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Relations of limited errors and uncertainties type B of the bridge as conditioner circuit for sensors of broadly variable resistances

Streszczenie. *Podano funkcje rozwarciowych współczynników przetwarzania rezystancji na napięcie i ich postacie iloczynowe dla przetwornika w postaci mostka rezystancyjnego (4R) o zasilaniu prądowym lub napięciowym i dowolnych przyrostach rezystancji jego ramion. Przedstawiono dwa zaproponowane ujęcia błędów i niepewności: jednorodne, w wartościach względnych odniesionych do czułości początkowych i dwuskładnikowe – z wyodrębnieniem składników dla stanu początkowego i dla przyrostu funkcji przetwarzania. Rozpatrzono przetworniki rezystancji mostki o jednakowych rezystancjach początkowych i kilku najczęściej stosowanych wariantach ich przyrostów. Przedstawiono przebiegi ich rozwarciowych funkcji przetwarzania oraz błędów granicznych i niepewności standardowych typu B przy zasilaniu prądowym i napięciowym (**Funkcje rozwarciowe współczynników przetwarzania rezystancji na napięcie i ich postacie iloczynowe dla przetwornika w postaci mostka rezystancyjnego**).*

Abstract*. The functions of transfer coefficients and their rationalized forms for the unloaded four arms bridge of arbitrary variable arm resistances, supplied as two-port from current or voltage source, are given together with their error propagation formulas for arbitrary (general) case. Two types of description of errors and uncertainties as accuracy measures, i.e. related to initial sensitivities of functions of bridge transfer coefficients and their rationalized double component forms for zero and increments of these transfer function, are introduced. Metrological properties of the commonly used bridge of similar initial arm resistances in balance and some variants of their increments are discussed. Transfer coefficient, limited error and standard uncertainty type B as functions of relative increments in broad range are shown for these variants. Generalized formulas for bridges of the linear transfer function are introduced. The relations of limited errors and uncertainties are presented in graphical form.*

Słowa kluczowe: mostek sensorowy, błąd, niepewność, metrologia.
Keywords: sensor bridge circuit, error, uncertainty, metrology.

doi:10.12915/pe.2014.12.19

Introduction

Accuracy of the current and voltage supplied strain bridges has been analyzed in [5], but only for very small sensor increments. This paper is based on earlier author proposals given in papers [2] - [4], [7]. As it has been pointed there, the generalized accuracy description of the 4R bridge of arbitrary variable arm resistances was not existing in the literature before above papers, but is urgently needed mainly for:
- initial conditioning circuits of analogue signals from broadly variable impedance sensors [1],
- identification of the changes of several internal parameters of the equivalent circuit of the object working as twoport X, when it is measured from its terminals for testing, monitoring and diagnostic purposes.

Near the bridge balance state, an application of relative errors or uncertainties is useless, as they are rising to $\pm\infty$. In [2-4], this obstacle was bypassed by relating the absolute value of any bridge accuracy measure to the initial sensitivity of the current to voltage or voltage to voltage bridge transfer function. The initial sensitivities are valuable reference parameters as they do not change within the range of the bridge imbalance. In paper [4] the new double component approach to describing the bridge accuracy is described. It has form of sum of the initial stage and of the bridge imbalance accuracy measures. Such method of describing accuracy is commonly used for the broad range instruments, e.g. digital voltmeters. A relation of each components to the accuracy measures of all variable and stationery bridge arm resistances have been developed.

In this paper the formulas of accuracy measures (the limited errors and uncertainties) of the sensor bridges, typically used for measurements of temperature, and many other variables, will be presented in the graphical form.

Bridge transfer functions

Four resistances (4R) connected in the closed loop can work as twoport type X with two pair of terminals A-B and C-D, shown on Fig. 1. If some of its internal resistances R_i are variable the output voltage U'_{DC} may change sign for some set of them. This circuit is used in measurements under the commonly known name - bridge. In order to work with sensors it is imbalanced and specially designed. The ideal supply is preferred to use: by current $I_{AB}\rightarrow J=\mathrm{const.}$, $R_G\rightarrow\infty$ or by voltage $U_{AB}=\mathrm{const.}$, $R_G=0$ and also the unloaded output, i.e.: $R_L\rightarrow\infty$, $U'_{DC}\rightarrow U_{DC}^{\infty}$. For single variable measurements it is enough to know the changes of one terminal parameter and the bridge output circuit voltage U'_{DC} is mostly used. With notations of Fig. 1 formulas (1), (3) of U_{DC}^{∞} and bridge transfer functions (2), (4) together with their rationalized two-factor product forms are given in Table 1 [4].

