

# Model of the heat energy transmission from deep rocks to the energy recovery system

**Abstract.** The article discussed a mathematical model of the flow of thermal energy from deep rocks to the heat energy recovery system, and presented the results of the simulation of the created model. The considered model of the system is an integral part of the production of electricity from thermal energy from hot rocks located at great depths. Recovery of the energy in the system is to be effected by means of the heat exchanger located in the rocks at great depths. In the article the results of simulation were presented in Matlab-Simulink. The maximum power of the system and the duration of the transition processes were estimated using the simulation model.

**Streszczenie.** W artykule został omówiony model matematyczny przepływu energii cieplnej z głęboko położonych skał do wymiennika systemu odzyskiwania energii oraz przedstawione wyniki symulacji zbudowanego modelu. Rozważany model jest składową częścią systemu wytwarzania energii elektrycznej z energii cieplnej gorących skał położonych na dużych głębokościach. Odzyskiwanie energii w tym systemie ma się odbywać przy pomocy wymiennika umieszczonego w skałach na dużych głębokościach. W pracy przedstawiono wyniki symulacji systemu w środowisku Matlab-simulink. Na podstawie symulacji wyznaczone zostało górne oszacowanie mocy całego systemu, oszacowano czas trwania procesów przejściowych. (Model przepływu energii cieplnej do wymiennika systemu odzyskiwania energii z głęboko położonych skał.)

**Keywords:** thermal energy, transition process, mathematical modeling, computer simulation

**Słowa kluczowe:** energia cieplna, proces przejściowy, modelowanie matematyczne, symulacja komputerowa

doi:10.12915/pe.2014.01.33

## Introduction

The development and functioning of the economy requires continuous access to energy. It is estimated that about the half of fossil energy resources found on Earth have been used up. It is necessary to attract new sources of energy, preferably the renewable.

The heat energy inside the Earth is virtually inexhaustible. It is believed that its sources are mainly radioactive decay. Currently used exploitation methods of the energy are based on the use of water in deep crevices, which has a temperature above the 65 Celsius degrees.

Researchers are studying the possibility of recovering heat from deep hot rock impervious to water. These rocks can be found at great depths exceeding 5,000 m (petrothermal resources). The advantage of this source of energy is the high temperature of the rock, which at this depth can be as high as 350 Celsius degrees.

## The mathematical model

In this part a mathematical model of energy flow to the heat energy conversion system was discussed. The mathematical model was developed for simulation in MATLAB Simulink. It was assumed that the rock around the heat exchanger is homogeneous in every direction. It follows that the temperature distribution around the heat exchanger does not depend on the direction, and it is dependent only on the distance from the heat exchanger.

(Fig.1) shows the distribution of layers 1,2,...,N of energy flow model. In the heat exchanger there is temperature  $T_0$ . It is assumed that the radius  $R_N$  is the boundary above which there is the fixed temperature and it equals  $T_{N+1}$ . Heat flow between successive layers  $i = 1, 2, \dots, N+1$ , also between a layer 1 and heat exchanger is the result from the temperature difference between the layers. In the figure it is indicated by  $Q_{i,i}$  energy that flows from layer  $i+1$  to layer  $i$  and by  $Q_{O,i}$  energy that flows from layer  $i$  to layer  $i-1$  in time interval  $\Delta t$ . Rays  $S_0, S_1, \dots, S_N, S_{N+1}$  are the radius of a centre of layers, the rays  $R_0, R_1, R_2, \dots, R_N$  are the dividing radius of layers.

Capacity and thermal conductivity of the rock material was signed by  $C_w$  [J/m<sup>3</sup>·K],  $C_p$  [J/m·K·s]. So the heat capacity of each layer for  $i=1, 2, \dots, N$  is given by the formula

