

Equivalent circuit of single-core cable line with ground return wire and variable earth resistivity along its path

Abstract. This paper presents an approach of modelling single-core cable line and a parallel laid additional conductor. Results are presented from various analyzed cases, which include situations when the additional conductor is not laid along full cable length. The proposed approach besides offering a possibility to reduce complex multiphase cable line to a single equivalent π -circuit enables decision making on the length of the additional conductor. Namely, its length could be reduced in order to reduce costs without compromising the safety levels of touch and step voltages.

Streszczenie. W artykule zaprezentowano podejście do modelowania linii kablowej jednorodzeniowej i równoległe położonego przewodu dodatkowego. Wyniki otrzymano z dla różnych przypadków analizowanych, włączających takie sytuacje kiedy dodatkowy przewód nie jest położony wzdłuż całej długości kabla. Zaproponowane podejście, oprócz dostarczenia możliwości redukcji wielofazowej linii kablowej do pojedynczego zastępczego π -obwodu, umożliwia podejmowanie decyzji co do długości dodatkowego przewodu. Mianowicie, jego długość może być zredukowana w celu redukcji kosztów bez zmniejszenia poziomów bezpieczeństwa w zakresie napięcia dotykowego i krokowego. (Obwód zastępczy jednorodzeniowej linii kablowej z przewodem ziemnopowrotnym i zmienną rezystywnością ziemi wzdłuż jego ścieżki)

Keywords: modelling, single-core cable, equivalent π -circuit, touch and step voltages.

Słowa kluczowe: modelowanie, jednorodzeniowe kable, zastępczy π -obwód, napięcie dotykowe i krokowe.

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Introduction

The main concern in the distribution networks in cases with ground faults is to obtain safe level of step and touch voltage. Because the grounding systems of these networks are so complex, interconnected substation grounding systems by overhead and cable lines, the fulfilment of these requirements it is a complex task. To obtain accurate results from the analysis, real models of the network elements must be used. In this paper we focus on the underground cable lines as components of grounding systems which are practically most common elements in the distribution networks of large cities [1], [3-4] and [7].

Any proposed grounding system as conceptual engineering solution as an important part of every power facility is subjected under severe economic evaluation. The proposed conceptual solution must provide safe levels of touch and step voltage in the power facilities in cases of ground faults at any point in the system, and that the relay protection will function properly under any conditions, before the solution is implemented in real life. For that purpose, vast number of simulations are made, using as much as it can, realistic model of all the elements that effects at the grounding system.

Specifically we consider single-core underground cables with metallic sheath insulated from ground (coated cables) which in many cities are laid out in same trench in parallel with an additional conductor. This conductor is a bare conductor directly buried into the ground and it is in a good contact with it in its full length. Sometimes, this additional conductor, from economic reasons or by other reasons, is not laid by the full length of the single-core cable but by some point of its length, and this is the case that is analysed in the paper.

Modelling of grounding system elements was the subject in many papers and methods for complex systems analysis with underground cables were proposed. Following the idea of [7] two approaches were developed in [1], where the single-core cable line can be reduced to a simple equivalent π -circuit containing an active element beside the usual passive impedances. This equivalent circuit can be further utilized to solve complex grounding systems and calculate step and touch voltages, fault current reduction factors, etc. The proposed methods are compared against previously published calculated or measured data. The comparison is made with [9] where experimental measurements are performed on a real cable line, which

radially supplies two substations connected in series. We also compare our results with the calculated values of [9].

General considerations

For the purpose of this paper, let us consider an underground cable line comprising of three single-core cables, whose metallic sheaths are isolated from the ground (coated cables). Usually, in the same trench with the single-core cables, an additional uncoated cable is laid. The case where the additional conductor is not laid by its full length of the single-core cable we will analysed as special case that derives from the main case.

Commonly metallic sheaths of the single-core cables, as well as the additional conductor, are connected to the groundings at both ends of the cable line. Fig. 1 depicts the possible scenarios of layouts of three single-core cables and additional conductor. The single-core cables are marked as 1, 2 and 3, while the additional conductor is marked as 4.

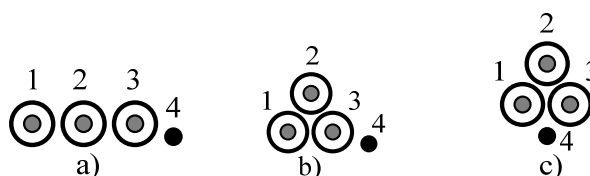


Fig. 1 Layout of the single-core cable and parallel laid additional conductor

Let us consider a case where the additional conductor is laid by its full length of the single-core cable and the conductor is in continuous contact with the ground. The four-wired configuration should be treated as a multiphase distributed parameters line whose equivalent circuit is given in Fig. 2, for a small segment with length Δx . In Fig. 2 only the metallic sheaths are presented while the phase conductors which are mutually coupled with them are modelled with appropriate induced voltages.

