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Working and reflected active powers of harmonics generating single-phase loads

Abstract. *Harmonics generating loads when supplied from a common source of a sinusoidal voltage cause reflection of a part of the energy delivered to the load back to the supply. Such a load has to be supplied therefore, with a power which is higher than the active power, even if the load is purely resistive, and consequently, the energy provider delivers more energy to such a load than that measured by an energy meter. This power is called "a working active power" of the load. The paper explains the concepts of the working and reflected active powers and presents results of measurement of these powers for common single-phase harmonics generating loads.*

Streszczenie. *Odbiorniki generujace harmoniczne w warunkach zasilania z powszechnie dostępnych źródeł napięcia sinusoidalnego, powodują odbicie do źródła części energii dostarczonej do odbiornika. W związku z tym, odbioniki takie muszą być zasilane z mocą wyższą od mocy czynnej, nawet wtedy, gdy są to odbiorniki czysto rezystancyjne. Moc tę nazano roboczą mocą czynną i jest ona większa od mocy czynnej o odbitą moc czynną. W wyniku tego odbicia energii, jej dostawca dostarcza do odbiornika generującego harmoniczne więcej energii, niż to wynika z jej pomiaru na zaciskach odbiornika. Niniejszy artykuł wyjaśnia szczegółowo koncepcję roboczej i odbitej mocy czynnej a także przedstawia wyniki pomiarów tych mocy dla pewnych generujących harmoniczne odbiorników jednofazowych. (**Robocza i odbita moc czynna odbiorników jednofazowych generujących harmoniczne).***

Key Words: Distortion, Currents' Physical Components, CPC, energy accounts, Advanced Metering Infrastructure, AMI, harmonics.
Słowa kluczowe: Przebiegi odkształcone, Składowe Fizyczne Prądów, CPC, rozliczenia energetyczne, harmoniczne.

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Introduction

Financial accounts for energy W are currently based on the cost of active energy, meaning on an integral of the active power P

$$\int_0^{\text{month}} P\,dt = W\ . \tag{1}$$

Large customers usually pay also for a low power factor, or an extra cost due to its low value is hidden inside of tariffs for energy.

Such accounts are based on standards and regulations derived from our century old understanding of the power properties of electrical systems. The majority of loads at the time when these standards were established were linear and time-invariant (LTI) and consequently, did not produce any waveform distortion. Therefore, voltages and currents in distribution systems could be regarded as sinusoidal. At the present time voltages and currents in distribution systems could be substantially distorted from a sinusoidal waveform, however.

This is because residential and commercial customers use nonlinear and/or electronically driven devices much more commonly now than before. Microwave ovens, televisions, fluorescent light bulbs and computer-based appliances are examples of such equipment. There is an innumerable amount of them in our homes and commercial buildings. All of them contribute to current and voltage distortion.

Distribution voltage and current distortion, commonly expressed in terms of harmonics, causes a number of detrimental effects for both customer loads and distribution system equipment. The issue of financial responsibility for these effects and economic incentives aimed at reduction of harmonic distortion are important both for the quality of the power system operation and for the fairness of energy accounts.

Having this fairness in mind, it might be disturbing that accounts based on the active energy (1) are applied now when the level of harmonic distortion could be much higher as compared to that when these accounts were established, more than a century ago.

This paper will show that harmonics generating loads (HGL), even purely resistive ones, have to be supplied with a power which is higher than the active power of such a load. Such power is referred to as the ***working active power.*** It is higher, because a part of energy delivered to the load is carried by harmonics back to the supply source. This energy has to be first delivered to the load from the supply, thus the HGL operates at the power higher than the active power by a ***reflected active power***. This may cause some loss of revenues for the energy provider, because conventional meters of the active energy are not capable of revealing this extra loading and consequently, an extra cost at the energy delivery to harmonics generating loads. Consequently, conventional accounts for energy create an unfair situation between the energy provider and the user of this energy in devices that generate current harmonics. This situation could be remedied if energy accounts would be based on the integral of the working active power, since the harmonics generated in the customer load would contribute to an increase of the bill for energy, thereby creating incentives for the customer for reduction of these harmonics.