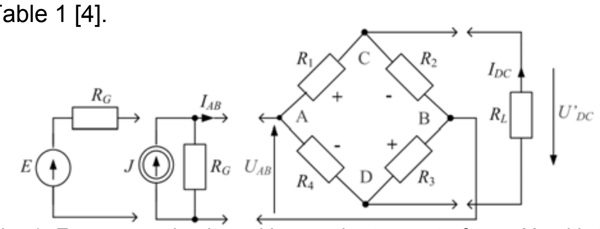


Fig. 1. Four arms circuit working as the twoport of type X with the voltage or current supply source branch

Table 1. Open circuit voltage of the resistance bridge and its transfer functions

a) current supply J	
(1)	$U'_{DC}\rightarrow U_{DC}^{\infty}=I_{AB}\,r_{21}$
(2)	$r_{21}\equiv\dfrac{U_{DC}^{\infty}}{I_{AB}}=\dfrac{R_1R_3-R_2R_4}{\sum R_i}\equiv t_0\,f(\varepsilon_i)$
where	$t_0\equiv\dfrac{m\,nR_{10}}{(1+m)(1+n)}$ $\quad f(\varepsilon_i)=\dfrac{\Delta L(\varepsilon_i)}{1+\varepsilon_{\Sigma R}}$ $\quad \varepsilon_{\Sigma R}=\dfrac{\varepsilon_1+m\varepsilon_2+n(\varepsilon_4+m\varepsilon_3)}{(1+m)(1+n)}$
b) voltage supply E	
(3)	$U'_{DC}\rightarrow U_{DC}^{\infty}=U_{AB}\,k_{21}$
(4)	$k_{21}\equiv\dfrac{U_{DC}^{\infty}}{U_{AB}}=\dfrac{R_1R_3-R_2R_4}{(R_1+R_2)(R_3+R_4)}\equiv k_0\,f_E(\varepsilon_i)$
	$k_0\equiv\dfrac{m}{(1+m)^2}$ $\quad f_E(\boldsymbol{\varepsilon}_i)\equiv\dfrac{\Delta L(\varepsilon_i)}{(1+\varepsilon_{12})(1+\varepsilon_{34})}$ $\quad \varepsilon_{12}\equiv\dfrac{\varepsilon_1+m\varepsilon_2}{1+m}$ $\quad \varepsilon_{43}\equiv\dfrac{\varepsilon_4+m\varepsilon_3}{1+m}$ *n* is arbitrary
a) and b):	$\varepsilon_i=\lfloor\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4\rfloor^{\mathrm{T}}$, $\quad \Delta L(\varepsilon_i)=\varepsilon_1-\varepsilon_2+\varepsilon_3-\varepsilon_4+\varepsilon_1\varepsilon_3-\varepsilon_2\varepsilon_4$

where:

- I_{AB}, U_{AB} -current and voltage of the supply terminals A B,
- $R_i \equiv R_{i0} + \Delta R_i \equiv R_{i0}\left(1+\varepsilon_i\right)$ - arm resistance of initial value R_{i0} and absolute ΔR_i and relative ε_i increments,
- r_{21}, k_{21} – current to voltage and voltage bridge transfer functions of the open-circuited output,
- $t_0 = \dfrac{R_{10}R_{30}}{\sum R_{i0}}$ $\quad k_0 = \dfrac{R_{10}R_{30}}{(R_{10}+R_{20})(R_{30}+R_{40})}$ - initial bridge open circuit sensitivities of r_{21} and of k_{21},
- $\sum R_i \equiv \sum_{i=1}^{4} R_i = \left(1+\varepsilon_{\Sigma R}\right)\sum R_{i0}$ - sum of arm resistances,
- $\varepsilon_{\Sigma R}(\varepsilon_i)$, ΣR_{i0} - its increment and initial value,
- $f(\varepsilon_i)$, $f_E(\varepsilon_i)$ - normalized bridge imbalance function of r_{21} and of k_{21},
- $\Delta L(\varepsilon_i), \varepsilon_{\Sigma R}(\varepsilon_i)$ - increment of the function $f(\varepsilon_i)$ numerator

