1-(1-j\frac{t_p}{t})^j].$$

Then we have

(7) $$P(A_s) = Q(s, n, A),$$

where A is given by the formula (4). In this paper we study the case when the parameters $n, k, x_1, ..., x_k, T_1, ..., T_k$ are not necessarily deterministic and can be random.

Let $N(t)$ be the time counter of n nodes, which are divided into k-groups (as described above), such that $x_1+...+x_k=1$, where $x_j \cdot 100\%$ is the percentage of nodes from the j-th group, T_j is the average time between transmissions of each node from the j-th group $(j = 1, 2, ..., k)$. Note that $N(t)$ can be written in the form

(8) $$N(t) = \sum_{j=1}^{k} N_{(j)}(t),$$

where $N_{(j)}(t)$ is the number of transmissions of all nodes from the j-th group $(1 \le j \le k)$. Then $N_{(j)}(t)$ is the Poisson process with the rate $\lambda_j = x_j n / T_j$ $(j = 1, 2, ..., k)$.

Let $j = 1, 2, ..., k$. We write $N_{(j)}(t)$ in the form

(9) $$N_{(j)}(t) = \sum_{i=1}^{n} N_{(j)i}(t),$$

where $N_{(j)i}(t)$ is the Poisson process with the rate x_j / T_j and all random variables $N_{(j)i}(t)$ $(j = 1, 2, ..., k,\ i = 1, 2, ..., n)$ are independent. By (6) and (7), we obtain that

(10) $$N(t) = \sum_{i=1}^{n} Y_i(t),$$

where $Y_i(t)$ $(i = 1, 2, ..., n)$ is a Poisson process with the rate A given by (4).

Random number of nodes

First consider the case when the number of nodes is a random variable, say U. Let U be the random variable such that $P(U=n) = p_n$, where $p_n \ge 0$ $(n = 1,2, ...)$ and $p_1+p_2+...=1$. Let $Y_1(t), Y_2(t), ...$ are Poisson processes with the rate A such that $U, Y_1(t), Y_2(t), ...$ $(t \ge 0)$ are independent random variables. Then by (8), $N(t)$ can be written in the form

(11) $$N(t) = \sum_{i=1}^{U} Y_i(t).$$

By (7), (10) and (11), we have $P(A_s/U = n) = Q(s, n, A)$. From the formula of total probability, we obtain that

(12) $$P(A_s) = \sum_{n=1}^{\infty} Q(s, n, A) p_n,$$

where $p_n = P(U = n)$.

Now we consider the particular cases of the random variable U.

(a) Let $P(U = n_0) = \varepsilon$, $P(U = n_1) = 1 - \varepsilon$, where $n_0, n_1 \in \mathbf{N}$, $0<\varepsilon<1$. Then

$$P(A_s) = \varepsilon \sum_{j=2}^{\infty} e^{-n_0 As} \frac{(n_0 As)^j}{j!}[1-(1-j\frac{t_p}{s})^j] +$$
$$+ (1-\varepsilon) \sum_{j=2}^{\infty} e^{-n_1 As} \frac{(n_1 As)^j}{j!}[1-(1-j\frac{t_p}{s})^j].$$

(b) Let U has the geometric distribution: $P(U = n) = p(1-p)^{n-1}$, for $n = 1, 2, ...$, where $0 < p \le 1$. Then we have

$$P(A_s) = \sum_{n=1}^{\infty} \sum_{j=2}^{\infty} e^{-nAs} \frac{(nAs)^j}{j!}[1-(1-j\frac{t_p}{s})^j] p(1-p)^{n-1}$$

(c) Let U has the binomial distribution

$$P(U = k) = \binom{n}{k} p^k (1-p)^{n-k},$$

for $k = 0, 1, ...n$, where $0 < p < 1$, $n = 1, 2, ...$. Then we have

$$P(A_s) = \sum_{k=0}^{n} \sum_{j=2}^{\infty} e^{-kAs} \frac{(kAs)^j}{j!} \cdot [1-(1-j\frac{t_p}{s})^j] \binom{n}{k} p^k (1-p)^{n-k}.$$

(d) Let U has the Poisson distribution, $U \sim Poiss(a)$, such that

$$P(U = n) = e^{-a} \frac{a^n}{n!},$$

for $n = 0, 1, 2, ..., (a > 0)$. Then taking into account that $Q(t, 0, s) = 0$, we obtain

$$P(A_s) = \sum_{n=1}^{\infty} \sum_{j=2}^{\infty} e^{-nAs} \frac{(nAs)^j}{j!}[1-(1-j\frac{t_p}{s})^j] e^{-a} \frac{a^n}{n!}.$$

For simplicity of calculations, there is no loss of generality in assuming that the random variable U takes values not necessarily integer.