This observation was first reported in [1, 3], but without any measurement data that could provide even a rough assessment of quantitative difference between the working active power and the active power.

This paper focuses on powers of resistive single-phase harmonics generating loads. It provides explanation of the concept of a working active power and the reflected active power. It also provides results of these power measurements for a few common loads that generate current harmonics.

Active power of resistive HGL

Assume that a voltage source with internal sinusoidal voltage $e(t)$ and purely resistive internal impedance R_S supplies, as shown in Fig 1, a purely resistive load of resistance R, which generates current harmonics without, for simplicity, any dc component.

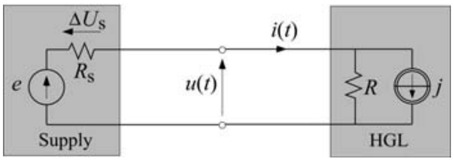

Fig. 1. Resistive circuit with harmonic generating load

The load current can be decomposed into the fundamental harmonic $i_1(t)$ and all higher order harmonics, $i_h(t)$, namely

$$i(t) = \sum_{n=1}^{\infty} i_n(t) = i_1(t) + i_h(t) \tag{2}$$

where n denotes the harmonic order. The load current harmonics flow through the supply impedance and as a result, the load voltage is distorted and can be presented in the form

$$u(t) = \sum_{n=1}^{\infty} u_n(t) = u_1(t) + u_h(t). \tag{3}$$

Because the internal voltage $e(t)$ is sinusoidal, the load voltage harmonics $u_n(t)$ are the supply response to the current harmonics injected to the source by the HGL, namely

$$u_n(t) = -R_s\, i_n(t). \tag{4}$$

The active power at the load terminals is equal to

$$P = \frac{1}{T}\int_0^T u(t)i(t)dt = \sum_{n=1}^{\infty} P_n = P_1 + \sum_{n=2}^{\infty} P_n = P_1 + P_h\ . \tag{5}$$

Because of property (4), the active power P_n of harmonics of the order n higher than n=1 is negative, since

$$P_n = \frac{1}{T}\int_0^T u_n(t)i_n(t)dt = -R_s \frac{1}{T}\int_0^T i_n^2(t)dt = -R_s I_n^2 \leq 0 \tag{6}$$

while the active power of the fundamental harmonic P_1 is positive. It means that the energy conveyed by harmonics of the order n >1 flows in the opposite direction to the direction of the energy flow at the fundamental frequency, i.e., back from the load to the supply source, where it is dissipated at the supply internal resistance, R_s.

The negative value of the sum of all active powers of the higher order harmonics

$$-\sum_{n=2}^{\infty} P_n = -P_h \overset{\text{df}}{=} P_r \tag{7}$$

is positive and will be referred to as a ***reflected active power***.

Thus, the active power of a harmonics generating load supplied with a sinusoidal voltage can be decomposed as follows

$$P = \frac{1}{T}\int_0^T u(t)i(t)dt = P_1 - P_r\ . \tag{8}$$

Decomposition (8) means that the active power P of an HGL is lower than the active power of the fundamental harmonic P_1 of the supply voltage and current. It also means that the HGL has the active power P on the condition that its power at the fundamental harmonic is equal to P_1. Thus, HGLs, even resistive ones, have to be supplied with higher power than their active power P. Therefore, this power is referred to as a ***working active power***, $P_w = P_1$, and the active power of a HGL can be decomposed into the working and reflected active powers:

$$P = P_w - P_r. \tag{9}$$

The relationship between the working, reflected and the common, P, active powers are illustrated in a diagram of energy flow shown in Fig. 2.