If transfer function $r_{21}=0$ or $k_{21}=0$ the bridge is in balance. From (2) and (4) the balance conditions of both supply cases, and generally for any single source, are the same: $R_1R_3=R_2R_4$. The balance of the bridge can occurs for many different combinations of R_i, but the basic balance state is defined for all $\varepsilon_i=0$, i.e. when:

(5) $$R_{10}R_{30} = R_{20}R_{40}$$

Bridge transfer functions (2), (4) can be simplified to products of their initial sensitivities t_0, k_0 in the balance and normalized unbalance functions (ε_i), $f_E(\varepsilon_i)$. Their formulas can be expressed by initial values R_{i0} and increments of all resistances, i.e. $R_i=R_{i0}(1+\varepsilon_i)$ and R_{i0} referred to one of the first arm, i.e.: $R_{20}\equiv mR_{10}$, $R_{40}\equiv nR_{10}$ and from (2) $R_{30}=mnR_{10}$, as is shown in Table 1.

Relations of transfer functions r_{21}, k_{21} on ε of the simplified A-E cases of 4R bridge of $R_{i0}=R_{10}$ and relative increments $\pm\varepsilon$ of sensor resistances are given in Fig. 2a,b.

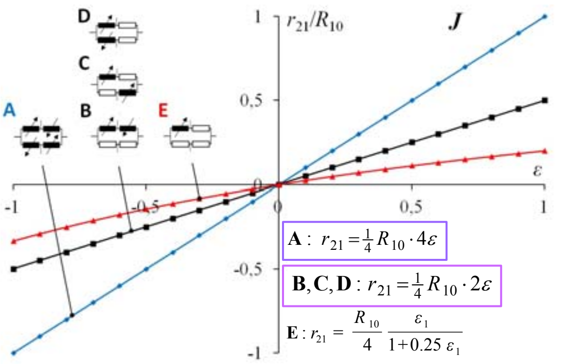


Fig. 2a. Relations of open circuit transfer functions r_{21} of A-E cases of $4R_{10}$ bridge on relative increment ε of the sensor resistances (m=1, n=1, $|\varepsilon_i|=\varepsilon$)

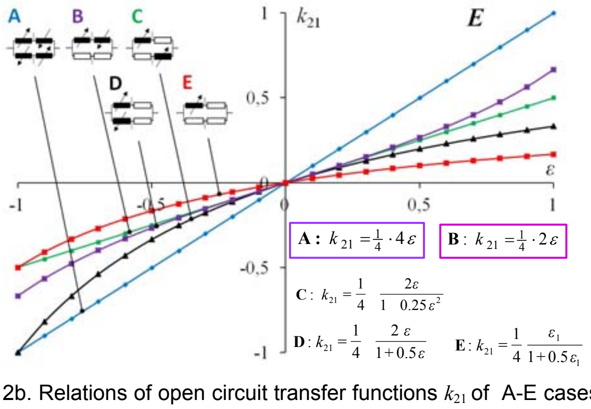


Fig. 2b. Relations of open circuit transfer functions k_{21} of A-E cases of $4R_{10}$ bridge on relative increment ε of the sensor resistances (m=1, n=1, $|\varepsilon_i|=\varepsilon$)

Description of the accuracy of bridge transfer functions

Instantaneous values of measurement errors of bridge transfer functions r_{21} and k_{21} result from the total differential of analytical equations (2) and (4) from Table 1. After ordering all components of δ_{Ri} absolute error of transfer function r_{21} is:

(6) $$\Delta_{r21} = R_1\frac{R_3 - r_{21}}{\Sigma R_i}\delta_{R1} - R_2\frac{R_4 + r_{21}}{\Sigma R_i}\delta_{R2} + R_1\frac{R_1 - r_{21}}{\Sigma R_i}\delta_{R3} - R_4\frac{R_2 + r_{21}}{\Sigma R_i}\delta_{R4} = \sum_{i=1}^{4} w_{Ri}\,\delta_{Ri}$$

where:

$w_{Ri} \equiv R_i\dfrac{(-1)^{i+1}R_j - r_{21}}{\sum R_i}$ - weight coefficients of δ_{Ri} components;

δ_{Ri} - relative error of R_i, subscript i =1,2,3,4 when j=3,4,1,2; multiplier $(-1)^{i+1}$=+1 if i is 1, 3 or –1 if i is 2, 4.