$$(1) C_i = \frac{4}{3} \pi (R_i^3 - R_{i-1}^3) C_w.$$

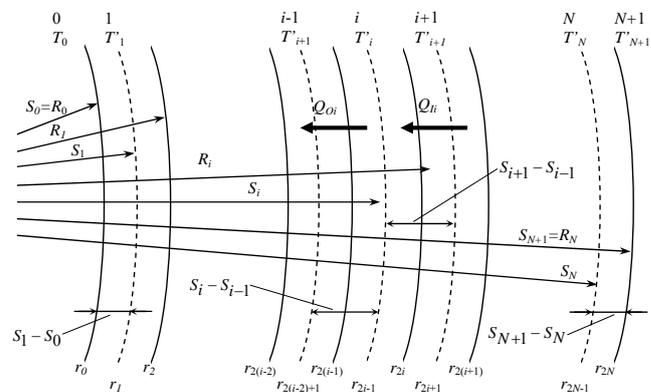


Fig.1. Layers of rocks around the heat exchanger

The thermal conductivity between the layers  $i+1$  and  $i$  for  $i=0, 1, 2, \dots, N$  is given by the formula

$$(2) K_i = 4\pi \frac{R_i^2}{S_{i+1} - S_i} C_p.$$

The energy that flows to the layer  $i$  in the time interval  $\Delta t$ , for  $i=1, 2, \dots, N$ , is given by the formula

$$(3) Q_{i,i} = K_i (T_{i+1} - T_i) \Delta t.$$

The energy that flows out of the layer  $i$  in the time interval  $\Delta t$ , for  $i=1, 2, \dots, N$  is given by the formula

$$(4) Q_{O,i} = K_{i-1} (T_i - T_{i-1}) \Delta t.$$

The change in internal energy of the layer  $i=1, 2, \dots, N$  is given by formula

$$(5) \Delta Q_i = \Delta T_i \cdot C_i = Q_{i,i} - Q_{O,i}.$$

On the basis of (1), (2), The energy that flows to the layer  $i$  in the time interval  $\Delta t$ , for  $i=1, 2, \dots, N$ , is given by the formula

(3), (4), (5)  $i=1, 2, \dots, N$  the following equation was derived

$$(6) \frac{\Delta T_i}{\Delta t} = \frac{3}{(R_i^3 - R_{i-1}^3)} \frac{C_p}{C_w} [R_i^2 \frac{T_{i+1} - T_i}{S_{i+1} - S_i} - R_{i-1}^2 \frac{T_i - T_{i-1}}{S_i - S_{i-1}}].$$

Equation (6) is a differential equation of heat balance in the layers.

## Derivation of layers

the temperature distribution at steady state given in article [7] was used in determining the radius. The layer thickness was derived in this way that in the steady state of the system the temperature difference between the layers  $r_i$ ,  $i=1,2,\dots,2N$  was constant. Rays  $R_0=S_0=r_0$ , and  $R_N=S_{N+1}=r_{2N}$  are given. The remaining rays of the model (6) were appointed. As a result of the use the temperature distribution given for the system in steady state was derived recursive relationship for the rays as shown below

$$(7) \quad r_i = \frac{2N}{\frac{2N}{r_{i-1}} - \frac{1}{r_0} + \frac{1}{r_{2N}}}$$

The rays  $S_0, S_1, \dots, S_N, S_{N+1}$  of the centre of layers, and the rays  $R_1, R_2, \dots, R_N$  of dividing of layers can be determined from the equation

$$(8) \quad R_i = r_{2i}, \quad S_i = r_{2i-1}, \quad i = 1, 2, \dots, N.$$

## System power

The maximum power of the system (6) can be calculated from the equation (9). This power is the amount of the heat that flows from the first layer to the heat exchanger at the time interval  $\Delta t$ .

$$(9) \quad P = Q_{O1} / \Delta t = \frac{4\pi R_0^2}{S_1 - S_0} C_p (T_1 - T_0) = E(T_1 - T_0)$$

Taking into account the conductivity of the walls of the heat exchanger  $C_{pw}$  [J/m<sup>2</sup>] the maximum power of the system can be determined from the equation

$$(10) \quad P = \frac{4\pi R_0^2}{S_1 - S_0} \frac{C_p}{1 + \frac{C_p}{C_{pw}} \frac{2}{S_1 - S_0}} (T_1 - T_0).$$