The analysed segment is composed of serial and two shunt elements. The serial element consists of four generators connected in series with the line impedances, which are represented with 4×4 impedance matrix $\underline{z} \cdot \Delta x$. The elements of the impedance matrix are calculated with the analytical expressions (1) and (2) as in [6], [8], which are based on Carson's theory of the ground fault current return path through earth [10]:

$$(1) \quad \underline{z}_{ij} = \omega \cdot \frac{\mu_0}{8} + j\omega \cdot \frac{\mu_0}{2\pi} \cdot \ln \frac{D_e}{d_{ij}},$$

$$(2) \quad z_{ii} = R_i' + \omega \cdot \frac{\mu_0}{8} + j\omega \cdot \frac{\mu_0}{2\pi} \cdot \ln \frac{D_e}{e^{-\mu_r \frac{a}{4}} \cdot r_i},$$

where: R_i' is a resistance per length of conductor i (in Ω/m), $\omega = 2\pi f$, $\mu_0 = 4\pi \cdot 10^{-7} H/m$, μ_r is a relative permeability of conductor material, r_i is radius of conductor i (in meters), d_{ij} is distance between conductors i and j (in meters) and D_e is the equivalent earth penetration depth (in meters). The equivalent earth penetration depth is calculated by Eq. (3):

$$(3) \quad D_e = 658 \cdot \sqrt{\frac{\rho}{f}},$$

where ρ is earth resistivity (in Ωm). If round conductors are used parameter $a = l$, and for the hollow conductors this parameter is calculated by Eq. (4):

$$(4) \quad a = \left(1 - 4k^2 + (3 - \ln k) \cdot k^4\right) \cdot (1 - k^2)^{-2},$$

where k is the ratio of the inner and the outer conductor radiuses [11]. The Eq. (1)-(3) are just an approximation and they are quite accurate at power frequencies (50/60 Hz).

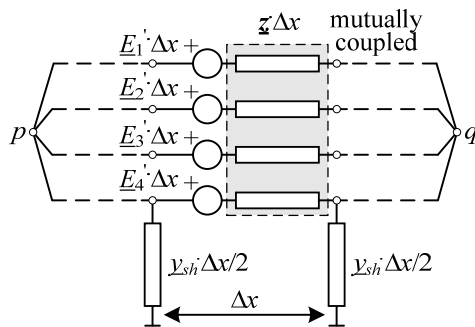


Fig. 2 A segment of the distributed parameters underground four-wire line

Besides the four conductor representation in Fig. 2, there are three more conductors, namely the phase conductors. Under normal circumstances they are not connected to the grounding systems with direct metallic connections in any way, but they are mutually coupled to the cable sheaths and the additional conductor. This coupling induces voltages in the cable sheaths and the additional conductor, which are proportional to the intensity of the phase currents. When grounding systems are analysed, as in this case, point of interest are the situations when single ground faults occur, because the severity of the grounding potentials might yield possible danger from touch and step voltages. In such cases, we may consider that there is a current in just one phase conductor and neglect the currents in the other two since they are significantly smaller than the fault current.

The shunt elements in Fig. 2 are due to the current leakage from the additional conductor into the ground which is modelled with shunt admittance $\underline{y}_{sh} \cdot \Delta x$ (per unit length) and the admittance is calculated as in [12]:

$$(5) \quad \underline{y}_{sh} = \left[\frac{\rho}{4\pi} \cdot \left(2 \ln \frac{l}{r_4} + \ln \frac{\sqrt{16h^2 + l^2} + l}{\sqrt{16h^2 + l^2} - l} \right) \right]^{-1},$$

where l is the cable length (in meters), h and r_4 (both in meters) are burial depth and radius of the additional conductor.

Proposed model

Defining the equivalent π -circuit

As we observe the Fig. 2, we can transform the four voltage generators into current generators. The currents of the current generators are calculated with Eq. (6), from where it is obvious that they do not depend on the segment length Δx .

$$(6) \quad \underline{J} = (\underline{z} \cdot \Delta x)^{-1} \cdot \underline{E}' \cdot \Delta x = \underline{z}^{-1} \cdot \underline{E}' = \underline{y} \cdot \underline{E}',$$

where with \underline{y} is cable admittance matrix, which is derived from the impedance matrix $\underline{y} = \underline{z}^{-1}$.

