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An innovative approach to optimisation of E-I core inductor

Abstract*. In the paper a new approach for optimisation and designing an E-I inductor, by using the Taguchi Design of Experiments (DoE) technique is presented. The proposed technique is applied on an existing iron core inductor. The target of optimisation is to reduce the total mass of active materials, preferably focussing on copper mass, while the required value of magnetising inductance is kept inside the prescribed bounds. Additional constraints are also considered. The inductor lamination geometry and properties of active materials (iron core and copper wire) are kept the same. The method incorporates application of 3D FEM as virtual design tool, to prove the optimisation results*

Streszczenie. *W artykule przedstawiono nowe podejście w optymalizacji i projektowaniu induktora z rdzeniem E-I przy wykorzystaniu techniki Design of Experiment (zaproponowanej przez Dr. Taguchi. Proponowana technika zastosowana jest dla istniejącego rdzenia induktora. Celem optymalizacji jest redukcja całkowitej masy materiałów aktywnych ze szczególnym uwzględnieniem masy miedzi przy zachowaniu wymaganej wartości indukcyjności w przewidzianych granicach. Dodatkowe ograniczenia również zostały rozważone. Warstwowa geometria induktora i właściwości materiałów aktywnych (rdzeń i przewody miedziane)zostały zachowane bez zmian. Metoda wykorzystuje trójwymiarową analizę metodą elementów skończonych do potwierdzenia wyników optymalizacji. (****Innowacyjne podejście do optymalizacji induktora z rdzeniem E-I****)*

Keywords: E-I core inductor, Optimisation, Taguchi design of experiments (DoE), Finite element analysis,
Słowa kluczowe: induktor z rdzeniem E-I, optymalizacja, Design of Experiments, analiza elementowo-skończeniowa.

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Introduction

Recently, within the engineering community, there is a significant interest for optimal designing of electromagnetic devices. In engineering practice the concept of optimization procedure means creating better and more economical product or solution, by using existing resources. On the other hand, when an electromagnetic device is in use, the main engineering task is to improve the performance characteristics by simple design modifications. Research in this area demonstrated successful solution of optimisation problems using various techniques, like pattern search deterministic methods [1], population based stochastic methods, as genetic algorithms [2] and particle swarm optimization [3], as well as neural networks and artificial intelligence based methodologies [4].

In the case of generic inductor design, there is a large number of design parameters to be chosen including: all of the magnetic core dimensions, the type and grade of magnetic material, air gap length, copper wire quality and its cross-section, number of turns and winding area, as well as the required value of the inductance. Hence, the inductor design can present a challenging optimization problem. However, when there is a need to improve the design of an already existing inductor, additional constraints should be taken into account. In the paper the optimisation of an E-I core inductor is addressed.

The proposed innovative optimisation approach is based on the principles of Design-of-Experiments (DoE) and application of Taguchi methodology. Taguchi constructed a special set of general design guidelines for factorial experiments that cover many applications, including optimisation problem solutions.

Principles of Taguchi Design of Experiments

Taguchi method is a statistical method that has been developed in 1980 by Dr. Genichi Taguchi of Japan, and originally was used to improve the quality of manufactured goods. More recently it is successfully applied to conduct optimisation in various scientific areas. Sometimes, the method is used to find near-optimum design before proceeding with another optimisation technique. When the intention of a designer is to understand the relationship between causes and effects, rather than just mathematical equations for their mutual relations, then the approach by *Taguchi Design of Experiments* (*DoE*) is the best suited.

The methodology, when applied as an optimisation technique, represents a series of tests in which purposeful changes are made to the input variables of an objective function, so that the changes of the output responses can be observed, identified and analysed. The investigator selects an arbitrary number of inputs – *design factors*, sets their values, and simultaneously changes the factors' settings from experiment to experiment in a specified manner; the result is maximum amount of information about the effect of all input variables on the output response, at the least number of accomplished experiments. It has been also found that Taguchi method is useful for 'fine tuning' of a given process, or optimisation procedure for 'best' results.

The standard procedure, when optimising any process, by applying this method, is consisted of a number of steps: (1) define the objective function of optimisation, which is the problem dependent; (2) identify and specify the constraints of optimisation; (3) identify the design variables and their levels of change; (4) select and set up the orthogonal array matrix experiment; (5) conduct the experiments; (6) analyse the data, determine the optimum levels of design variables and predict the optimum value of the objective function; (7) perform the verification of the optimum solution and, where appropriate, plan the future actions.

