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Modeling of four electrode measuring system for determination the resistivity of thin antistatic coatings

***Abstract**. In the paper, the mathematic model for determine the resistivity of thin antistatic coating was presented. The model based on four-points method and method of images. In the model, the parameters of measuring system and coating was described. It depend on thickness of the coating and the relative permeability of the bulk.*

Streszczenie.** W artykule opisano model matematyczny do wyznaczania rezystancji cienkich warstw antyelektrostatycznych. Model opiera się na metodach czteropunktowej oraz odbić zwierciadlanych. Wyznaczony limit rozmiaru układu - odstępu pomiędzy elektrodami pomiarowymi - uzależniony jest od grubości warstwy i stosunku rezystywności warstwy/podłoże. (**Modelowanie czteroelektrodowego systemu pomiarowego do wyznaczania rezystancji cienkich powłok antyelektrostatycznych.)

Słowa kluczowe: antyelektrostatyczność, modelowanie, rezystancja elektryczna, powłoki antyelektrostatyczne
Keywords: anti-electrostatic, modeling, electrical resistance, anti-electrostatic coatings

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Introduction

Risks caused by electrostatic discharge make it necessary to protect products, facilities and equipment from excess electric charge. One of the techniques for this purpose in common use is the application of conductive antistatic coatings. In order to make the protection reliable, it is necessary to verify the quality of electric properties of these coatings [1, 2]. However, the measurements carried out in accordance with the IEC 61340-2-3:2000 standard feature low repetitiveness and have to be made at a specially equipped lab stand [3].

The proposed description of the four-electrode system is a modified four-point method supplemented with the method of images [4, 5]. Theoretical calculations for determination of the parameters of the measurement system and the examined coating were used for design of the control-measurement system. The designed system, based on the presented equations, makes it possible to carry out *in-situ* measurements of anti-electrostatic coatings. The system also increases to a large extent the repetitiveness of the obtained results [6]. The model considered in the paper describes one of the chosen issues concerning resistivity measurements of anti-electrostatic coatings. The problem under study in the present paper is focused at correct determination of the parameters of the measurement system.

Model of the multi-electrode system with point electrodes on a coating expanding to infinity

The presented theoretical considerations refer to a simplified four-electrode model. The current distribution in the coating with resistivity ρ_1 placed on the bulk layer with resistivity ρ_2, cf. Fig. 1.

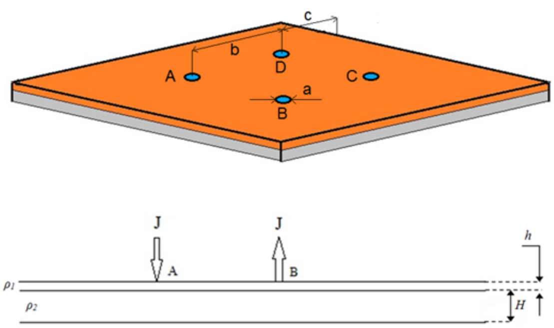

Fig. 1. Setup: coating-bulk layer with marked locations of current (A, B) and voltage (C, D) electrodes.

It is assumed that the coating layer is constant, equal to *h*. Under certain assumptions the model may be useful for determination of volume and surface resistivities. It should be noticed, though, that the surface resistivity determined with this method is an equivalent quantity, which describes the whole system: anti-electrostatic layer + bulk volume [7, 8].

In the analysis the following simplifying assumptions are made:

1. The coating has the thickness *h* and it covers an extensive planar bulk layer with thickness *H*. This assumption makes it possible to treat the considered system in the first approximation as expanding to infinity
2. The current electrodes *A* and *B* are point-like and the distance between them is *b*, the current *I* flows into point *A* and the sink point is *B*
3. The voltage electrodes *C* and *D* are point-like and together with the current electrodes make up a square with dimension *b*. These electrodes do not affect the potential distribution.
4. The upper surface of the coating *(z=0)* and the lower layer of the bulk volume *(z = -h - H)* are in the contact with the non-conductive region (usually this is the air), whereas the surface of the contact coating-bulk volume is placed at depth *z = -h*. Accordingly, on the surfaces *z =0* and *z= -h - H* the normal component of the current density vector is equal to zero (except from the points of contact with current electrodes).