Furthermore, there will be such quadruplets of current generators for each segment of the cable line and their currents will be the same. The orientation of these current generators in all segments of the cable line is the same. As a consequence of that we will have a single quadruplet of current generators going from point q to point p . These four generators can be represented by a single equivalent generator defined by following expression:

$$(7) \quad \underline{J}_e = \sum_{i=1}^4 \underline{J}_i = \sum_{i=1}^4 \sum_{j=1}^4 \underline{y}_{ij}' \cdot \underline{E}_j'$$

The rest of the circuit from Fig. 2, which is comprised of passive elements only, can be represented by an equivalent π -circuit so that the equivalent circuit for the four-wired cable line will get the form given in Fig. 3, which is also used as a modelling basis of the EMTP [12-13].

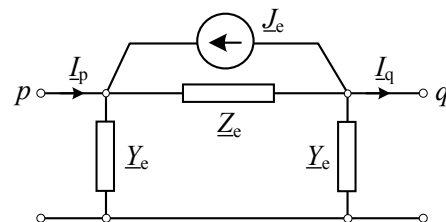


Fig. 3 Equivalent π -circuit for the proposed model

Circuit theory approach for calculating the elements of the proposed equivalent π -circuit when the additional conductor is laid in full cable's length

Commonly used procedure for calculation of the elements of equivalent π -circuit \underline{Z}_e and \underline{Y}_e is the wave propagation theory. That approach requires usage of complex mathematical apparatus, including hyperbolic functions, to obtain \underline{Z}_e and \underline{Y}_e . In this paper we would like to present another approach in which we shall use the circuit theory approach for obtaining \underline{Z}_e and \underline{Y}_e .

The passive part of the circuit given in Fig. 2 comprises of large number of segments with small lengths which in the limiting case leads to an infinite number of infinitesimally short segments. However, for practical approach even with a modest number of segments we will get quite accurate results. For now, let us assume that the number of segments is known and equal to n_s . For the sake of simplicity let us split the full cable length in $n_s=3$ segments, shown in Fig. 4.

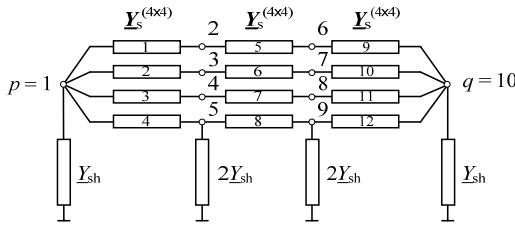


Fig. 4 Passive part of the cable line divided in finite number of segments

The number of nodes n and the number of branches m in the circuit is given by $n = 4 \cdot (n_s - 1) + 2$ and $m = 4 \cdot n_s$. The admittance matrix for the serial parts of the segments is:

$$(8) \quad \underline{Y}_s = (\underline{z} \cdot l_s)^{-1} = \underline{y} / l_s,$$

while the shunt admittance of the segments are calculated as:

$$(9) \quad \underline{Y}_{sh} = \underline{y}_{sh} \cdot l_s,$$

where $l_s = l / n_s$ is the segment length. For now, let us assume that the earth resistivity is equal along the line. In real life this is not a case, especially for a cable length up to several kilometres. In these cases we should split the cable in sections where the earth resistivity is constant and apply the same approach on a section base.

For the circuit of Fig. 4 we construct the bus admittance matrix in two steps. In the first step we consider the serial branches only while the shunt branches which are not mutually coupled will be added in the second step. Throughout this section for all matrices we use sparse storage and operations which are readily available in MATLAB™ or GNU Octave.

The branch admittance matrix is with dimension $m \times m$ and it is block diagonal with number of blocks equal to the number of segments. In this case with n it has the following structure:

$$(10) \quad \underline{Y}_{branch} = \begin{bmatrix} \underline{Y}_s & 0 & 0 \\ 0 & \underline{Y}_s & 0 \\ 0 & 0 & \underline{Y}_s \end{bmatrix}_{m \times m}.$$

The bus admittance matrix for the serial branches is obtained as:

$$(11) \quad \underline{Y} = \mathbf{D}^T \cdot \underline{Y}_{branch} \cdot \mathbf{D},$$

where \mathbf{D} is branch-bus incidence matrix with size $n \times m$.