If resistances are expressed as $R_i=R_{i0}\,(1+\varepsilon_i)$, $R_j=R_{j0}\,(1+\varepsilon_j)$ formula (6) is

(6a) $$\Delta_{r21} = \frac{t_0}{1+\varepsilon_{\Sigma R}}\sum_{i,j}\left[(-1)^{i+1}\left(1+\varepsilon_j\right) - \frac{r_{12}}{R_{j0}}\right]\left(1+\varepsilon_i\right)\delta_{Ri}$$

Absolute error of transfer function k_{21} has other forms, i.e.:

(7) $$\Delta_{k21} = \frac{R_1R_2}{\left(R_1+R_2\right)^2}\left(\delta_{R1}-\delta_{R2}\right) + \frac{R_3R_4}{\left(R_3+R_4\right)^2}\left(\delta_{R3}-\delta_{R4}\right)$$

or with relative increments ε_i

(7a) $$\Delta_{k21} = k_0\left[\frac{(1+\varepsilon_1)(1+\varepsilon_2)}{\left(1+\varepsilon_{12}\right)^2}\left(\delta_{R1}-\delta_{R2}\right) + \frac{(1+\varepsilon_3)(1+\varepsilon_4)}{\left(1+\varepsilon_{34}\right)^2}\left(\delta_{R3}-\delta_{R4}\right)\right]$$

From (6) and (7) one could see that if errors δ_{Ri} of the neighboring bridge arms have the same sign they partly compensate each other.

If relative errors δ_{Ri} of resistances R_i are expressed, by their initial errors δ_{i0} and incremental errors $\delta_{\varepsilon i}$, i.e.

(8) $$\delta_{Ri} = \delta_{i0} + \frac{\Delta_{\varepsilon i}}{1+\varepsilon_i} = \delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}$$

then

(9) $$\Delta_{r21} = \sum w_{Ri}\,\delta_{Ri} = \sum w_{Ri}\left(\delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\,\delta_{\varepsilon i}\right)$$

where:

(9a) $$w_{Ri} = \frac{t_0}{1+\varepsilon_{\Sigma R}}\left[(-1)^{i+1}(1+\varepsilon_j) - \frac{R_{i0}}{R_{j0}}\Delta L\,(\varepsilon_i)\right](1+\varepsilon_i)$$

From (9a) it is seen the surprise not commonly known relation that even if arm resistance R_i is constant ($\varepsilon_i=0$, $\delta_{Ri}=\delta_{i0}$), weight coefficient w_{Ri} of its component in Δ_{r21} still depends on other arm increments ε_j.

In initial balance state, i.e. when all arm increments $\varepsilon_j=0$, the nominal transfer function $r_{21}(0)\equiv r_{210}=0$, but if resistances R_i usually have some initial errors δ_{i0} and $\Delta_{r210}\neq 0$, $\Delta_{k210}\neq 0$

(10a) $$\Delta_{r210} = t_0\,\delta_{210}$$

(10b) $$\Delta_{k210} = k_0\,\delta_{210}$$

where: $\delta_{210} = \delta_{10} - \delta_{20} + \delta_{30} - \delta_{40}$

Relative errors are preferable in measurement practice, but it is not possible to use them for transfer functions r_{21}, k_{21} near the bridge balance as the ratio of absolute error $\Delta_{r21}\to\Delta_{r210}\neq 0$ and the nominal $r_{21}\to r_{210}=0$ (or for the voltage supply ratio of $\Delta_{k21}/k_{21}\to k_{210}=0$) is rising to $\pm\infty$. Then other possibilities should be applied. There are two possible ways to describe accuracy of the transfer function r_{21} (or k_{21}) in the form of one or of two related components:

– absolute error of the bridge transfer function may be referenced to initial sensitivity factor t_0 of r_{21} (or to k_0 of k_{21}) or to the range of transmittance r_{21max} - r_{21min} (or k_{21max} - k_{21min});

– initial error Δ_{r210} have to be subtracted from Δ_{r21} and then accuracy could be described by two separate terms: for zero and for transfer function increment, as it is common for digital instrumentation (Fig. 3a,b).

