Six-phase symmetrical induction machine under fault states-modelling, simulation and experimental results

Introduction

Power electronics has been utilized in many fields of industry for over several decades now, and due to its continuous development many areas have been deeply investigated. One of the earliest field of application for power electronics were an electrical drives, what resulted in a well-known and widely used technology today. It is worth mentioning that modern civilization demands more and more for electric machines in vast range of applications. There are several reasons for this situation among which a searching for an alternative energy sources [1], introducing more efficient solution for automotive [2] and aircraft industry [3], can be listed.

Many of applications in mentioned fields such an offshore wind turbines, vehicles drives and aircraft generators or aileron actuators, require devices with an ability to uninterruptable work. For remote applications such an offshore wind energy generation this kind of systems are motivated mainly due to maintenance costs whereas in both: electric vehicles and more electric aircraft safety reason is a number one objective. Additionally, among many other fields where higher reliability is required a ship propulsion, military, or nuclear energy systems could be listed.

Unfortunately, a ubiquitous 3-phase machines suffer from a huge shortcoming in this aspect. Namely, during one phase failure the 3-phase machine is unable to work properly anymore without additional neutral wire and divided DC-link in power electronics converter [4]. Moreover, in order to reinforce reliability issues, some redundant solutions are applied such those described in [5]. However, these solutions are occupied by a necessity of extra space which is needed to install other machines-converter set, and yield higher costs.

All these mentioned factors caused that many researchers focus on multiphase machines and their inherent ability for post-fault operation.

Dealing with fault states is a risky task, thus a proper machine model operating under these conditions is required first. Then, following paper focuses mainly on a basic approach for modelling 6-phase induction machine connected to 2-level Voltage Source Inverter (VSI) performing under chosen fault states such an open-phase. Moreover, other possible scenarios are mentioned. This paper is an important part for analyzing so-called fault-tolerant control strategies.

Multiphase machine

As it was mentioned in previous section, the increased interest in uninterruptable operation of electrical machines can be solved by introducing a multiphase machines. It should be noticed that term ‘multiphase’ refers to machines with phase number higher than 3. Historically, those devices were used for high power application to decrease current values [6] in one phase by splitting them for a larger number of phases. It was caused by a low rated current values of early semiconductors. The main advantage of multiphase machines over 3-phase counterparts is that after failure of one phase, the machine is still able to generate rotating vector of magnetic flux. Additionally, those machines have other features such as:
• higher power density coefficient (PDC),
• lower electromagnetic torque ripples,
• lower audible noise.

It is worth mentioning that most of typical 3-phase machines such an induction synchronous and asynchronous, permanent magnet or reluctant ones have their multiphase counterparts. Another issue is a phase number and its winding displacement. Most common types are machines with odd number of phases or with number of \( N = k \cdot 3 \), where \( k \) belongs to \( \{2,3,4,\ldots\} \) what corresponds to \( k \) sets of 3-phase windings. In this latter case, it is important what is an angle \( \gamma \) of two adjacent windings. This results in so-called symmetrical (where \( \gamma \) is constant between each phase) or asymmetrical (where each set of 3-phase winding is shifted by angle \( \gamma \) in respect to the other 3-phase set) machines (Fig 1). Moreover, for machines with winding number being multiple of three can be connected into single or separated \( k \) neutral points. More detailed work about multiphase machines in general is published in [7].

![Figure 1. Asymmetrical and symmetrical six-phase machines](image-url)
In following paper a single neutral point, six-phase symmetrical induction machine under fault condition is modelled, simulated and experimentally verified. To control this machine a 6-phase, 2-level VSI is connected as presented in Fig. 2. Similarly to 3-phase machines, in this case many different control algorithms can be utilized such as: DTC [8], FOC [9] or a predictive control [10]. However, due to simplicity the simple $U/f =$ const method is applied to model and analyze chosen fault states.

![Diagram of a six-phase machine](image)

**Figure 2. Simulated machine-converter system**

It has to be noted that multiphase machines can be utilized even more efficiently with advanced fault-tolerant control methods that can maintain a circular trajectory of stator magnetic flux which and that is described deeply in [11]. However, this is out of scope of this paper.