In the second step, we add the shunt branches to the bus admittance matrix as follows:

$$(12) \quad \underline{Y}_{ii(new)} = \underline{Y}_{ii(old)} + 2 \cdot \underline{Y}_{sh}, \quad i = 5, 9, \dots, n-1,$$

$$(13) \quad \underline{Y}_{ii(new)} = \underline{Y}_{ii(old)} + \underline{Y}_{sh}, \quad i = 1, n,$$

where the subscript "old" denotes the bus admittance matrix obtained in the first step and the subscript "new" denotes the same matrix at the end of step two.

In order to calculate the elements values of the equivalent π -circuit of Fig.4, we need to calculate the input and mutual impedances for buses 1 and n , i.e. $\underline{Z}_{11}, \underline{Z}_{1n}, \underline{Z}_{n1}$ and \underline{Z}_{nn} . The most obvious way to calculate them is to perform matrix inverse on the bus admittance matrix, which is not efficient since it is both memory and time consuming. Here, we introduce different approach by solving the following system of equations:

$$(14) \quad \underline{Y} \cdot \underline{U} = \underline{I}.$$

We apply Eq. (14) twice with two different vectors on the right side of the expression. In the first case we set all elements of \underline{I} to zero, except for $I_1 = 1$ and in the second case, we set all elements of \underline{I} to zero, except for $I_n = 1$. In such a way we get the first and the last column of the inverse of the bus admittance matrix. Having columns 1 and n of \underline{Z} we can calculate the reduced bus admittance matrix:

$$(15) \quad \underline{Y}' = \begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{1n} \\ \underline{Z}_{n1} & \underline{Z}_{nn} \end{bmatrix}^{-1}.$$

This reduced admittance matrix is of size 2×2 since it is related only to buses 1 and n , meaning that all other buses are eliminated which was our goal since we are aiming for circuit as in Fig. 4. On other hand, the reduced admittance matrix can be easily constructed from the Fig. 4 as:

$$(16) \quad \underline{Y}' = \begin{bmatrix} \underline{Y}_e + \underline{Z}_e^{-1} & -\underline{Z}_e^{-1} \\ -\underline{Z}_e^{-1} & \underline{Y}_e + \underline{Z}_e^{-1} \end{bmatrix}.$$

Finally, we can calculate the values of the elements from the π -equivalent circuit (Fig. 4), by using Eq. (16):

$$(17) \quad \underline{Z}_e = -(\underline{Y}'_{12})^{-1},$$

$$(18) \quad \underline{Y}_e = \underline{Y}'_{11} - \underline{Z}_e^{-1} = \underline{Y}'_{11} + \underline{Y}'_{12}.$$

Besides the parameters of the equivalent π -circuit it is useful to calculate the distribution of voltages and currents across the cable line. Assume that the voltages \underline{U}_p and \underline{U}_q , with reference to remote earth, which depend on the situation outside the cable line such as the location and the intensity of the fault current are known. Then the voltages of all inner nodes in Fig. 4 can be calculated from the bus admittance matrix which is known through Eq. (11) – (13).

We start with Eq. (14) by omitting the first and the last row in matrix equation since the voltages for the first and the last node are known. Then in the remaining rows we transfer the terms from the first and the last column on the right hand side of the expression so that we get the following system of linear equations:

$$(19) \quad \underline{Y}_{inner} \cdot \underline{U}_{inner} = -(\underline{Y}_1 \cdot \underline{U}_p + \underline{Y}_n \cdot \underline{U}_q),$$

where \underline{Y}_{inner} is a sub-matrix of \underline{Y} comprised of rows and columns from 2 to $n-1$, \underline{U}_{inner} is a vector of voltages for the inner nodes, \underline{Y}_1 and \underline{Y}_n are the first and the last column of \underline{Y} where the row number also goes from 2 to $n-1$. Vector of all voltages \underline{U} is defined as:

$$(20) \quad \underline{U} = \begin{bmatrix} \underline{U}_p & \underline{U}_{inner} & \underline{U}_q \end{bmatrix}^T.$$

Since we know the voltage vector Eq. (20), the branch currents are obtained by the following expression:

$$(21) \quad \underline{I}_{branch} = \underline{Y}_{branch} \cdot \mathbf{D}^T \cdot \underline{U} - \mathfrak{R}(\underline{J}),$$

where \mathfrak{R} is an operator that creates a vector consisting of an $n_s \times 1$ tiling copies of \underline{J} . The currents leaking from the additional conductor to the ground are:

$$(22) \quad I_{ground} = 2 \cdot Y_{sh} \cdot U_4,$$

where U_4 is a voltage vector for the additional conductor comprised of every fourth element of U .