Two component absolute error of transfer function r_{21} (6) after subtracting its initial value is

(11) $$\Delta_{r21} - \Delta_{r210} = \sum_{i=1}^{4}\left[w_{Ri} - (-1)^{i-1}\right]\delta_{i0} + \sum_{i=1}^{4} w_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}$$

And after referenced it to r_{21}, and substitution w_{Ri} from (8a)

(12) $$\delta_{r21r} \equiv \frac{\Delta_{r21} - \Delta_{r210}}{r_{21}} = \frac{\delta_{r21} - \delta_{210}}{f(\varepsilon_i)} = \sum_{i=1}^{4} w'_{r\,i0}\,\delta_{i0} + \sum_{i=1}^{4} w'_{r\,\varepsilon i}\,\delta_{\varepsilon i}$$

where:

(12a) $$w'_{r\,i0} = (-1)^{i-1}\frac{\varepsilon_i + \varepsilon_j + \varepsilon_i\varepsilon_j - \varepsilon_{\Sigma R}}{\Delta L(\boldsymbol{\varepsilon}_\mathbf{i})} - t_0\frac{1+\varepsilon_i}{R_{j0}(1+\varepsilon_{\Sigma R})}$$

(12b) $$w'_{r\,\varepsilon i} = \left[\frac{(-1)^{i-1}(1+\varepsilon_j)}{\Delta L(\boldsymbol{\varepsilon}_\mathbf{i})} - \frac{t_0}{R_{j0}(1+\varepsilon_{\Sigma R})}\right]\varepsilon_i$$

Weight coefficients (12a,b) are finite for any value of r_{21} including r_{21}= 0 because if all $\varepsilon_i \to 0$ also $\Delta L \to 0$.

Error δ_{r21r} is equivalent to error $\delta_{\varepsilon i}$ of the resistance R_i increment ε_i given in relative error formulas (8a,b)

From (9a) and (11) the absolute error is:

– for current to voltage transfer function r_{21}

(13a) $$\Delta_{r21} = t_0\,\delta_{210} + r_{21}\,\delta_{r21r}$$

– and similarly for the voltage transfer function k_{21}

(13a) $$\Delta_{k21} = k_0\,\delta_{210} + k_{21}\,\delta_{k21k}$$

where: $t_0\,\delta_{210} = \Delta_{r210}$, $k_0\,\delta_{210} = \Delta_{k210}$ – absolute errors of initial value r_{21} or k_{21} e.g. $r_{210} = 0$ or $k_{210} = 0$; δ_{r21r} and δ_{k21k} – related errors of increments r_{21} - r_{210} or k_{21} - k_{210} from the initial stage.

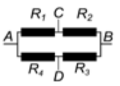
Two-component accuracy equation of k_{21} transfer function was found in the same way as for r_{21}.

Accuracy measures of the open-circuit 4R bridge in general case were placed in Table 2. Formulas for limited errors $|\delta_{r21}|$ and random measures (uncertainties) $\bar{\delta}_{r21}$ for the 3 chosen below the simpler cases of 4R bridge: "A" - jointed all arm resistances, "B" – jointed R_1 and R_2 or "D" – jointed R_1 and R_4 are given in [6, 7].

On Fig. 3a,b given are relations of limited errors on relative increment $\boldsymbol{\varepsilon}$ of case A (measurements with strain gauges).

The accuracy measures (16-19) in function of resistance increment $\pm\varepsilon$ of differential sensor for two kinds of bridge supply are presented in Fig. 4a and Fig. 4b (next page)..

Table 2. Accuracy measures of the open-circuit 4R bridge in general case [6]