Along with the working active power P_w, also a ***working active current*** $i_w(t)$ can be defined. The working active current is the active component of the supply current fundamental harmonic $i_1(t)$, i.e.,

$$i_w(t) = i_{1a}(t) = \frac{P_w}{U_1^2} u_1(t) = G_1 u_1(t)\ . \tag{10}$$

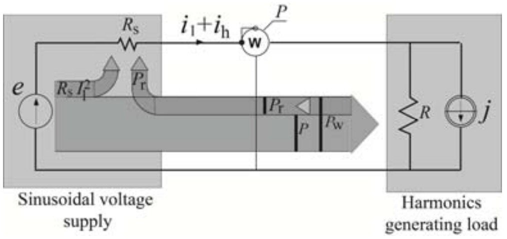

Fig. 2. Diagram of energy flow in a resistive circuit with harmonics generating load

One should observe, that even if the load is purely resistive, i.e., without any capability of energy storage, the supply current fundamental harmonic $i_1(t)$ can be shifted with respect to the voltage fundamental harmonic $u_1(t)$. In such a case, the active component of the current fundamental harmonic $i_{1a}(t)$, i.e., the working active current $i_w(t)$, is not identical to the supply current fundamental harmonic $i_1(t)$, since this harmonic also contains a reactive component $i_{1r}(t)$. A resistor connected in series with a TRIAC, as shown in Fig. 3, is a common example of such a situation.

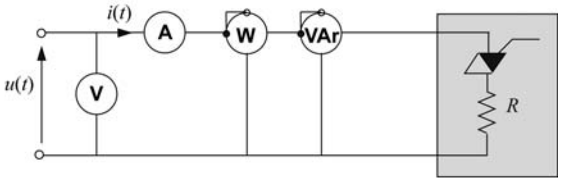

Fig. 3. Circuit with purely resistive load and TRIAC

In such a circuit, supplied from a source of a sinusoidal voltage, the current fundamental harmonic $i_1(t)$ is shifted with respect to the voltage as shown in Fig. 4. This phase-shift depends on the TRIAC firing angle α.

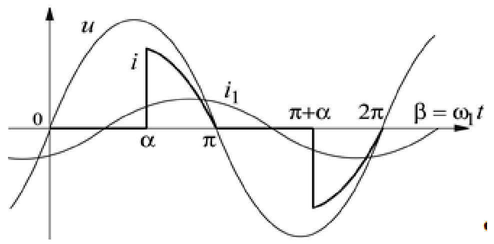

Fig. 4. Voltage and current in circuit shown in Fig. 3

In effect of this phase-shift, the supply is loaded in this purely resistive circuit not only with the active power P, but also with the reactive power Q.

Equation (8) and the diagram in Fig. 2 show that HGLs have to be supplied with higher active power than the active power P measured at the load terminals; in fact, such a load has to be supplied by the sum of the active power P and the rejected active power P_r, but this power is not visible by the energy meter connected at the load terminals. Consequently, the energy provider is not paid for the rejected energy. When the energy provider supplies HGLs, he has some losses in revenues. To avoid this loss, he should be paid for the energy delivered at the working active power, i.e.,

$$\int_{0}^{\text{month}} P_{\text{w}}\,dt = W_{\text{w}} \geq W \tag{11}$$

which can be called a ***working energy***.

The increase in the energy demand by an HGL as compared to that of an LTI load, which is equivalent as to the active power P, is even higher than the difference between the working and common active energies. This is because delivery of the working energy W_{w} requires higher supply current rms value $||i||$ and consequently, the energy loss at the supply resistance increases as compared to the loss at delivery of the same energy to LTI loads. This is explained in more details in Fig. 5 which shows two loads supplied from the same sinusoidal voltage source. The first load is a resistive HGL and the second is a resistive linear time-invariant (LTI) load, equivalent as to the active power P to the first one.