After taking heat exchanger resistance it is necessary to modify the heat balance equation of the first layer

$$(11) \quad \frac{\Delta T_1}{\Delta t} = \frac{3R_1^2}{(R_1^3 - R_0^3)(S_2 - S_1)} \frac{C_p}{C_w} (T_2 - T_1) - \frac{3R_0^2}{(R_1^3 - R_0^3)(S_1 - S_0)} \frac{1}{1 + \frac{C_p}{C_{pw}} \frac{2}{S_1 - S_0}} \frac{C_p}{C_w} (T_1 - T_0)$$

For analysis of the duration of the system at a given fixed power  $P_{const}$  it is also necessary to modify the heat balance equation of the first layer

$$(12) \quad \Delta Q_1 = \Delta T_1 \cdot C_1 = Q_{I1} - P_{const} \Delta t.$$

Hence, after substituting (1), (2), The energy that flows to the layer  $i$  in the time interval  $\Delta t$ , for  $i=1,2,\dots,N$ , is given by the formula

$$(13) \quad \frac{\Delta T_1}{\Delta t} = \frac{3R_1^2}{(R_1^3 - R_0^3)(S_2 - S_1)} \frac{C_p}{C_w} (T_2 - T_1) - P_{const} \frac{3}{4\pi(R_1^3 - R_0^3)C_w}$$

If a model with fixed power of equation (6), (13) for power  $P_{const}$  is substitute the equation (10) this provides to the model described by equations (6) and (11). In the model of the fixed set power when maximum power calculated from equation (11) falls below the set point of power  $P_{const}$  then maximum power received through the heat exchanger is calculated from the equation (11).

## The simulation of the model in Matlab simulink

For the purpose of simulation in Matlab Simulink the following equations were used (6) for  $i=2,\dots,N$ , and equation

(11) for the first layer. After turning off the constants  $V_i, X_i$  the following formulas were received

$$(14) \quad \frac{dT_i}{dt} = X_i T_{i-1} - (V_i + X_i) T_i + V_i T_{i+1},$$

$$(15) \quad \frac{dT_1}{dt} = V_1 (T_2 - T_1) - X_1 P_{const} = -X_1 P_{const} - V_1 T_1 + V_1 T_2.$$

where:

$$(16) \quad V_i = \frac{3R_i^2}{(R_i^3 - R_{i-1}^3)(S_{i+1} - S_i)} \frac{C_p}{C_w}$$

$$(17) \quad X_i = \frac{3R_{i-1}^2}{(R_i^3 - R_{i-1}^3)(S_i - S_{i-1})} \frac{C_p}{C_w}$$

$$(18) \quad X_1 = \frac{3}{4\pi(R_1^3 - R_0^3)C_w}$$

The system of equations (14), (15) can be written in matrix form

$$(19) \quad \begin{aligned} \dot{X} &= AX + BU \\ Y &= CX + DU \end{aligned}$$

The matrices state  $A$ , input  $B$ , output  $C$ , feedforward  $D$ , and control vector  $U$  were determined as follows

$$(20) \quad A = \begin{bmatrix} -V_1 & V_1 & 0 & \dots & 0 \\ X_2 & -(V_2 + X_2) & V_2 & \dots & 0 \\ 0 & X_3 & -(V_3 + X_3) & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & V_{N-1} \\ 0 & 0 & 0 & \dots & -(V_N + X_N) \end{bmatrix},$$

$$(21) \quad B = \begin{bmatrix} -X_1 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & V_N T_{N+1} \end{bmatrix}, \quad C = I_{N \cdot N}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \quad U = \begin{bmatrix} P_{const} \\ 1 \end{bmatrix}.$$

In the model in the Matlab Simulink environment matrices (20), (21) were used. An example of Matlab Simulink modeling equations of this type can be found in [6].

The system parameters in the model were assumed: density of rock material  $\rho=2800$  [kg/m<sup>3</sup>], specific heat volume of rock  $\lambda=790$  [J/Kg·K], the conductivity of rocks  $C=3.5$  to  $3.7$  [J/m·K·s], hence the heat capacity of the rock is  $C_w=\lambda \cdot \rho=2212000$  [J/m<sup>3</sup>·K]. It was assumed that the time interval  $\Delta t$  is equal to 1 [h], so the thermal conductivity of rocks is equal  $C_p=C \cdot \Delta t=3.6 \cdot 60 \cdot 60=12960$  [J/m·K·h]. It is assumed that the heat exchanger is made of steel with a heat conductivity of  $C=58$  [J/m·K·s], and the thickness is equal 0.02[m], so the coefficient  $C_{pw}=(58/0.02) \cdot 60 \cdot 60$  [J/m<sup>2</sup>·K·h]. The minimum temperature at the surface of the heat exchanger at a maximum power consumption is 423.15[K], the initial temperature of the rocks 600[K]. Scheme of the model of the system described by equations, (14), (15) in MATLAB Simulink is shown at (Fig.2).