Bridge type		a) Accuracy measures δ_{r21}, $\lvert\delta_{r21}\rvert$, $\bar{\delta}_{r21}$ of function r_{21}	b) Accuracy measures δ_{k21}, $\lvert\delta_{k21}\rvert$, $\bar{\delta}_{k21}$ of function k_{21}
General case 	Actual errors: absolute and relative	$\Delta_{r21} = R_1\frac{R_3 - r_{21}}{\Sigma R_i}\delta_{R1} - R_2\frac{R_4 + r_{21}}{\Sigma R_i}\delta_{R2} + R_1\frac{R_1 - r_{21}}{\Sigma R_i}\delta_{R3} - R_4\frac{R_2 + r_{21}}{\Sigma R_i}\delta_{R4}$ (6) $\boxed{\delta_{r21} \equiv \frac{\Delta_{r21}}{t_0} = \sum_{i=1}^{4} w'_{Ri}\delta_{Ri} = \sum_{i=1}^{4} w'_{Ri}\left(\delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\right)}$ (14) where: $w'_{Ri} = \frac{1+\varepsilon_i}{1+\varepsilon_{\Sigma R}}\left[(-1)^{i-1}(1+\varepsilon_j) - \frac{r_{21}}{R_{j0}}\right]$ subscript j=3, 4, 1, 2 when i =1, 2, 3, 4	$\Delta_{k21} = \frac{R_1 R_2}{(R_1+R_2)^2}(\delta_{R1} - \delta_{R2}) + \frac{R_3 R_4}{(R_3+R_4)^2}(\delta_{R3} - \delta_{R4})$ (7) $\boxed{\delta_{k21} \equiv \frac{\Delta_{k21}}{k_0} = \sum_{i=1}^{4} w'_{ki}\,\delta_{Ri} = \sum_{i=1}^{4} w'_{ki}\left(\delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\,\delta_{\varepsilon i}\right)}$ (15) where: n – arbitrary; $w'_{k1} = -w'_{k2} = \frac{(1+\varepsilon_1)(1+\varepsilon_2)}{1+\varepsilon_{12}}$; $w'_{k3} = -w'_{k4} = \frac{(1+\varepsilon_3)(1+\varepsilon_4)}{1+\varepsilon_{34}}$
$R_1 = R_{10}(1+\varepsilon_1)$ $R_2 = R_{20}\,(1+\varepsilon_2)$ $R_3 = R_{30}\,(1+\varepsilon_3)$ $R_4 = R_{40}\,(1+\varepsilon_4)$ $\varepsilon_i \geq -1$	Limited errors	$\boxed{\lvert\delta_{r21}\rvert = \sum_{i=1}^{4}\lvert w'_{Ri}\rvert\,\lvert\delta_{Ri}\rvert = \sum_{i=1}^{4}\lvert w'_{Ri}\rvert\left(\lvert\delta_{i0}\rvert + \frac{\lvert\varepsilon_i\rvert}{1+\varepsilon_i}\lvert\delta_{\varepsilon i}\rvert\right)}$ (16)	$\boxed{\lvert\delta_{k21}\rvert = \sum_{i=1}^{4}\lvert w'_{ki}\rvert\lvert\delta_{Ri}\rvert = \sum_{i=1}^{4}\lvert w'_{ki}\rvert\left(\lvert\delta_{i0}\rvert + \frac{\lvert\varepsilon_i\rvert}{1+\varepsilon_i}\lvert\delta_{\varepsilon i}\rvert\right)}$ (17)
	Standard uncertain-ties type B	$\boxed{\bar{\delta}_{r21} = \sqrt{\sum_{i=1}^{4} w'^2_{Ri}\,\bar{\delta}^2_{Ri}} = \sqrt{\sum_{i=1}^{4} w'^2_{Ri}\left(\bar{\delta}^2_{i0} + \frac{\varepsilon_i^2}{(1+\varepsilon_i)^2}\bar{\delta}^2_{\varepsilon i}\right)}}$ (18) correlation coefficients k_{ij}=0	$\boxed{\bar{\delta}_{k21} = \sqrt{\sum_{i=1}^{4} w'^2_{ki}\,\bar{\delta}^2_{Ri}} = \sqrt{\sum_{i=1}^{4} w'^2_{ki}\left(\bar{\delta}^2_{i0} + \frac{\varepsilon_i^2}{(1+\varepsilon_i)^2}\bar{\delta}^2_{\varepsilon i}\right)}}$ (19) correlation coefficients k_{ij}=0
Measures of r_{210}= 0	actual error	$\delta_{210} = \delta_{10} - \delta_{20} + \delta_{30} - \delta_{40}$ (20) | limited error $\lvert\delta_{210}\rvert_m = \sum\lvert\delta_{i0}\rvert$ (21)	Uncertainty type B, k_{ij}=0 $\bar{\delta}_{210} = \sqrt{\sum\bar{\delta}^2_{i0}}$ (22)