The conditions for *z = -h* depend on the resistivity of the bulk volume. Generally, these take the form:

$$(1)\quad V_1\big|_{z=-h} = V_2\big|_{z=-h}, \qquad \frac{1}{\rho_1}\frac{\partial V_1}{\partial z}\bigg|_{z=-h} = \frac{1}{\rho_2}\frac{\partial V_2}{\partial z}\bigg|_{z=-h}$$

The afore-given conditions are significantly simplified for two cases:

I. the bulk volume is significantly less conductive than the coating ($\rho_1 << \rho_2$),
II. the bulk volume is significantly more conductive than the coating ($\rho_1 >> \rho_2$).

These conditions are considered first, whereas the intermediate, general case (III) is considered next.

The bulk less conductive then the coating (ρ1/ρ2 → 0)

In the present case it can be roughly assumed that the normal component of the current density at the surface of the contact coating-bulk volume is zero. After taking $\rho_1 << \rho_2$ into account the formula (1) is obtained:

(2) $$\left.\frac{\partial V_1}{\partial z}\right|_{z=-h} = \underbrace{\frac{\rho_1}{\rho_2}}_{\to 0}\left.\frac{\partial V_2}{\partial z}\right|_{z=-h} \approx 0$$

Such an effect can be obtained by considering a two-dimensional model, which ignores the dependence of the z-coordinate. The current electrodes are then considered as fibers through the entire thickness of the coating h. This - of course - does not reflect the reality, but it makes it possible to derive the aforementioned approximate formula (3).

(3) $$R_{AB,CD}^{2D,0} = \frac{U_{AB,CD}^{2D,0}}{I} = \frac{\rho_1}{2\pi h}\ln 2$$

A more realistic model is obtained if instead of current electrodes in the form of fibers. The current electrodes are considered as touching the surface of the coating pointwise.

Taking into account the finite thickness of the coating h, requires the use of the fictitious current sources, which perform condition of zeroing for the normal component of current density on the surface $z = -h$ (method of images). When you take into account the single point source of current at the point P, is the realization of that condition requires making an infinite number of fictitious sources of performance $2I$ in points (x_P, y_P, $2ih$), where $i = \pm1, \pm2, \pm3, \ldots$ (fig.2).

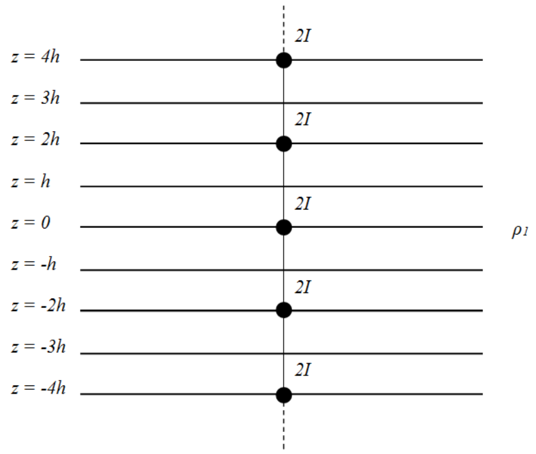

Fig. 2. Example section of the fictitious scheme of current sources in the method of mirror.

In this 3D case, expression for the resistance R is obtained

(4) $$R_{AB,CD}^{3D,0} = \frac{U_{AB,CD}^{3D,\infty}}{I} = \frac{\rho_1}{2\pi h} f_1\left(\frac{h}{b}\right)$$

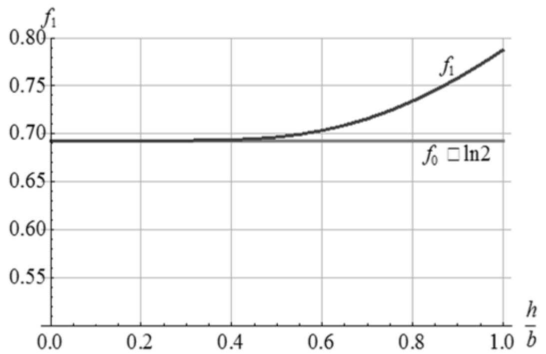

Fig. 3. Graphs of functions *$f_1(h/b)$*, and errors in the relative ratio of *h/b.*

