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Methodological analysis of PEEC and MOM techniques for determination of current along a conductor placed in a conductive medium

***Abstract**. This paper presents a methodological analysis of differences between the classical method of moments (MoM) approach and the partial element equivalent circuit (PEEC) approach to determination of current along a conductor. A step by step investigation is performed of the two approaches while they are applied on a perfect conductor placed in a conductive environment, excited by a current source at one end. The fundamental discrepancy points of the two methods are marked and theoretical explanations of those discrepancies are offered during the analysis. Finally, the numerical results for the current along the conductor obtained by the two methods are compared.*

***Streszczenie.** W artykule zaprezentowano analizę metodologiczną różnicy między klasyczną metodą momentów MoM a metodą elementu zastępczego PEEC przy wyznaczaniu prądu w przewodniku. Analizowano dobrze przewodzący przewodnik umieszczony w przewodzącym środowisku i zasilany na jednym końcu. (**Analiza metodologiczna metody momentów i metody obwodu zastępczego w określaniu prądu w przewodniku położonym w środowisku przewodzącym**).*

Keywords: methodological analysis, PEEC, MoM, current distribution.
Słowa kluczowe: metoda momentów MoM, metoda element zastępczego PEEc, przewodzący element.

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The method of moments basics

The mathematical concept of matrix methods is to reduce a functional equation to a matrix equation. The method of moments (MoM) is a procedure for obtaining the mentioned matrix equations. This is presented in the classic book by Harrington [1]. The creator of the MoM, Harrington explains: "It is more a concept than a method. Almost any approximate solution can be interpreted according to the method of moments." That is why, in order to compare MoM with another numerical approach, it has to be applied on a certain example.

Let the deterministic equation to be solved be

$$L(f)=g \tag{1}$$

where: L is a linear operator, g is the excitation function, and f is the response (unknown to be determined). Eq.(1) is transformed into a matrix equation by the following procedure: f is represented by a set of functions:

$$f=\sum_n \alpha_n f_n \tag{2}$$

where: α_n are scalars to be determined and f_n are called basis functions. A set of linear equations is obtained through the inner product functions:

$$\sum \alpha_n \langle w_m, Lf_n \rangle = \langle w_m, g \rangle \tag{3}$$

where: w_n are the test functions chosen appropriately for each case. The matrix form of these equations is:

$$[l_{mn}][\alpha_n]=[g_m] \tag{4}$$

where: l_{mn} are the members of the obtained matrix. If that matrix is not singular, the unknown coefficients α_n are obtained straightforwardly as

$$[\alpha_n]=[l_{mn}]^{-1}[g_m] \tag{5}$$

Partial element equivalent circuit method basics

The partial element equivalent circuit (PEEC) method [2] was introduced by Ruehli in 1974. A major advantage of PEEC is the fact that it provides a way to transform electromagnetic problems into electric circuit theory problems. This transformation is performed by creating a heterogeneous mixed circuit from a full-wave solution of the Maxwell's equations for an electromagnetic problem. A typical example of the equivalent circuit created by PEEC would contain a series of self-partial and mutual-partial inductances together with shunt capacitances and current controlled current sources attached to each node, as it may be observed in Fig.1.

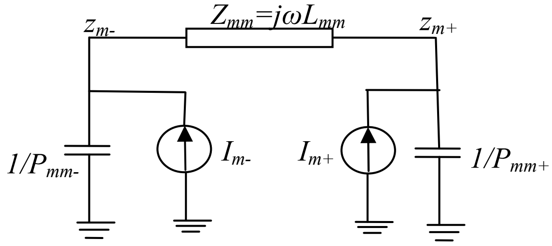

Fig.1. Equivalent circuit of one segment obtained by PEEC

Methodological differences between the two approaches

In this section a methodological investigation of the two methods is performed and theoretical explanations of their differences are offered. In order to compare MoM and PEEC approach we applied them to determine the current along a perfect conductor. The conductor is placed in an unbounded conductive environment with relative permittivity ε_r and conductivity σ. The geometry of the system used for the application of the two methods is presented in Fig. 2.