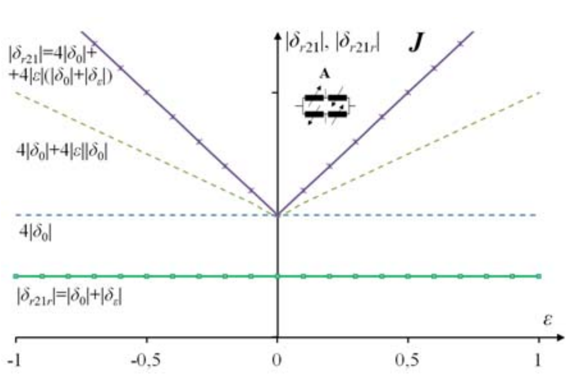


Fig. 3a. Limited errors of the current J supplied open-circuit 4R bridge of equal all arm initial resistances $4R_{10}$ and ε_1=ε_3 =-ε_2= - $\varepsilon_4 \equiv \varepsilon$ (case A) Their errors are:$\lvert\delta_{10}\rvert$=$\lvert\delta_{20}\rvert$=$\lvert\delta_{30}\rvert$=$\lvert\delta_{40}\rvert \equiv \lvert\delta_0\rvert$, $\lvert\delta_{\varepsilon i}\rvert$=$\lvert\delta_\varepsilon\rvert$, $\lvert\delta_\varepsilon\rvert$=$\lvert\delta_0\rvert$.

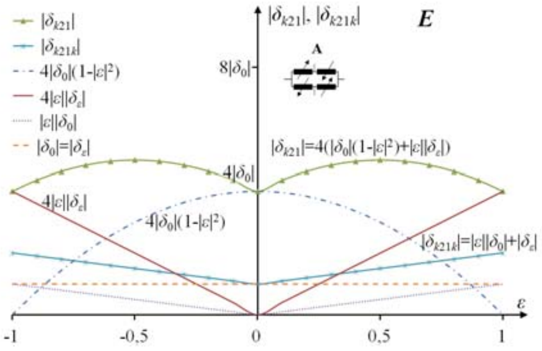


Fig. 3b. Limited errors of the voltage E open-circuit 4R bridge of equal all arm initial resistances $4R_{10}$ and increments ε_1=ε_3=-ε_2=-$\varepsilon_4 \equiv \varepsilon$ (case A).Their errors are:$\lvert\delta_{10}\rvert$=$\lvert\delta_{20}\rvert$=$\lvert\delta_{30}\rvert$=$\lvert\delta_{40}\rvert \equiv \lvert\delta_0\rvert$, $\lvert\delta_{\varepsilon i}\rvert$=$\lvert\delta_\varepsilon\rvert$, $\lvert\delta_\varepsilon\rvert$=$\lvert\delta_0\rvert$

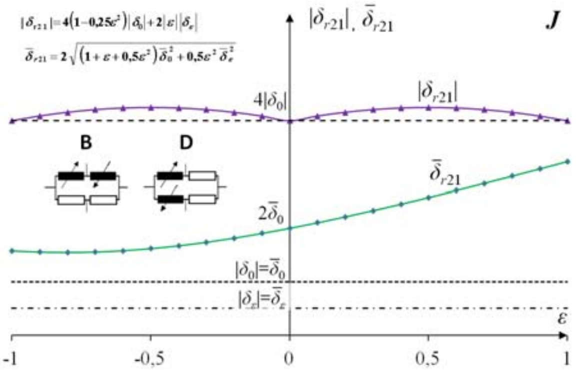


Fig. 4a. Limited error and uncertainty of the open-circuit 4R bridge of equal all arm initial resistances $4R_{10}$ ($|\delta_{10}|=|\delta_{20}|=|\delta_{30}|=|\delta_{40}|\equiv|\delta_0|$, $|\delta_{\varepsilon i}|=|\delta_\varepsilon|$, $|\delta_\varepsilon|=0.5|\delta_0|$; $\varepsilon_1=-\varepsilon_2\equiv\varepsilon$ and $\varepsilon_3=\varepsilon_4=0$ (case B); $\varepsilon_1=-\varepsilon_4\equiv\varepsilon$ and $\varepsilon_2=\varepsilon_3=0$ (case D); current supply)

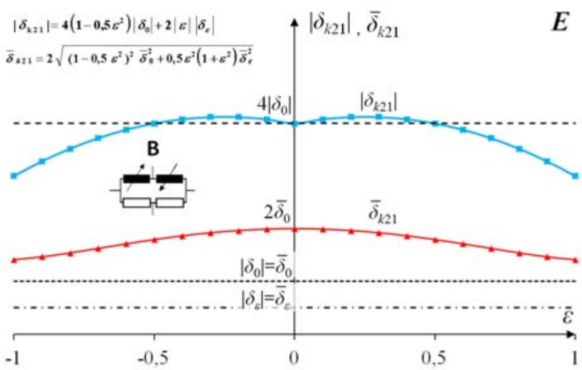


Fig. 4b. Limited error and uncertainty of the open-circuit 4R bridges of equal all arm initial resistances $4R_{10}$ ($|\delta_{10}|=|\delta_{20}|=|\delta_{30}|=|\delta_{40}|\equiv|\delta_0|$, $|\delta_{\varepsilon1}|=|\delta_{\varepsilon2}|=|\delta_\varepsilon|$, $|\delta_\varepsilon|=0.5|\delta_0|$, voltage supply)

The accuracy measures of function k_{21} are diminishing for higher values of negative and positive increments ε of sensor rresistances (Fig. 4b). In the same range of increments ε, the values of accuracy measures of r_{21} (Fig. 4a) are higher than corresponding values of accuracy measures of k_{21} (Fig. 4b).