(e) Let $0 < a < b$. We assume now that all numbers from some k size set $(k \in \mathbf{N})$ are equally likely to occur: $P(U=x) = 1/k$, for $x=a+i(b-a)/(k-1)$ $(i = 0, 1, 2, ..., (k-1))$. Then we have

$$P(A_s) = \frac{1}{k} \sum_{i=1}^{k-1} \sum_{j=2}^{\infty} e^{-\left(a+i\frac{b-a}{k-1}\right)As} \frac{\left[\left(a+i\frac{b-a}{k-1}\right)As\right]^j}{j!}[1-(1-j\frac{t_p}{s})^j].$$

(f) Let U has the continuous uniform distribution on the interval $[a, b]$, $U \sim U(a,b)$. Then we have

$$P(A_s) = \frac{1}{b-a} \int_a^b Q(s, x, A) dx =$$
$$= \frac{1}{b-a} \int_a^b \sum_{j=2}^{\infty} e^{-Asx} \frac{(Asx)^j}{j!}[1-(1-j\frac{t_p}{s})^j] dx =$$
$$= \frac{1}{b-a} \sum_{j=2}^{\infty} [1-(1-j\frac{t_p}{s})^j] \cdot \frac{1}{As\, j!} \int_{Asa}^{Asb} e^{-y} y^j dy.$$

(g) Let U has the exponential distribution, $U \sim Exp(a)$, with the density function

$$f(x, a) = \begin{cases} \frac{1}{a} e^{-\frac{x}{a}} & for \quad x \ge 0, \\ 0 & for \quad x < 0. \end{cases}$$

Then, after some calculations, we obtain

$$P(A_s)=\int_0^\infty \frac{1}{a}e^{-\frac{x}{a}}Q(s,\,x,\,A)\,dx=$$

$$=\int_0^\infty \frac{1}{a}e^{-\frac{x}{a}}\sum_{j=2}^{\infty}e^{-Asx}\frac{(Asx)^j}{j!}[1-(1-j\frac{t_p}{s})^j]\,dx=$$

$$=\frac{As}{\frac{1}{a}+As}-\frac{\frac{1}{a}As}{\left(\frac{1}{a}+As\right)^2}-\sum_{j=2}^{\infty}\frac{(As)^j}{\left(\frac{1}{a}+As\right)^{j+1}}\left[(1-j\frac{t_p}{s})\right]^j.$$

(h) Let U has the Gaussian distribution, $U\sim N(m,\,\sigma)$ $(m,\,\sigma>0)$, with the density function

$$f(x)=\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{(x-m)^2}{2\sigma^2}}\quad(x\in R).$$

Then we have

$$P(A_s)=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{(x-m)^2}{2\sigma^2}}Q(t,\,x,\,A)\,dx=$$

$$=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{(x-m)^2}{2\sigma^2}}\sum_{j=2}^{\infty}e^{-Asx}\frac{(Asx)^j}{j!}[1-(1-j\frac{t_p}{s})^j]\,dx=$$

$$=\sum_{j=2}^{\infty}[1-(1-j\frac{t_p}{s})^j]\cdot\frac{(As)^j}{j!}\int_{-\infty}^{\infty}e^{-Asx}x^j\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-m)^2}{2\sigma^2}}\,dx\,.$$

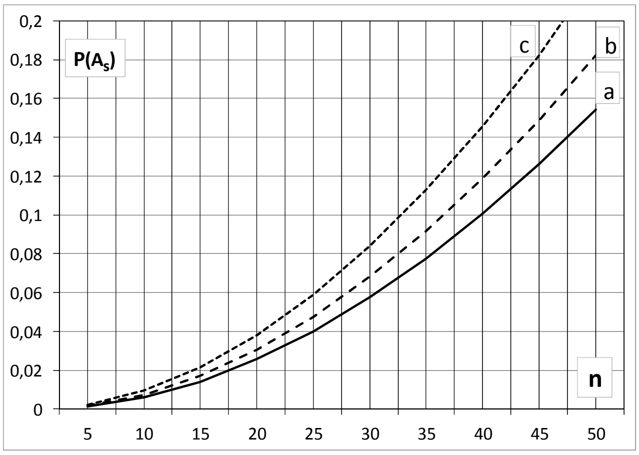

Fig. 2. Collision probability depending on the number of nodes. Continuous line – deterministic case of the number of nodes. The dashes lines – random case of the number of nodes. a – $P(U=n)=1$, b - $P(U=n)=0,8$, $P(U=n+0,1n)=0,2$, c - $P(U=n)=0,5$, $P(U=n+0,1n)=0,5$