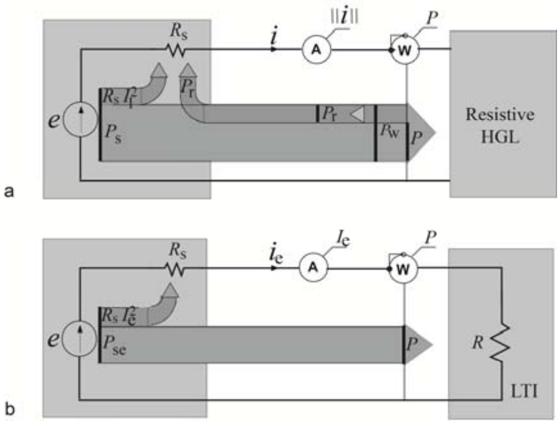

Fig. 5. (a) Circuit with resistive HGL and (b) with LTI load equivalent as to the active power P

The active power P_{s} of the internal voltage source $e(t)$ which supplies the HGL in Fig. 5(a) is the sum of the load active power P and the power of the supply resistance, i.e.,

$$P_{\text{s}} = P + R_{\text{s}}||\,i\,||^2 = P + R_{\text{s}}||\,i_{\text{h}}||^2 + R_{\text{s}} I_{\text{w}}^2 = P_{\text{w}} + R_{\text{s}} I_{\text{w}}^2. \tag{12}$$

When the same source supplies a resistive LTI load with the same active power P, then the source active power is

$$P_{\text{se}} = P + R_{\text{s}}\,||i_{\text{e}}||^2 = P + R_{\text{s}} I_{\text{e}}^2\,. \tag{13}$$

Thus, it is lower than P_{s}. Comparison of formulae (12) and (13) shows that $P_{\text{s}} \geq P_{\text{se}}$ for two reasons

1. $P_{\text{w}} \geq P$
2. $I_{\text{w}} \geq I_{\text{e}}$.

These two factors, which contribute to energy demand of HGL increase, might have a different effect upon the bill for energy, however. When a customer pays for energy delivered, the cost of energy lost at its delivery is automatically included in such a bill, through the tariff for energy.

Quantitative difference between active power P and working active power P_{w}

Since the working active power is a power of the fundamental harmonic of the supply voltage and current, a meter for its measurement has to be able of calculating the complex rms (crms) value of the supply voltage and current fundamental harmonic $\boldsymbol{U}_1$ and $\boldsymbol{I}_1$. Having these crms values, the working power is equal to

$$P_{\text{w}} = \text{Re}\{\boldsymbol{U}_1 \boldsymbol{I}_1^*\}. \tag{12}$$

Such measurement cannot be done, of course by conventional analog meters. A meter with a capability of digital signal processing (DSP) is needed for that.

To provide quantitative data for this paper on the difference of the active power and the working active power, National Instruments sampling A/D converters and LabView DSP software was used in lab experiments. The voltage and current sensors were isolated from the circuit under investigation by optoelectronic analog converters, which provided up to 10 V input signals for simultaneous A/D converters of 14 bit resolution. A diagram of the DSP-based instrument developed for experiments is shown in Fig. 6.

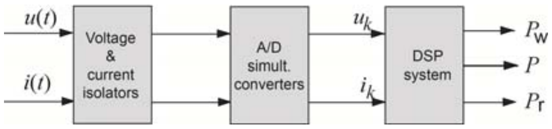

Fig. 6. Diagram of signal flow of the DSP-based instrument for powers measurement

Although the measurement accuracy was not a central issue of these experiments, the results obtained from the DSP-based instrument LabView, were compared, with respect to the voltage and current rms values, with results provided by Weston analog meters of 0.25% accuracy. The accuracy (ε_{d}) of the DSP-based measurement system compared to accuracy the Weston analog meters (ε_{a}) were recorded and the measurement error was evaluated with the formula

$$\varepsilon = \sqrt{\varepsilon_{\text{a}}^2 + \varepsilon_{\text{d}}^2}\,. \tag{11}$$

Combined accuracy of the voltage and current measurement are compiled in Table 1.