Circuit theory approach for calculating the elements of the proposed equivalent π -circuit when the additional conductor is not laid in full cable's length

In practice, there are cases when the additional conductor is not laid in full cable's length. This case looks similar to the previous one, and we shall use the same approach here as well. There are certain differences which are explained as follows.

Let us assume that we have the same cable layout as in the previous case with only difference that the additional conductor is not laid in full cable's length. To derive the model for this case, we divide the full cable's path, into two major sections. The first section includes the part of cable's length where the additional conductor is laid beside the cable, and the second section the rest of the cable's length. When we apply the circuit theory approach separately to each of the segments, we obtain equivalent model comprised of two equivalent π -circuits connected serially. The first equivalent π -circuit represents the first section, and as the other one represents the second section of the cable's length. Fig. 5 depicts the sections equivalent π -circuit.

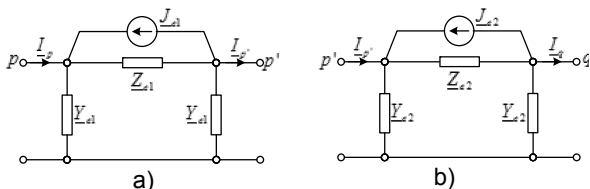


Fig. 5 Equivalent π -circuits for the first and the second section, respectively

The equivalent circuit for the first part of the cable is obtained with the same approach as before. The second equivalent π -circuit is of same form as the first equivalent π -circuit with different parameters. If we use Fig. 4 for depicting the second section, we see that elements 4, 8, 12, ..., 4n ($n = 1, 2, 3, \dots$), as well as all parallel branches Y_{sh} are not present, because there is no additional conductor next to the single core cables. The absence of the additional conductor can be modelled by putting zeros in the last column and last row in impedance matrix Z , while element Z_{44} is represented by a very big serial impedance (e.g. 10^6 Ohms/m). To determining J_{e2} in the second equivalent π -circuit, we use the Eq. (7) with minor modification:

$$(23) \quad J_e = \sum_{i=1}^3 J_i = \sum_{i=1}^3 \sum_{j=1}^3 y_{ij} \cdot E'_j.$$

By using the circuit theory approach we are able to obtain the parameters of both equivalent π -circuits. Furthermore by combining both equivalent circuits we are able to define a single model for both sections. The equivalent circuit parameters Y_{e1} and Y_{e2} of the composite equivalent π -circuit are not equal. The same is the case for the two current generators which are with different currents enabling simulation of different working conditions on both ends of the power cable.

Similar to this we can analyse cases when the earth resistivity is not constant along the cable's path. In that case we have to divide the cable's path in as many segments as

there are earth's resistivity changes and by applying the same circuit theory approach to each segment we would get the resultant model for the analysed case.

Results

The proposed method in [1] was tested for accuracy. For that purpose, various case scenarios were analysed where the additional conductor was laid to the full length of the cable line. Satisfactory results were obtained, with less than 1% error for Z_e and Y_e of the equivalent π -circuit compared to the results obtained using the wave propagation theory. In this paper, our goal was to apply the proposed method for case scenario where the additional conductor is not laid in the full length of the cable line.

Case 1: We first present results for a base case. In this case the additional conductor is laid in the full length of the cable line. We have used 110 kV single-core copper cable with XHPL insulation. The additional conductor is made of copper with cross-section of 50 mm^2 buried at a depth of 0.8 m. The cable's copper sheath is with a cross-section of 95 mm^2 with inner and outer cable's sheath radiuses of 45.166 mm and 45.5 mm respectively. The radius of the additional conductor is 4 mm. The power frequency is 50 Hz. The configuration of the cable's layout and to the additional conductor is as in Fig.1a) with equal distance of 95 mm between the conductors. This power cable connects high voltage supply substations and medium voltage distribution substation at a distance of 2,5 km. The grounding resistance of the supply substation is $R_1 = 0,05 \Omega$, while $R_2 = 0,5 \Omega$ for the distribution substation. The earth resistance is constant through the full cable's length, $\rho = 100 \Omega\text{m}$. A single line to ground fault occurs on phase 1 in the distribution substation with fault current $I_f = 10 \text{ kA}$. Fig. 6 depicts the connection between the substations.

The equivalent circuit for the supply system is given in Fig. 7. The elements of the equivalent π -circuit for the cable line are calculated with the proposed method ($n_s = 1000$) and given in Fig. 7.