Compared to equation (3) from the 2D model, the value ln2 was affected by the function $f_1(h/b)$, the graph shown in Figure 3. It can be concluded that for $h/b < 0{,}4$ the value of $f_1(h/b)$ is practically constant. It can be shown that $f_1(0)$ is ln2 and thus $f_1(h/b) \approx \ln 2$ for $h/b < 0{,}4$. In this case, the 3D model gives the result as a 2D model. This indicates that the formulas received from 2D analysis are sufficiently accurate when $b > 0{,}4h$, which is usually fulfilled in a wide range, as in practice $b \gg h$.

The bulk much better conductive than the coating ($\rho 1/\rho 2 \to \infty$)

In the present case it can be assumed approximately that the base is equipotential, i.e. that surface of contact of the coating-bulk *($z = -h$)* is equipotential, that is the first condition (1) takes the form $V_1(z = -h) = \text{const}$. The condition on the surface $z = 0$ is as previously. The fulfillment of the above conditions can be achieved by banded placement of fictitious point sources and current $2I$ and $-2I$ points on the coordinate $z = 2ih$ (Fig. 4).

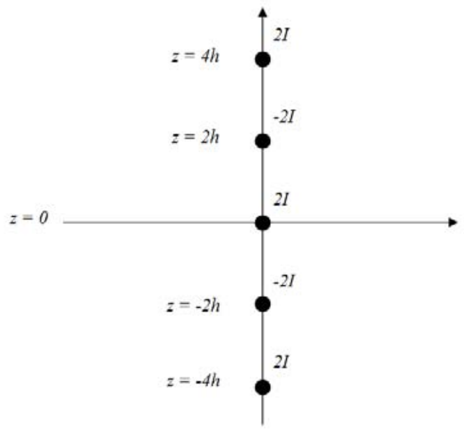

Figure 4. The system shows the distribution of current sources.

In this 3D case, expression for the resistance R is obtained

(5) $$R_{AB,CD}^{3D,\infty} = \frac{U_{AB,CD}^{3D,\infty}}{I} = \frac{\rho_1}{2\pi h} f_2\left(\frac{h}{b}\right)$$

Graph of the function $f_2(h/b)$ is shown in Figure 5

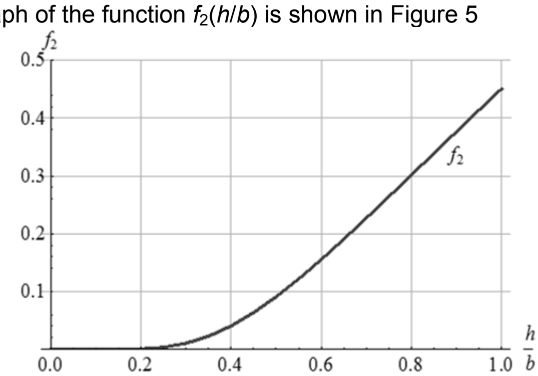

Fig. 5. Graph $f_2(h/b)$ for $h/b \to 0$, it tends to zero, b - it is the distance between the electrodes, h - thickness of the coating.

The General case

The third considered case is the most complicated. The resulting expression for the resistance R depends on the coating thickness h, bulk H and the distance between the electrodes b and ratio (ρ_1/ρ_2).

Let us first consider a single electrode giving a current at the point (0, 0, 0). In this case, the potential of the coating fulfills the Poisson equation

(6) $$\nabla^2 V_1(X) = -\frac{2I\rho_1}{4\pi}\delta(X), \qquad 0 \geq z \geq -h$$

where $X = (x, y, z)$, and $\delta(X)$ is a three-dimensional Dirac delta. The potential in the area of the bulk while fulfills Laplace's equation, because there are no field sources

(7) $$\nabla^2 V_2(X) = 0, \qquad -h \ge z \ge -(h+H)$$

On the surfaces $z = 0$ and $z = -(h + H)$ Neumann's zero conditions are fulfilled. They result zeroing of normal component of the current density vector:

(8) $$\left.\frac{\partial V_1}{\partial z}\right|_{z=0} = 0, \qquad \left.\frac{\partial V_2}{\partial z}\right|_{z=-(h+H)} = 0$$

At surface boundary z =-h continuity conditions fields are fulfilled (1).