Table 1. Meters accuracy

Measured Quantity	Analog meter (ε_{a})	DSP meter (ε_{d})	Combined Accuracy
Voltage	0.25%	0.12%	0.28%
Current	0.25%	0.15%	0.29%

Reference experiment: Purely Resistive LTI Load

Before the instrument was used for measuring the active powers of HGLs, it was tested in a circuit with a purely resistive load of resistance R = 98.0 Ω, supplied from a source of a sinusoidal voltage with the internal resistance R_{s} = 6.0 Ω. The supply resistance was chosen to have the voltage drop up to 5% of the open circuit voltage rms value, U = 120 V. The measured active power was P = 139.0 W. The measured working active power P_{w} was equal to the active power P =139.0 W and consequently, the reflected active power P_{r} = 0.0 W.

After the instrument was tested, four experiments with common HGLs such as a microwave oven, fluorescent bulbs with electronic ballasts, a PC game console and a single-phase rectifier were run. Measurement results are compiled in Table 2.

Table 2. Measurement results

Exp. #	R_{S}	$\|\|i\|\|$	$\|\|i_{\text{e}}\|\|$	P	P_{w}	$P_{\text{s}}/P_{\text{se}}$
–	Ω	A	A	W	W	–
Ref.	6.0	1.15	1.15	131.0	131.0	1.000
No. 1	0.5	12.1	10.4	1280	1310	1.015
No. 2	4.0	1.62	1.15	132.0	139.0	1.036
No. 3	7.0	0.90	0.63	74.0	78.0	1.039
No. 4	3.0	2.10	1.54	178.0	185.0	1.032

Details of experiments are presented below.

Experiment #1. Microwave Oven

A microwave oven type GE 1300 W, was supplied from a voltage source of internal resistance R_s = 0.5 Ω and the voltage of rms value U = 126 V. The waveforms of the voltage and current at the microwave oven terminals are shown in Fig. 7.

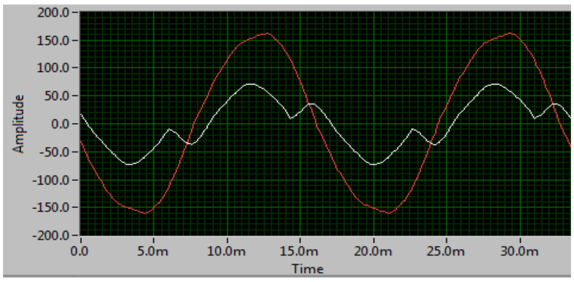

Fig. 7. Voltage and current waveforms at microwave oven terminals

Experiment #2. CFL Fluorescent Bulbs

Ten of GE compact fluorescent light (CFL) bulbs with electronic ballasts were supplied from a voltage source of internal resistance R_s = 4.0 Ω and the voltage of rms value U = 120 V. The waveforms of the voltage and current at bulbs terminals are shown in Fig. 8.

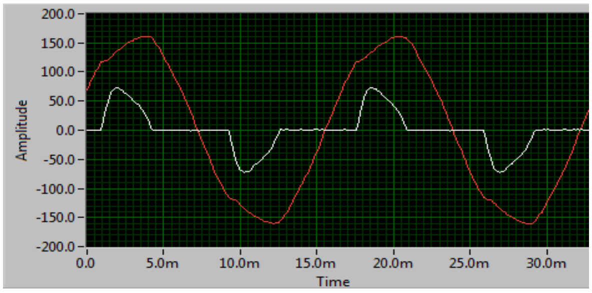

Fig. 8. Voltage and current waveforms at terminals of CFL bulb

Experiment #3. PC Game Console

A Microsoft Xbox360 game console was supplied from a voltage source of internal resistance R_s = 7.0 Ω and the voltage of rms value U = 120 V. The waveforms of the voltage and current at console terminals are shown in Fig. 9.