**Machine model**

A basic set of electrical equations for modelling induction machine of any phase number is presented in (1):

\[
\begin{align*}
\mathbf{v}_s &= \mathbf{R}_s \cdot \mathbf{i}_s + \frac{d}{dt}[\mathbf{L}_m \cdot \mathbf{i}_s + \mathbf{L}_w \cdot \mathbf{i}_r] \\
\mathbf{v}_r &= \mathbf{R}_r \cdot \mathbf{i}_r + \frac{d}{dt}[\mathbf{L}_r \cdot \mathbf{i}_r + \mathbf{L}_w \cdot \mathbf{i}_s]
\end{align*}
\]

where: $\mathbf{v}$ - phase voltages vector (length $N$), $\mathbf{i}$ - phase currents vector (length $N$), $\mathbf{L}$ - inductances matrix ($N \times N$) and $\mathbf{L}_w$ and $\mathbf{L}_r$ describes stator and rotor values. Additionally, indices $ss$ and $rr$ describes self-inductance and mutual inductances within stator and rotor respectively whereas indices $sr$ and $rs$ correspond to stator-to-rotor and rotor-to-stator mutual inductances, respectively.

Phases notation in this paper, for six-phase induction machine is following: $a_1, a_2, b_1, b_2, c_1, c_2$ where indices denotes a typical 3-phase winding with $2\pi/3$ phase shift. This notation is used for all quantities such a voltages, currents and fluxes - denotes as in (2). Similarly to 3-phase machine it is possible to present machine model by a set of other equations which simplify analysis and application of control algorithms. For this reason a Space Vector Decomposition (SVD) method is applied:

\[
\begin{bmatrix}
\mathbf{g}_{a1} \\
\mathbf{g}_{a2} \\
\mathbf{g}_{b1} \\
\mathbf{g}_{b2} \\
\mathbf{g}_{c1} \\
\mathbf{g}_{c2}
\end{bmatrix}
= \mathbf{T}_6
\begin{bmatrix}
\mathbf{g}_a \\
\mathbf{g}_b \\
\mathbf{g}_c \\
\mathbf{g}_s \\
\mathbf{g}_r \\
\mathbf{g}_i
\end{bmatrix}
\]

with proper transformation $\mathbf{T}_6$ (3):

\[
\mathbf{T}_6 = \begin{bmatrix}
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

where: $\gamma$ denotes the angle between adjacent phases and equals $2\pi/6$ for six-phase symmetrical machine. As it can be noticed, the SVD (2) produces another set of 6 differential equations for both rotor and stator expressions. However, now they creates 3 sets of orthogonal complex planes $a-\beta$, $x-y$, $x-y$. Often, the third complex plane is named zero sequence part. Even if it can be difficult to interpret these planes it simply says that vectors placed on each do not affect each other. Nevertheless, an advantage of (2) is that during balanced conditions, components of (2) are orthogonal to each other. Nevertheless, an advantage of (2) is that during balanced conditions, components of (2) are orthogonal to each other.

Executing (2), (3) for both sides of equations (1) it has to be pointed out that there are separate complex planes for stator and rotor expressions. Customarily it is denoted with an apostrophe sign, however in this paper it is omitted and all equations in (5) refer to stator rotating reference frame.