Random intertransmission times

We consider now the case when the average time between transmissions (intertransmission times) of each node from the i-th group is a random variable, say τ_i $(1\le i\le k)$, such that all $\tau_1,\ \ldots,\ \tau_k$ are independent random variables. Putting in the formulas (4) and (6) the random variable τ_i in place of Ti $(1\le i\le k)$, we have

(13) $$A=A(x_1,\ldots,x_k;\,\tau_1,\ldots,\tau_k)=\sum_{i=1}^{k}\frac{x_i}{\tau_i}\ ,$$

(14) $$Q(t,n,\,A)=\sum_{j=2}^{\infty}e^{-nt\sum_{i=1}^{k}\frac{x_i}{\tau_i}}\cdot\frac{(nt\sum_{i=1}^{k}\frac{x_i}{\tau_i})^j}{j!}[1-(1-j\frac{t_p}{t})^j].$$

From (14), by taking the expected value, we obtain the probability of collisions

(15) $$P(A_s)=\int_0^\infty\ldots\int_0^\infty\sum_{j=2}^{\infty}e^{-ns\sum_{i=1}^{k}\frac{x_i}{y_i}}\cdot\frac{(ns\sum_{i=1}^{k}\frac{x_i}{y_i})^j}{j!}\cdot$$
$$[1-(1-j\frac{t_p}{s})^j]\,\mu_{\tau_k}(dy_k)\ldots\mu_{\tau_1}(dy_1),$$

where $\mu_{\tau_1}\ldots\mu_{\tau_k}$ are the distributions of the random variables $\tau_1,\ \ldots,\ \tau_k$, respectively.

A randomization of $x_1,\ \ldots,\ x_k$.

We define the set B_k in the following way

$$B_k=\{(\,y_1,\ldots,y_k)\in R^k:\ \ y_1,\ldots,y_k>0,\ \ y_1+\ldots+y_k=1\}.$$

We consider a random vector $(\xi_1,\ \ldots,\ \xi_k)$ taking values in $\boldsymbol{R}^k$ with the joint probability distribution $\mu_{(\xi1,\ \ldots,\ \xi k)}(d(x_1,\ \ldots,\ x_k))$ concentrated on the set B_k. Putting in (4) and (6), the random variables $\xi_1,\ \ldots,\ \xi_k$ in place of $x_1,\ \ldots,\ x_k$, respectively, we obtain that

(16) $$A=A(\xi_1,\ldots,\xi_k;\ T_1,\ldots,T_k)=\sum_{i=1}^{k}\frac{\xi_i}{T_i}\,,$$

(17) $$Q(t,n,\,A)=\sum_{j=2}^{\infty}e^{-nt\sum_{i=1}^{k}\frac{\xi_i}{T_i}}\cdot\frac{(nt\sum_{i=1}^{k}\frac{\xi_i}{T_i})^j}{j!}[1-(1-j\frac{t_p}{t})^j].$$

Taking the expected value, we obtain the formula for the probability of collisions

(18) $$P(A_s)=\int_0^\infty\ldots\int_0^\infty\sum_{j=2}^{\infty}e^{-ns\sum_{i=1}^{k}\frac{x_i}{T_i}}\cdot\frac{(ns\sum_{i=1}^{k}\frac{x_i}{T_i})^j}{j!}\cdot$$
$$[1-(1-j\frac{t_p}{s})^j]\,\mu_{(\xi_1\ldots\xi_k)}(d(x_1,\ldots,x_k)).$$

Random number of groups

We consider now the case when the number of groups is a random variable, say κ. We assume that κ takes values positive and integer. Putting in (4) and (6) the random variable κ in place of k, and normalizing (respectively), we obtain

$$A=A(x_1,\ldots,x_\kappa;T_1,\ldots,T_\kappa)=(x_1+\ldots+x_\kappa)^{-1}\sum_{i=1}^{\kappa}\frac{x_i}{T_i}\ \ ,$$

$$Q(t,n,\,A)=$$

$$\sum_{j=2}^{\infty}e^{-nt(x_1+\ldots+x_\kappa)^{-1}\sum_{i=1}^{\kappa}\frac{x_i}{T_i}}\cdot\frac{(nt(x_1+\ldots+x_\kappa)^{-1}\sum_{i=1}^{\kappa}\frac{x_i}{T_i})^j}{j!}[1-(1-j\frac{t_p}{t})^j].$$

Taking the expected value, we obtain the formula for the probability of collisions

$$P(A_s)=\sum_{k=1}^{\infty}\sum_{j=2}^{\infty}e^{-ns(x_1+\ldots+x_\kappa)^{-1}\sum_{i=1}^{k}\frac{x_i}{T_i}}\ \cdot$$

$$\frac{(ns(x_1+\ldots+x_k)^{-1}\sum_{i=1}^{k}\frac{x_i}{T_i})^j}{j!}[1-(1-j\frac{t_p}{s})^j]P(\kappa=k).$$