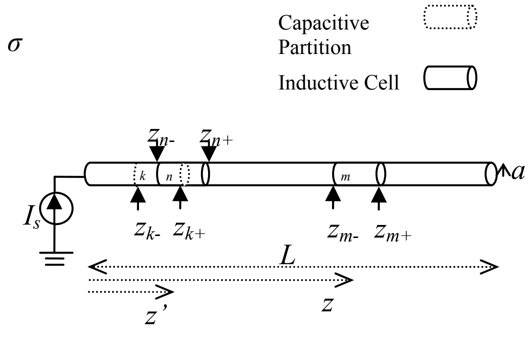

Fig.2. The perfect conductor placed in a conductive medium

The first methodological discrepancy between the two approaches is found in the way the segmentation of the conductor is performed. The MoM approach requires only one type of segmentation, segments m and n, visible in Fig. 1. PEEC method, on the other hand, besides segments m and n, includes another type of segments that are shifted towards the beginning end of the conductor for half a segment's length (notated as k in Fig. 1). The mentioned new type of segments is needed to locate the charge per unit length density given further in this paper in Eq. (13).

In general, the analysis starts with the boundary conditions for the tangential electric field on the surface of the perfect conductor

$$E_z^t = E_z^i + E_z^s = 0 \tag{6}$$

where: E_z^i is an incident electric field and E_z^s is the scattered electric field. The scattered electric field is expressed through the magnetic vector potential, A_z, and the electric scalar potential, φ, and equals zero

$$E_s = \frac{\partial\varphi}{\partial z} + j\omega A_z = 0 \qquad \Rightarrow \tag{7a}$$

$$\frac{\partial\varphi}{\partial z} = -j\omega A_z \tag{7b}$$

Propagation effects are taken into account by both approaches through the Green's function for the magnetic vector potential, G_A, and the Green's function for the electric scalar potential, G_V. The right side of Eq. (7b) is treated analogously by both methods:

$$-j\omega A_z = -j\omega \int_L G_A I(z')dz' \tag{8}$$

In the next step both methods expand the current to be determined, $I(z')$, into a linear combination, according to Eq. (2). That is the main reason for the PEEC method to be notated as "MoM based method".

A major methodological difference between the MoM and PEEC approaches appears in the treatment of the left side of Eq. (7b), i.e. the application of the continuity equation. If the calculations are further continued using MoM, the continuity equation is applied in the very next step of the analysis, yielding

$$\frac{\partial\varphi}{\partial z} = \frac{1}{j\omega}\frac{d}{dz}\int_L G_V \frac{dI(z')}{dz'}dz' \tag{9}$$

where the Green's functions in an unbounded conductive medium have the scalar form

$$G_V(z,z') = \frac{1}{4\pi\underline{\varepsilon}}\frac{e^{-\gamma R}}{R} \tag{10}$$

and

$$G_A(z,z') = \frac{\mu}{4\pi}\frac{e^{-\gamma R}}{R} \tag{11}$$

In Eq. (11), R is the distance between the segment of interest and the source segment, while γ is the propagation constant of the conductive medium

$$R = \sqrt{a^2 + (z-z')^2} \quad , \qquad \gamma = \sqrt{-\omega^2 \underline{\varepsilon}\mu} \tag{12}$$

If the PEEC approach is implemented, the potential difference is integrated along one segment and expressed through the charge per unit length density

$$\int_{z_{m-}}^{z_{m+}} \frac{\partial\varphi}{\partial z}dz = \int_L Q'(z')\cdot\left[G_V(z_{m+},z') - G_V(z_{m-},z')\right]dz' \tag{13}$$

while the continuity equation is taken into account later by applying Kirchhoff's current law on every node in the equivalent circuit analysis. The resulting circuit can be solved with any network method or solver. In this case specifically, we solved it in the frequency domain with the modified nodal approach (MNA) [3].

Another difference in the implementation of these two methods occurs while calculating the potential in the analyzed system. The potential is calculated by integrating the electric field using MoM. On the other hand, the potential is directly determined by solving the equivalent circuit using PEEC, which is another one of the method's advantages.

Numerical results

The two numerical methods investigated in this methodological analysis are applied on the system illustrated in Fig.1: a 10 m long perfect conductor placed in an unbounded conductive medium with relative permittivity ε_r =10 and specific conductivity σ=0.1 S/m.