\[
\begin{align*}
\mathbf{v}_{sa} &= \mathbf{R}_s \cdot \mathbf{i}_{sa} + \left(\mathbf{L}_s + \mathbf{L}_w\right) \frac{d}{dt}\mathbf{i}_{sa} + \mathbf{L}_w \frac{d}{dt}(\mathbf{i}_{sa} \cos \theta - \mathbf{i}_{sb} \sin \theta) \\
\mathbf{v}_{sb} &= \mathbf{R}_s \cdot \mathbf{i}_{sb} + \left(\mathbf{L}_s + \mathbf{L}_w\right) \frac{d}{dt}\mathbf{i}_{sb} + \mathbf{L}_w \frac{d}{dt}(\mathbf{i}_{sa} \sin \theta + \mathbf{i}_{sb} \cos \theta) \\
\mathbf{v}_{sa}^{\prime} &= \mathbf{R}_s \cdot \mathbf{i}_{sa} + \left(\mathbf{L}_s + \mathbf{L}_w\right) \frac{d}{dt}\mathbf{i}_{sa}^{\prime} + \mathbf{L}_w \frac{d}{dt}(\mathbf{i}_{sa} \sin \theta + \mathbf{i}_{sb} \cos \theta) \\
\mathbf{v}_{sb}^{\prime} &= \mathbf{R}_s \cdot \mathbf{i}_{sb} + \left(\mathbf{L}_s + \mathbf{L}_w\right) \frac{d}{dt}\mathbf{i}_{sb}^{\prime} + \mathbf{L}_w \frac{d}{dt}(\mathbf{i}_{sa} \cos \theta - \mathbf{i}_{sb} \sin \theta)
\end{align*}
\]

where $T_6^{-1} = T_6^T$.

Executing (2), (3) for both sides of equations (1) it has to be pointed out that there are separate complex planes for stator and rotor expressions. Customarily it is denoted with an apostrophe sign, however in this paper it is omitted and all equations in (5) refer to stator rotating reference frame.
where: $L_s$ – stator leakage inductance, $L_r$ – rotor leakage inductance, $L_m$ – per-phase equivalent circuit magnetizing inductance, $\theta_r$ – rotor electrical position.

To complete machine equations two additional equations are required. Namely, production of electromagnetic torque:

$$T_e = P L_m \left[ \cos \theta_r \left( i_{\alpha_r} i_{\beta_r} - i_{\alpha_p} i_{\beta_p} \right) - \cos \theta_s \left( i_{\alpha_s} i_{\beta_s} - i_{\alpha_p} i_{\beta_p} \right) \right]$$

where: $P$ – is a poles pair number and a mechanical equation:

$$T_L - T_e = J \frac{d\omega_m}{dt} + h_{om}$$

where: $T_L$ – is a load torque, $J$ – inertia of rotating mass, $h_{om}$ – friction coefficient, $\omega_m$ – rotor mechanical speed.

Equations (6)-(7) can be implemented in Matlab/Simulink environment. Additionally, to be able to utilize SimPowerSystem power electronics components such the ‘Universal Bridge’ converter it is necessary to connect this machine model via simple interface consisted of ‘Current Control Sources’ that are controlled by machine phase currents.

**Fault states analysis**

Figure 2 presents a simulated machine-converter system. It is rather obvious that a higher number of switches and phases increases a probability that any of this part can be broken. Following fault states can be considered as the simplest ones:

- switch fault (A)
- machine phase fault (B)

A) Due to the fact that any typical power electronic switch consist of IGBT transistor with anti-parallel diode different cases should be considered. Both fault such short-circuit and open-switch can happen for both upper and lower transistor in converter’s branch. Nonetheless, anti-parallel diode is still able to conduct current and block lower transistor in converter’s branch. Nonetheless, anti-parallel diode is still able to conduct current and block lower transistor in converter’s branch. Nonetheless, anti-parallel diode is still able to conduct current and block lower transistor in converter’s branch. Nonetheless, anti-parallel diode is still able to conduct current and block lower transistor in converter’s branch. Nonetheless, anti-parallel diode is still able to conduct current and block lower transistor in converter’s branch.

B) Second fault state is more complex from the modeling as well as a performance point of view. Namely, phase lost can be result of disconnection of phase cable or failure of whole inverter’s leg. This influences significantly the machine’s phase currents, and in turn, on trajectory of a rotating vector of stator magnetic flux. This flux is presented in Figure 3.

Gray color depicts the magnetic vector trajectory during healthy, whereas the black color – failure case. This is only simplified hodograph based on setting one ideal current source to 0. However, in simulation and real test bench cases, the six phase system becomes unbalanced and magnitude in remaining phases differs from each other. It can be briefly mentioned that one of the main goals of fault-tolerant control methods is to keep circular trajectory with reduced phase number.