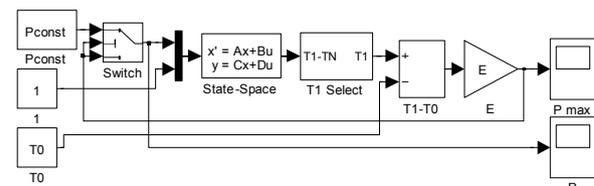


Fig.2. Model scheme of heat flow in Matlab Simulink

Simulation of the model (14), (15) at maximum power was done. Diagrams of temperature in function of time were obtained as shown at (Fig.3). The model assumes 60 layers,  $R_0=50$ [m],  $R_N=1000$ [m]. Layers 1, 2, 4, 8, 16, 32 have radii: 1 - (50-50.8[m]), 2 - (50.8-51.6[m]), 4 - (52.4-

53.4[m]), 8 - (56.2-57.2[m]), 16 - (65.6-67.0[m]), 32 - (98.2-101.3[m]).

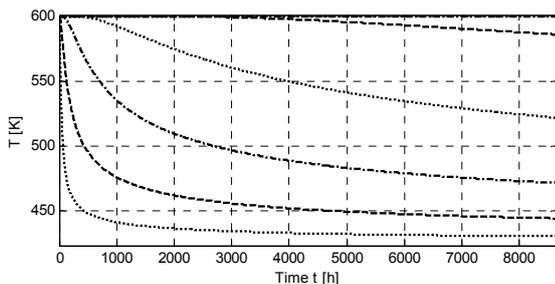


Fig.3. The course of change in temperature as a function of time in the layers 1, 2, 4, 8, 16, 32, model with 60 layers

Simulation of the model was conducted for  $N=60, 120, 180$  layers. As a result of the simulation waveforms (Fig.4) of system power were obtained. The model assumes  $R_0=50[m]$ ,  $R_N=1000[m]$ . The dashed line, continuous, dotted in the figure corresponds to the models of 60, 120, 180 layers.

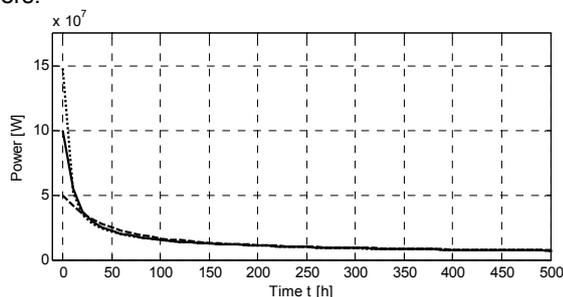


Fig.4. The maximum power changes as a function of time, simulation 20 days, models of 60, 120, 180 layers

Simulations were run for 50 years for heat exchangers with different radii, values of the radii  $R_0=10, 20, 30, 40, 50[m]$  were adopted. As the simulation result charts (Fig.5) of system power were obtained. On the figure lines of the less value of power correspond to the heat exchangers of smaller diameter.

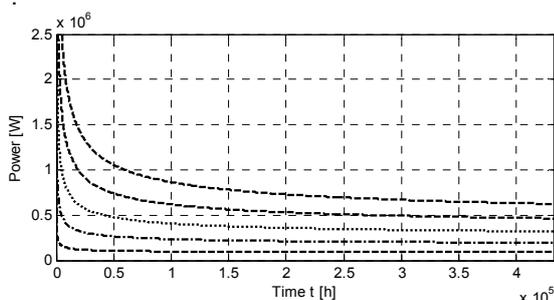


Fig.5. Mileage changes of maximum power received through the exchanger as a function of time, simulation for 50 years

For smaller diameter heat exchangers i.e: 10, 20[m] can be assumed that final output power this is the power in steady state. The final values for the heat power for exchangers of diameter 10, 20, 40, 50, 100 [m] were, respectively, 0.0889, 0.1958, 0.3206, 0.4634, 0.6242 [MW]. The power in steady state of the system is growing more than twice when radius increases twice, which confirms the results reported in the article [7].