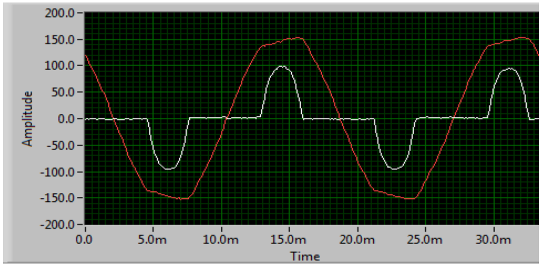

Fig. 9. Voltage and current waveforms at game console terminals

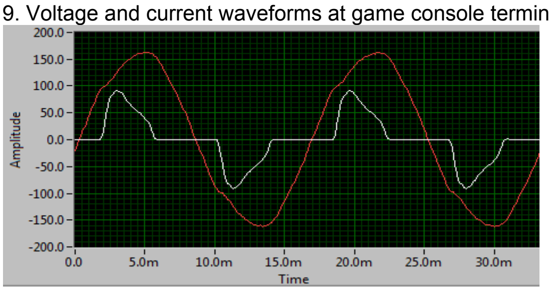

Fig. 10. Voltage and current waveforms at terminals of single-phase rectifier with capacitive filter

Experiment #4. Single-phase rectifier

A single-phase rectifier used as a HGL was built as a full-wave rectifier with capacitive filter with capacitance C = 100.8 μF and loaded with resistance, R = 152.1 Ω. The supply source resistance was R_s = 3.0 Ω.

Recorded waveforms of the voltage and current at the rectifier terminals are shown in Fig. 10.

It should be observed that in this experiment the HGL is not a resistive load, however. This is because of the capacitive filter.

Conclusions

The analysis of power properties of single-phase resistive harmonics generating loads, presented in this paper, shows that such loads require more energy than measured by common energy meters. This is because the working active power P_w of such loads is higher than their active power P. The supply current rms value of such loads is also higher as compared to LTI loads equivalent as to their active power, which causes additional loss of energy in the supply. Measurements of the working active power and the active power for common HGLs show that the difference between them is not high, just a single or even part of a percent. Nonetheless, due to a growing number of such loads, the energy providers can face some loss of revenue.

Currently, when customers are billed for energy measured by conventional energy meters, a customer with HGLs pays exactly the same bill as a customer with the same active power, but consumed by linear loads. Such a customer transfers, along with generated current harmonics, all harmonics related problems to the energy provider. The provider has to pay for extra energy losses and for the waveform distortion related disturbances in his equipment. Therefore, charging customers for the working instead of the active energy might be a step in the right direction.

Such accounts based on the amount of the working energy might create financial incentives, both at the provider and the user side, for improvement of the supply and the loading qualities. Switching to such accounts fit well present trends of changes in power systems and a concern with the supply and loading quality degradation, as well as increase in the cost of energy. Such accounts can be implemented in “smart meters,” developed under the concept of the Advanced Metering Infrastructure (AMI) [3, 4]. Now, communication with energy meters and cost reduction in data collecting, are the primary concerns at AMI development. Such meters are based on the DSP technology, which can be easily implemented also for more advanced tasks, such as working energy measurement, however.

REFERENCES

[1] Czarnecki L.S., Working, reflected and detrimental active powers, *IET on Generation, Transmission and Distribution,* (2012) *Vol. 6, No. 3, pp. 233-239*

[2] Czarnecki L.S., Toups T., Working energy-based economic incentives for the supply and loading quality improvement in isolated micro-grids” *Proc. of the Int. Conf. on Harmonics and Quality of Power, (ICHQP)* (2012) Hong Kong

[3] Mauri G., D. Moneta D., C. Betoni C., Energy conservation and smart grids: new challenge for multimetering infrastructures, *IEEE Power Tech. Conf.*, Bucharest, (2009) pp. 1-5

[4] Sui H., at all, An AMI system for the deregulated electricity market, IEEE Trans. on Industry Appl., (2009) Vol.45, No. 6., pp. 2104-2108

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