Defining Objective Function and Specifying Constraints

First, depending on the optimisation problem, the objective function and the target of optimisation have to be defined. The constraints of optimisation and requirements should be also specified. Simplifying the statement, the task is to find the minimum (optimum) of an objective function F,

$$\text{Min}\ \{F = f(\mathbf{X}, C)\} \tag{1}$$

which is subject to the design constraints:

$$g_i(\mathbf{X}, C, R) = 0\ , \quad i = 1, 2, \dots, j \tag{2}$$

where: $\mathbf{X} = (x_1, x_2, \dots, x_n)$ is an n-dimensional vector of the design variables to be determined; their selection is a problem dependent task. C represents various constraints from common knowledge and designer's experience; they can be either geometrical or physical design constraints. R are requirement specifications which the optimum design must meet; these functions are depending on the particular optimisation problem to be solved.

Selecting Design Variables and Levels of Change

The Taguchi technique of laying out the conditions of experiments involving multiple factors is popularly known as the *factorial design of experiments*. A full factorial design will identify all possible combinations for a given set of factors. Since most optimisation problems usually involve a significant number of factors, the full factorial design results in large *Number of Experiments – NoE*; thus, for N selected variables, with L levels of change, the full factorial number of experiments would be:

$$NoE = L^N \tag{3}$$

The bigger is the number of design variables (N) and the more levels (L) of change are selected, the total number of experiments (NoE) will be dramatically increased, and NoE will reach such figures that it will practically become impossible to be fully conducted. To reduce the number of experiments to a practical level, only a small *sub-set* from all the possibilities is selected. The method of selecting a limited number of experiments which produces the most information is known as a *partial fraction experiment*.

The first task is to select the reasonable number of independent variables – *design factors N*, taking into consideration the most relevant parameters to the particular optimisation problem, which is user and problem dependent task. The next task is to define the bounds within which the design factors will be changed, and also, inside the design space to select a reasonable number of *level changes L*. It is the designers' responsibility to set the proper level values. Finally, depending on N and L, to select and establish the best fitted *Taguchi Orthogonal Array* – OA_x of experiments, where x is with exactly prescribed value (6, 9, 12, 16, 18 ...).

Setting Orthogonal Array and Conducting Experiment

The Taguchi optimisation technique uses a special set of arrays called *orthogonal arrays*, where it is assumed that there is no interaction between any two factors. These standard arrays stipulate the way of conducting the minimal number of partial fraction experiments which would give the full information of all factors that affect the optimal design. Depending on the specified number of independent design variables N and their levels of change L, there are many *standard* orthogonal arrays available. Once the orthogonal array is selected, the experiments are conducted as per the level combinations. It is necessary to ensure that all experiments from the array are conducted.

Analysis of Data and Prediction of Optimal Design

Since each experiment from the orthogonal array is the combination of different factor levels, it is essential to segregate the individual effect of each independent variable on the objective function and to perform the analysis of the collected output data. For this purpose the Taguchi method incorporates: (1) *ANOM* – ANalysis Of Means of each level for each design variable; (2) *ANOVA* – ANalysis of VAriance test; (3) *Sensitivity analysis* of design variables.

Analysis of means – ANOM allows ranking of the contribution of each variable to the optimal solution. It may be noted that from the *ANOM* the significance of individual design variables is found from their percentage contribution. Using *analysis of variance test – ANOVA* one can accept or reject an independent variable and to calculate the objective function. The data from the array can be also analysed by plotting the data and by using the graphical outputs easy to visualise the experimental results. By conducting *sensitivity analysis*, it is defined whether the particular design variable is significantly influencing the objective function during the optimisation process. The sensitivity plots indicate changes of the objective function due to the change of the level settings for each design variable. It may be noted that the mean level values are centred around the grand mean value. If the mean values of all the levels for a particular design variable are very close to the grand mean line, this variable is not sensitive to changes in the level settings, and hence, it is without any significant influence.

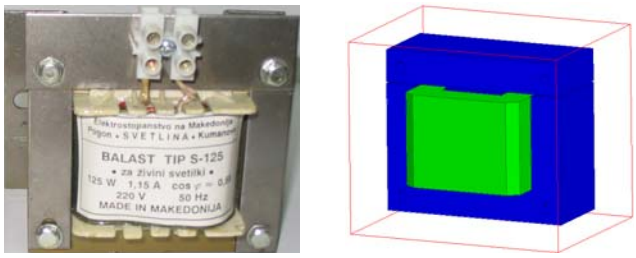
Once the sensitivity analysis is carried out, the optimum level plots are used to determine the optimum level value for each design variable. The plots are combined with the values from orthogonal array table for each experiment, and the optimum value of the objective function is calculated.

Case Study – an E-I Core Inductor

Laminated iron-core inductors are widely used in various electrical engineering applications and their optimisation is still a challenging task [2], [3] and [5]. A critical point in the design procedure of core inductors is accurate estimation of the total inductance, a parameter that mostly influences its performance. It is on the designer responsibility to develop the mathematical model of the inductor, as accurate as possible, which will best represent the real electromagnetic phenomena. The example considered here deals with design optimisation of an existing E-I core inductor. The case study starts with detailed investigation of the given inductor. At the beginning the mathematical model for analytical calculation of performance characteristics of the inductor is derived. After, the numerical FE model is developed, and the relevant magnetic characteristics of the studied inductor are computed. The inductor was tested in the laboratory. Where appropriate, the accuracy of the models, as well as both analytical and numerical results were confirmed and verified by measurements.