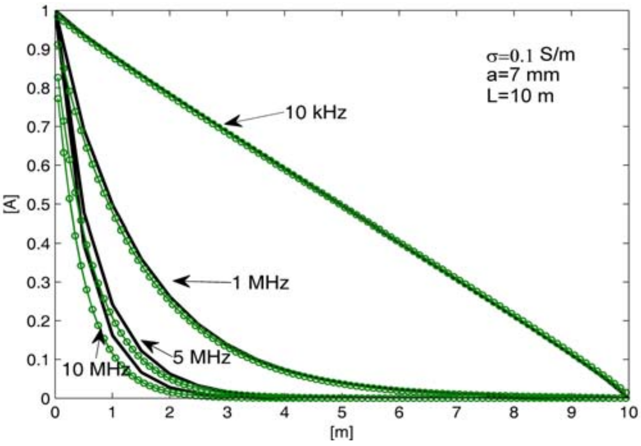

Fig.3. Comparison of the current magnitude along the conductor placed in an unbounded conductive medium with specific conductivity of 0.1 S/m ºººº PEEC [4], −−− Ref. [5]

The obtained values for the current magnitude along the conductor, calculated in the frequency domain by the PEEC method [4] are compared with results calculated by MoM, found in [5]. Figure 3 shows the compared results for several frequencies.

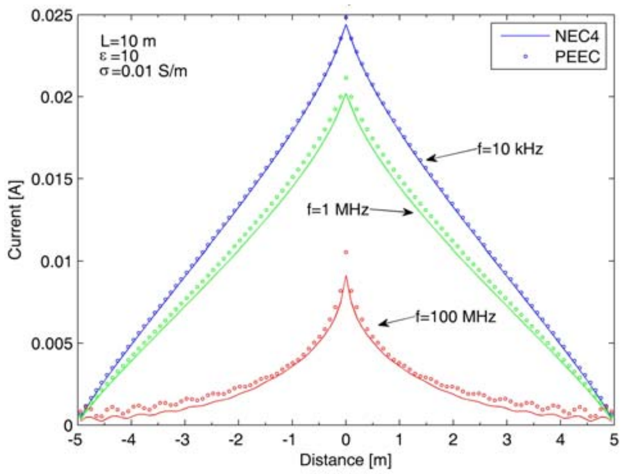

Fig.4. Comparison of the current magnitude along the conductor placed in imperfect ground with specific conductivity of 0.01 S/m

Figure 4 presents another case of implementation of the two methods that are being compared in this paper: a 10 m centre-fed (with 1 V voltage source) dipole antenna buried in imperfect ground at depth 0.4 m. The MoM calculations were carried out using the well known MoM based NEC4 solver [6]. The PEEC calculations take into account the presence of air half space by implementation of the quasi-static image theory, which leads to including the reflection coefficient [7]. The comparison of the values for the current obtained by implementation of the two approaches are shown for specific conductivity of the ground half space σ=0.01 S/m and relative permittivity ε_r =10.

The authors have performed thorough comparison of results obtained by the two methods using the rms error parameter in [8].

Conclusions

In this paper a methodological analysis of differences between the classical method of moments (MoM) approach and the partial element equivalent circuit (PEEC) method is performed. The two approaches were applied on a perfect conductor placed in a conductive environment, excited by a current source at one end. Theoretical explanations of the methodological discrepancies between the investigated approaches are offered during the analysis. Numerical results of the values determined by both approaches for the current along the conductor are presented.

REFERENCES

[1] Harrington R. F., Field computations by moment methods, Macmillan, New York, 1968

[2] Ruehli A., Partial element equivalent circuit (PEEC) method and its application in the frequency and time domain, *International Symposium on Electromagnetic Compatibility* (1996), 128-133

[3] Ho C. W., Ruehli A., Brennan P. A., The Modified Nodal Approach to network analysis *IEEE Transactions on Circuits and Systems,* Vol. CAS-22 (1975), No. 6

[4] Kuhar A., Jankoski R., Toseva A. V., Gagoska O. L., Grcev D. L., Partial element equivalent circuit model for a perfect conductor excited by a current source, *11th International Conference on Applied Electromagnetics – PES,* Nish, Serbia, 2013

[5] Grcev D. L., Markovski B., Toseva A. V., Kuhar A., Drissi K. E., Kerroum K., Modeling of horizontal grounding electrodes for lightning studies, *EUROEM 2012, European electromagnetics*, Toulouse, France, 2012.

[6] Burke J. G., Numerical Electromagnetic Code-NEC4, 1992

[7] Grcev D. L., Computer analysis of transient voltages in large grounding systems *IEEE Transactions on Power Delivery*, Vol. 11, (1996) No. 2, 815-822

[8] Kuhar A., Jankoski R., Toseva A. V., Gagoska O. L., Grcev D. L., Comparison of PEEC and MoM approaches to modelling of current distribution along conductors, *11th International Conference – ETAI,* Ohrid, Macedonia, 2013

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