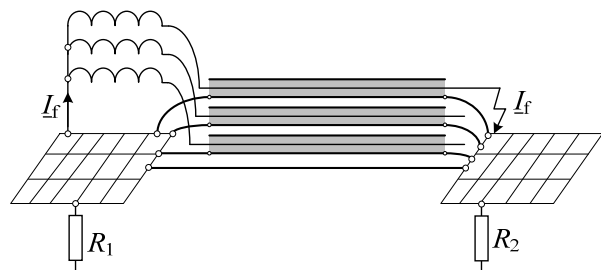


Fig. 6 Cable line connecting two substations

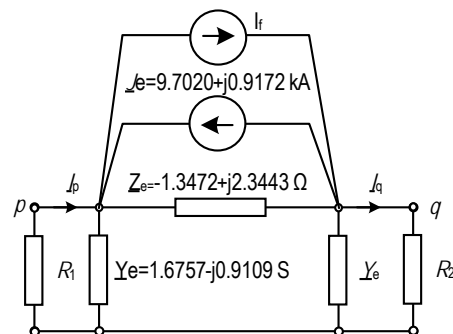


Fig. 7 Equivalent circuit for the system of Fig. 6

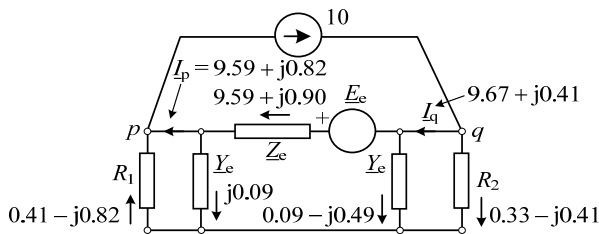


Fig. 8 Currents in the grounding system of the cable line

If we apply Eq. (20) for calculating the voltages on both ends of the cable lines, the result is $\underline{U}_p = 45.8 \cdot e^{j116.9 \text{ deg}} \text{ V}$ and $\underline{U}_q = 262.2 \cdot e^{-j51.6 \text{ deg}} \text{ V}$. The calculated currents are presented in kA in Fig. 8, where the current generator \underline{J}_e is converted into voltage generator $\underline{E}_e = (-15.221 + j21.508) \text{ kV}$.

Case 2: In this case we present the results, for the case when the additional conductor is not laid in the full cable's length. We use the same data as in case 1, with different arrangement of the additional conductor. Several sub-cases were analyzed with different conductor's length and mode of connection (connected only at one of the substations). The results which are presented are from the sub-case when the additional conductor length is 750 m and it is laid from the distribution substation towards the high voltage supply substation. By applying the proposed method we were able to calculate the elements from the equivalent π -circuits: $Z_c = (-0.159 + j1.669) \Omega$, $Y_{e1} = (0.220 - j0.058) \text{ S}$, $Y_{e2} = (1.467 - j0.415) \text{ S}$, $J_{e1} = (9.757 + j1.041) \text{ kA}$ and $J_{e2} = (-10.024 - j0.854) \text{ kA}$. The calculated voltages on both ends of the cable lines are $\underline{U}_p = 53.65 \cdot e^{-j67.52 \text{ deg}} \text{ V}$ and $\underline{U}_q = 237.47 \cdot e^{-j73.12 \text{ deg}} \text{ V}$ and they are still acceptable even though the additional conductor is almost three times shorter compared to case 1 (which is very cost effective).

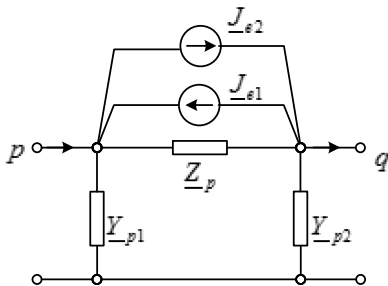


Fig. 9 Equivalent circuit for the system of case2

Conclusion

In this paper we present another approach, where by applying the well-known circuit theory we could obtain the elements of the equivalent π -circuit for complex grounding system. This approach uses a simple mathematical apparatus with acceptable level of error. By applying this approach to the analysed cases we were able to conclude that in such complicated cases, when the additional conductor is not in full cable's length or with no constant earth resistivity, we are still able to obtain the elements from the equivalent π -circuit and even to show that not always we need to put additional conductor in full cable's length, from economic perspective. Even though we shorten the additional conductor we are able to determine its length in

such a way that safety levels for touch and step voltages are met.

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