All random parameters

We consider now the case when in place of all considered parameters above we take random variables, which are not necessarily independent. We consider a random vector $(U,\ \kappa,\ \tau_1,\ \tau_2,\ldots,\ \xi_1,\ \xi_2,\ \ldots\ )$ such that all distributions of the real valued random variables $U,\ \tau_1,\ \tau_2\ \ldots,\ \xi_1,\ \xi_2,\ \ldots$ are concentrated on the set $(0,\ \infty)$, and the distribution of the real valued random variable κ is concentrated on the set $\{1,\ 2,\ \ldots\}$. Moreover, we assume that $\xi_1+\xi_2+\ldots\le 1$. Then, taking in (4) and (6) the random variables $U, \kappa, \tau_1, \tau_2, ..., \xi_1, \xi_2, ...$ in place of parameters, respectively, normalizing (respectively) and taking the expected value, we obtain the formula for the probability of collisions

(19)

$$P(A_s) =$$

$$E\left\{\sum_{j=2}^{\infty} e^{-Us(\xi_1+...+\xi_\kappa)^{-1}\sum_{i=1}^{\kappa}\frac{\xi_i}{\tau_i}} \cdot \frac{[Us(\xi_1+...+\xi_\kappa)^{-1}\sum_{i=1}^{\kappa}\frac{\xi_i}{\tau_i}]^j}{j!} \cdot [1-(1-j\frac{t_p}{s})^j]\right\}.$$

In the paper [13] we investigated the suitability of using WSN with random access and deterministic parameters, to the monitoring of patients (hospital ward). In the following example, which could be useful for monitoring of patients, we consider the case when both the number of nodes and the number of groups are random, as well as the percentages $x_1, x_2, ...$ are random.

Example 1: Let U has the Poisson distribution, $U \sim Poiss(\lambda)$. Let $P(\kappa=k)=p_k$ $(k=1, 2,...)$. Let $P((\xi_1, \xi_2, ...)=(\alpha, \alpha^2, ...))=p$, $P((\xi_1, \xi_2, ...)=(\beta, \beta^2, ...))=1-p$, where $0 < \alpha, \beta, p < 1$. We assume that U and κ are independent, and that they are all independent of $(\xi_1, \xi_2, ...)$. We put

$$P(\lambda, n) = e^{-\lambda}\frac{\lambda^n}{n!}.$$

Then, by (19) we obtain

$$P(A_s) = \sum_{n=1}^{\infty} P(\lambda, n)\{\sum_{k=1}^{\infty} p_k[\sum_{j=2}^{\infty}[1-(1-j\frac{t_p}{s})^j] \cdot$$

$$[pP(ns(\alpha+...+\alpha^k)^{-1}\sum_{i=1}^{k}\frac{\alpha^i}{T_i}, j) + (1-p)P(ns(\beta+...+\beta^k)^{-1}\sum_{i=1}^{k}\frac{\beta^i}{T_i}, j)]\}.$$

Conclusions

In this paper we give the formulas for the probability depending on: the random total number of the random nodes, the random number of groups, the random number of nodes in each group, the random average time between transmissions in each group. We also obtain the collision probability in the general case, when more than one parameter is random.

In Figure 2, we compare the collision probability depending on the random total number of nodes for different two-point distributions as well as deterministic. We see the influence of the increased percentage of a large number of nodes on the collision probability.

The obtained formulas for collision probability allow to analyze it, which depend on the random variable distributions type, as well as parameters of the distributions. This also applies to the case when random ones are more than one parameter at the same time.

In Example 1, we consider the case when there are random both a number of nodes and the number of groups as well as nodes percentage in groups. In this example, the random variables $\xi_1, \xi_2 ...$, which means nodes percentage in the subsequent groups are not independent. It can be seen that the use of the Poisson distribution as a random number of nodes distribution, may be justified for example in the case of the patients number in a hospital, which results from properties of the Poisson distribution [13].

Randomization consideration of parameters characterizing the network is a consequence resulting from the practical applications of this type of network. As a result of such action we can significantly improve the transmission quality in comparison to the solutions with deterministic network model [11, 12, 13, 14]. We obtain network solution with random access better adapted for exact applications.

In further research papers concern this topic we expect, on the basis of the obtained formula (19) of the collision probability in the general case, an increase a range and the number of possible applications which runs on the basis of random access in wireless measurement networks, in order to align them better to the practical needs and to improve their operation efficiency.

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