There are several methods to simulate an open-phase fault for multiphase machine in Matlab/Simulink. Due to the fact that model described by (5-7) is the voltage based representation, setting voltage in the phase input terminal to 0 would result in short-circuiting this terminal. Therefore, other approaches can be considered. One of solutions bases on putting very large resistor values in series into chosen phase. Similar behavior can be obtained by using ‘break’ element from SimPowerSystem library. Unfortunately, latter method does not allow to choose any time moment to apply fault because ‘break’ switch, turns itself off only in case of current zero-crossing condition. The other approach, presented in this paper, bases on calculating back-EMF induced in faulted phase and feeding it back into input voltage terminal. This can be obtained using dependencies in equations obtained by multiplying phase current and voltages by transformation (2) with zeroing current in faulted phase \[13\]. For an example the $a_1$ phase is taken as a faulty one. Following equations can be presented for back-EMF calculation. After multiplying equations (2) and (3) with faulty $a_g$ phase it can be noticed that variables $g_{sa}$, $g_{da}$, $g_{a-g}$ and $g_{a-g}$ will be affected. Moreover, following dependencies can be derived:

$$g_{sa} - g_{stl} = \frac{\sqrt{2}}{6} (g_{sa} - 2 g_{da} + g_{s-g})$$

$$g_{sa} + g_{stl} = \frac{\sqrt{6}}{6} (2 g_{sa} - g_{da} + g_{s-g})$$

It is known that in case of fault the broken phase current equals to 0 and substituting it into the second equation of (8). It can be seen that

$$i_{sa} + i_{stl} = -\frac{\sqrt{2}}{6} (i_{sa} + i_{stl})$$

Similar dependency can be addressed for $v_{sa}$ voltage:

$$v_{sa} = \frac{\sqrt{2}}{6} (v_{sa} - 2 v_{da} + v_{s-g} + v_{stl})$$

Then, knowing (9) and the voltage can be calculated knowing basic equation of (1) $v_{stl}$ can be calculated:

$$v_{stl} = -R_s \left[ \frac{\sqrt{2}}{6} (v_{sa} - 2 i_{sa} + i_{stl}) - L_m \frac{d}{dt}\left( \frac{\sqrt{2}}{6} (i_{sa} + i_{stl}) + i_{sa} \right) \right]$$

Finally, it can be written that:

$$v_{s-g} = \frac{\sqrt{2}}{2} v_{sa} - \left[ v_{s-g} \cos \gamma + v_{s-g} \cos 2\gamma + v_{sa} \cos 3\gamma \right]$$

$$+ v_{s-g} \cos 4\gamma + v_{sa} \cos 5\gamma$$

It can be noted that back-EMF (12) can be calculated using mainly phase voltages of remained phases. However in practical model application and utilization of power interface of SimPowerSystem it is convenient to apply following equation in $\alpha$-$\beta$ reference frame.

**Simulation results**

As it was mentioned earlier, simulation model was built in Matlab/Simulink environment. Additionally,
SimPowerSystem library was involved for modelling power electronic devices. Following assumptions were made:

- machine core saturation and core losses are neglected
- machine electrical parameters are time and temperature independent
- converter ideal switches

Parameters of simulated system are based on parameters of 2.5kW 6-phase machine that was obtained by rewinding a 3-phase induction motor. This approach can result in slightly worse performance than in case of dedicated 6-phase machine design. Mentioned parameters are presented in Table I.

In following simulations an operating point with 40% of rated torque is used during testing fault states. Simulation results for the a1 open-phase failure are shown in Figure 4. As it was assumed, after fault at time $t_f = 0.5s$ phase current reaches zero. Moreover, torque ripples can be observed, what has negative influence for machine performance and can shorten life of the machine’s bearings.

Additionally, magnitudes of currents in remained phases increase, what can be dangerous if their rated values are exceeded. Worth noting is a fact that due to presented currents, machine heats in a non-uniform manner what can drastically shorten life time of windings insulation.

Experimental results

Figure 6 shows a trajectory of the stator rotating magnetic flux vector in both healthy and failure cases for the operating point presented in Figure 4. As it was mentioned in previous section post-fault trajectory slightly differs from it simplified counterpart in Figure 3.