Simulation of the model described by equations (14) and (15) at a given fixed output power  $P_{const}$  was conducted. As the simulation result the following chart of system power was obtained (Fig.6). The model assumes: heat exchanger

of radius  $R_0=50[m]$ , simulation time of 15 years, fixed power equal to 5, 4, 3, 2, 1 [MW].

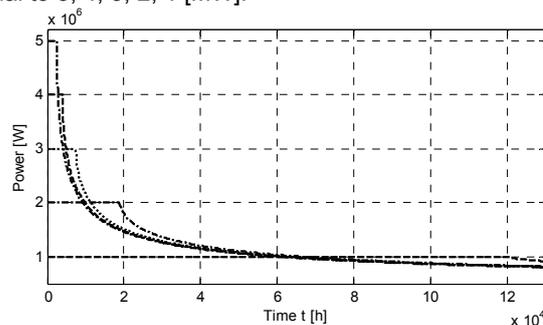


Fig.6. Changes in the level of power consumption for a fixed maximum power 5, 4, 3, 2, 1 [MW]

For the system with fixed power that is equal to 5, 4, 3, 2, 1 [MW], the power fall appears after the time respectively 2400[h]≈3[months], 3900[h]≈5[months], 7400[h]≈10[month], 19000[h]≈3[years], 120350[h]≈13[years].

Under the adopted assumptions for the heat exchanger with a diameter of 50[m], the power available in steady state is less than 1[MW]. It is possible to draw higher power in a state of transition for example: 1[MW] may be charged for more than 13 years.

### Summary and Conclusions

The article discussed a mathematical model of the flow of thermal energy from deep rocks that are the source to the heat energy recovery system, and presented the results of the simulation created model in the Matlab Simulink environment.

The maximum power of the system was estimated for the exchangers of various diameters based on the simulation model. The maximum time of power taken on a given level was also estimated.

The flow model will be used in the future work to build a model of the system of electricity production.

### REFERENCES

- [1]. Leja Franciszek: Rachunek różniczkowy i całkowy ze wstępem do równań różniczkowych, Wydawnictwo Naukowe PWN, Warszawa (2012)
- [2]. Kozłowski R.H., Żakiewicz B.M., Large capacity energy from Geo - Plutonic formation for power plants with zero CO2 emissions, *Journal of Achievements in Materials and Manufacturing Engineering*, 43/2 (2010), p. 790-795
- [3]. Kozłowski R.H., Geothermic Power Plants of high capacity - how far?, *Journal of Achievements in Materials and Manufacturing Engineering*, 49/2 (2011), p. 573-576
- [4]. Polyanin, A. D., Handbook of Linear Partial Differential Equations for Engineers and Scientists, *Chapman & Hall/CRC*, (2002)
- [5]. Mac S., Termodynamika, *Wydawnictwa Szkolne i Pedagogiczne*, Warszawa (1980)
- [6]. Szostek K., Sterowanie napędem siłownikiem przy zmiennych parametrach zasilania sprężonym powietrzem. *Pneumatyka*, (2008), nr. 1, s. 72-77
- [7]. Szostek R., Oszacowanie mocy źródła geotermicznego, *Modelowanie Inżynierskie (Modelling in Engineering)*, (2013), nr. 46, t. 15, s. 117-120

**Authors:** dr inż. Karol Szostek, Politechnika Rzeszowska, Katedra Termodynamiki i Mechaniki Płynów, ul. Powstańców Warszawy 8, 35-959 Rzeszów, E-mail: kszostek@prz.edu.pl; dr inż. Roman Szosteki, Politechnika Rzeszowska, Katedra Metod Ilościowych, ul. Powstańców Warszawy 8, 35-959 Rzeszów, E-mail: rszostek@prz.edu.pl.