The solution of the above problem can be obtained by integral transformations [4, 9].

The potential is of the form:

(9)
$$V_{AB}^{3D}(x,y,z) = V_1\left(\sqrt{(x-\tfrac{b}{2})^2 + y^2}, z\right) - V_1\left(\sqrt{(x+\tfrac{b}{2})^2 + y^2}, z\right)$$

An image of a field for the exemplary set of parameters is shown(Fig. 6).

a)

b)

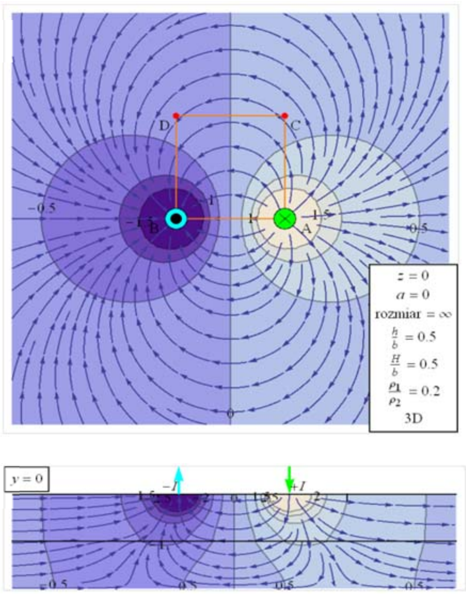

Fig. 6. The image of field by the formula 19 a) view from the "top", b) cross-sectional view. Where: *a* - the radius of the electrode, *b* - distance between the electrodes, *h* - thickness of the coating, *H* - base material thickness.

The voltage between the electrodes C and D is:

(10)
$$U_{AB,CD}^{3D} = V_{AB}^{3D}(\tfrac{b}{2}, b, 0) - V_{AB}^{3D}(-\tfrac{b}{2}, b, 0) = 2V_{AB}^{3D}(\tfrac{b}{2}, b, 0) = 2V_1(b,0) - 2V_1$$

thus

(11) $$R_{AB,CD}^{3D} = \frac{U_{AB,CD}^{3D}}{I} = \frac{\rho_1}{2\pi h} f_3\left(\frac{h}{b}, \frac{H}{b}, \frac{\rho_1}{\rho_2}\right)$$

where:

(12)
$$f_3 = h\cdot\frac{2-\sqrt{2}}{b} + 2\frac{h}{b}\int_0^\infty \frac{1-\frac{\rho_1}{\rho_2}\tanh(\frac{H}{b}\tau)}{\tanh(\frac{h}{b}\tau) + \frac{\rho_1}{\rho_2}\tanh(\frac{H}{b}\tau)} \frac{e^{-\frac{h}{b}\tau}}{\cosh(\frac{h}{b}\tau)} \left[J_0(\tau) - J_0(\sqrt{2}\tau)\right] d\tau$$

For $\rho_1/\rho_2 \to 0$, this function tends to $f_1(h/b)$, while for $\rho_1/\rho_2 \to \infty$ tends to $f_2(h/b)$ In Figure 7, 8, 9 shows graphs *f* depending on its arguments.

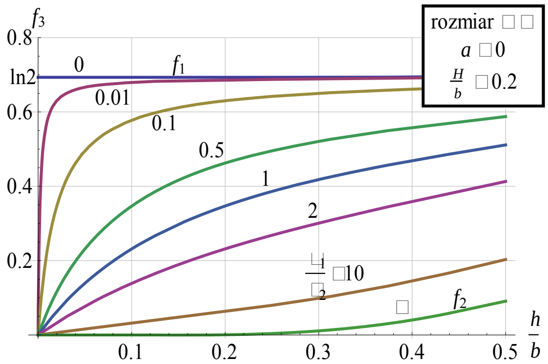

Fig. 7. The function values of f_3 in dependence on the used arguments, denotes: *a* - the radius of the electrode, *b* - distance between the electrodes, *H* - base material thickness.