Results of power utilization during pre- and post-fault cases are considered in Figure 5. A mechanical power is referred to the power supplied from DC-link circuit.

Presented waveform were obtained due to utilization of moving average filter which allows to remove switching noise as well as power ripples.

## Table I. Model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor rated power</td>
<td>$P$</td>
<td>2.65kW</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>$L_s$</td>
<td>41 [mH]</td>
</tr>
<tr>
<td>Rotor leakage inductance</td>
<td>$L_r$</td>
<td>41 [mH]</td>
</tr>
<tr>
<td>Magnetizing inductance</td>
<td>$L_m$</td>
<td>410 [mH]</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>$R_s$</td>
<td>10 [$\Omega$]</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>$R_r$</td>
<td>10 [$\Omega$]</td>
</tr>
<tr>
<td>Rotor inertia</td>
<td>$J$</td>
<td>0.03 [kg*m²]</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>$k$</td>
<td>0.0015</td>
</tr>
<tr>
<td>Pole pairs</td>
<td>$p$</td>
<td>2</td>
</tr>
<tr>
<td>Phase angle</td>
<td>$\gamma$</td>
<td>2m/6</td>
</tr>
<tr>
<td>DC-link voltage</td>
<td>$V_{dc}$</td>
<td>660 [V]</td>
</tr>
<tr>
<td>Control strategy</td>
<td>$\text{-}$</td>
<td>$U/f = \text{const}$</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>$f_s$</td>
<td>5 [kHz]</td>
</tr>
<tr>
<td>Reference frequency</td>
<td>$f_r$</td>
<td>50 [Hz]</td>
</tr>
</tbody>
</table>

Figure 4. Phase a1 (black) open-phase fault. Waveforms from top: electromagnetic torque, stator currents

Figure 5. Mechanical and total consumed power waveform in pre- and post-fault performance with half rated load torque

Figure 6. Stator magnetic flux vector trajectories for healthy (gray) and faulty (black) conditions

As it can be noticed the increase of consumed DC-link power occurs due to fault presence. This increase is about 2% for particular operation point and this method can be method utilized for examination of the post-fault characteristics.

The a1 open-phase fault was verified on a 2.5kW six-phase symmetrical, single neutral induction machine coupled to 3-phase 5.5kW induction machine test bench shown in Figure 7. Six-phase machine was connected to two typical 3-phase converters in parallel to the same DC-link. For simple open-loop control method a converter transistor gates were controlled be TMS28335 DSP.
Due to safety reason the multiphase machine was supplied with lowered DC-link voltage and frequency decreased proportionally to 25 Hz. Switching frequency was set to 5kHz. Figure 8a shows sampled phase currents after fault occurrence. Presented signals are available in TMS28335 DSP processor memory through AD converters. Figure 8b and 8c present waveforms measured with two 4-channel Tektronix oscilloscopes. A significant current distortion that can be observed is resulted by rewound six-phase stator winding distribution where different phases share the same slots. This can be clearly noticed when faulted a1 reduces distortions in phase b2. Nonetheless, it can be noticed that amplitudes of remaining currents in post-fault condition were simulated properly what was the main goal of presented paper. Simulation results differs slightly in currents amplitudes mainly due to real machine parameters that should be measured carefully. The latter issue is also crucial for more advanced control strategies such i.e Field Oriented Control. Worth noting is a fact that precise parameters measurement obtained by typical locked-rotor and no-load tests of chosen induction machine requires properly contracted symmetrical six-phase transformer.

Conclusion
Presented paper describes modelling of the symmetrical, six-phase induction machine under open-phase failure case. Both simulation and experimental results for both healthy and faulty conditions are shown. Negative impact of phase lost were explained and briefly analyzed. As it can be noticed a proper design of real machine is extremely crucial due to particular phases coupling. Presented paper presents a basic consideration for further work connected to fault-tolerant control algorithms.

Presented work was done at Institute of Control and Power Electronics at Warsaw University of Technology as a part of grant founded by Dean of Electrical Engineering Faculty.

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