As shown in Fig. 5a and Fig. 5b formulas and relations on ε of the limited error and their two bridge configurations (temperature measurement with the use of Pt sensors) have different values for both types (J, E) of the bridge supply.

Conclusions

Actual values of instantaneous errors of r_{21} or k_{21} could be calculated only if signs and values of errors of all resistances are known. In reality it happens very rare. More frequently are used or their limited systematic errors (of the worst case) or estimated from them statistical standard deviation measures as uncertainty type B.

Formulas of this accuracy measures of r_{21} or k_{21} could be obtained after transformation of error propagation formulas. All these accuracy measures is possible to find in one component or two component forms.

The two-component method of the bridge transfer function r_{21} accuracy representation, for its initial value (e.g. equal to zero) and for its increment is similar like commonly used formulas describing accuracy of digital instruments and of the broad range sensor transmitters.

As shown in Fig. 4a,b and Fig. 5a,b the accuracy measures (14 -19) have different values for current J or voltage type *E* of bridge supply.

The double component approach of the bridge accuracy may be used for all types of sensor conditioning circuits.

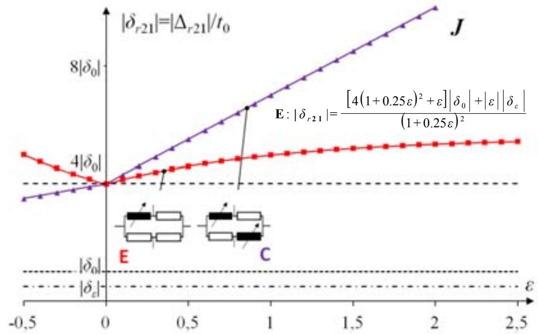


Fig. 5a. Limited error of the open-circuit 4R bridges of equal all arm initial resistances $4R_{10}$ ($|\delta_{10}|=|\delta_{20}|=|\delta_{30}|=|\delta_{40}|\equiv|\delta_0|$, $|\delta_{\varepsilon1}|=|\delta_{\varepsilon3}|=|\delta_\varepsilon|$, $|\delta_\varepsilon|=0.5|\delta_0|$; $\varepsilon_1=\varepsilon$ and $\varepsilon_2=\varepsilon_3=\varepsilon_4=0$ (case E); $\varepsilon_1=\varepsilon_3\equiv\varepsilon$ and $\varepsilon_2=\varepsilon_4=0$ (case C); current supply).

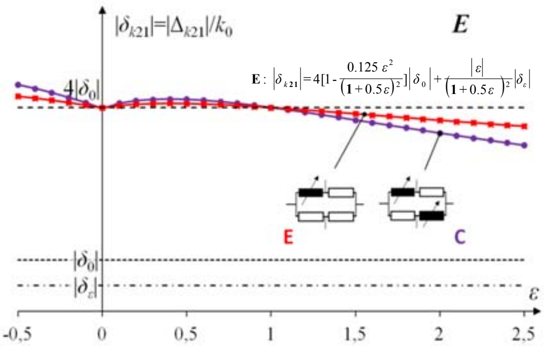


Fig. 5b. Limited error of the open-circuit 4R bridges of equal all arm initial resistances $4R_{10}$ ($|\delta_{10}|=|\delta_{20}|=|\delta_{30}|=|\delta_{40}|\equiv|\delta_0|$, $|\delta_{\varepsilon1}|=|\delta_{\varepsilon3}|=|\delta_\varepsilon|$, $|\delta_\varepsilon|=0.5|\delta_0|$; $\varepsilon_1=\varepsilon$ and $\varepsilon_2=\varepsilon_3=\varepsilon_4=0$ (case E) ; $\varepsilon_1=\varepsilon_3\equiv\varepsilon$ and $\varepsilon_2=\varepsilon_4=0$ (case C) ; voltage supply).

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