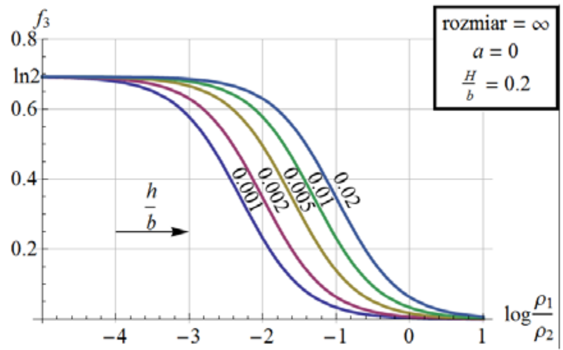

Fig. 8. Dependence of the function f_3 on the ratio of bulk/coating permittivity for different values of the ratio *h/b,* denotes: *a* - the radius of the electrode, *b* - distance between the electrodes, *h* - thickness of the coating, *H* - base material thickness.

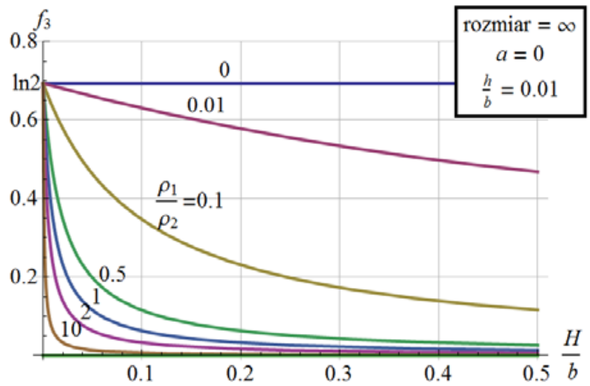

Fig. 9. Dependence of the f_3 function on the ratio of thickness *H* / size of the electrodes *b* for different ratio ρ_1/ρ_2, denotes: *a* - the radius of the electrode, *b* - distance between the electrodes, *h* - thickness of the coating, *H* - base material thickness.

Verification of the model

The considered problem relies in the size minimization for the set of measurement electrodes taking into account the varying ratio *h/b*. The analysis has been carried out at constant of values *a* and *c*

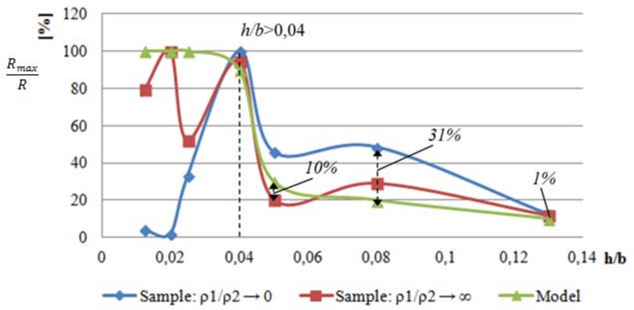

Fig. 10. The dependence of the relative resistance values and actual measurements of the model, for different ratios of *h/b*

The values shown in Figure 10 coincide to a large extent with the intermediate model. The discrepancies between real values and the model for a range of values from 0 to 0,04 *h/b* are due to the size of the electrodes used. As the result, the upper limit of the size of the electrodes is obtained. The results are not consistent with the values of the model. In other cases, the difference between the measured values and the model does not exceed 31% [5].

Summation

The presented models refer to the use of the system for measuring the resistance four-electrodes anti-static coatings on various types of bulk ($\rho_1/\rho_2 \to \infty$ and $\rho_1/\rho_2 \to 0$). A description of the phenomenon was obtained using 2D and 3D models taking into account the effect of the thickness of the test coating.

The presented model calculations allow one to determine the distance *b* between the point-measuring electrodes. The thickness *h* of antistatic coating has a substantial impact on the values taken by parameter *b*. Knowing the function of the ratio *b/h* it takes the possible to select the optimal values of the parameters, so that the resulting value of resistance $R_{AB,CD}$ with lowest possible error

Moreover, in cases where $b > 0{,}4h$ the simplified model may be used (eq.6). The next stages of modeling is to minimize the area of measuring *c* and maximizing the diameter of the measuring electrode *a*.

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