Generic Model Analysis

The front view of the generic E-I core inductor model is shown in Fig. 1 (a), while the 3D model is presented in Fig. 1 (b). The shell-type iron core is formed by silicon-steel laminations of magnetic material M 530-50A with relative permeability μ_r = 1630 at B_m=1.5 T. The main dimensions of the core lamination cross-section are sketched in Fig. 2.



(a) front view (b) 3D model

Fig.1. Generic model – original E-I core inductor

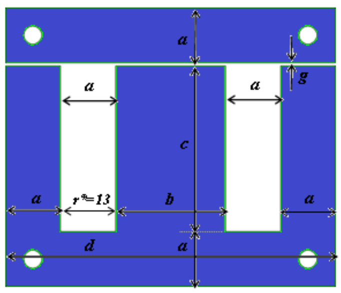


Fig.2. Geometrical cross-section of the core

The example geometry is composed of standardised E-I gapped sheet with rectangular laminated core cross-section spanning for 36 mm (70 layers) into-the-page direction. The dimensions in Fig. 2 are: a = 14 mm, b = 28 mm, c = 42 mm, d = 84 mm. The rated working value of the air-gap length is g = 0.815 mm. The excitation winding is consisted of 608 copper wire turns with heavy build insulation; the maximum possible winding width is r^* = 13 mm and the copper fill fraction of the window is 49%.

The inductor is connected to a lighting lamp. The rated data are: voltage supply U_n=220V@50Hz; current I_n=1.15A; power P_n=125W and power factor $\cos \varphi_n$=0.55; the starting voltage of firing the lamp is prescribed to be U_{min}=160V, while the maximum winding current is limited to I_{max}=1.75A. The magnetising inductance L_m and the power factor are variable, and dependent on the working conditions.

The 3D finite element analysis of the inductor provided an accurate prediction of its performance, including fringing effects, leakage fluxes and saturation. The 3D domain of the model, presented in Fig. 1 (b), is meshed with 87,724 tetrahedral finite elements. The magnetic flux density distribution, at rated current I_n = 1.15 A, is shown in Fig. 3.

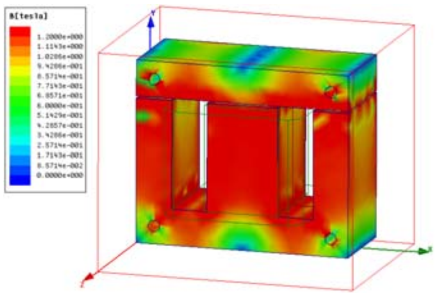


Fig.3. Magnetic flux density distribution, I_n = 1.15 A

The FEA was used as powerful tool for virtual testing of the prototype and a verification of the device modelling. In this case, the measured and FEM results of the generic model were used for verification of the mathematical model of the inductor. The results are presented in Table 1 comparatively. They show a resonable agreeement. After the performance analysis of the studied inductor was done, it was found that savings of active material mass, without deteriorating its main features, are possible.

Table 1 Comparative results for the generic inductor

Quantity	Measured	Analytical	3D FEM
Flux density in the core (T)	–	1.136	1.196
Magnetising inductance (H)	0.2825	0.3272	0.2929

Problem Definition

The focus of the work is setting forth an innovative optimisation technique based on Taguchi methodology. The target is to minimise the total mass of active materials of the inductor, with particular emphasis on the consumption of copper. For the *objective function* of optimisation the copper mass of winding is selected. The *constraints of optimisation* are divided into four groups: (1) geometrical – depending on the lamination cut and the window area, occupied by the winding; (2) functional – defining bounds of the interval where the value of the magnetising inductance must fall; (3) thermal – determining the maximum value of the winding's current density; (4) economical – when optimising the copper mass, the mass of iron core should not be increased. The *requirements of optimisation* are to keep the same shape of magnetic lamination and not to change the material specifications. In this way, the freedom in selection of the design variables is pretty reduced, but still there are possibilities to solve the defined optimisation problem and to improve the inductor design. Taking the same standard E-I lamination sheet, the only possible geometrical variable is the core depth D_{Fe} that is expressed through n – always an integer number of laminations. In accordance with the core sheet dimensions as seen in Fig. 2, the coil thickness is limited to a window width ($r^* < a$). Also, the technical constraints determine that the diameter of copper wire should be with standard WG value. The thermal constraint determines the maximum value of the winding current density (G_{cu} < 6.5 A/mm^2). From functional point of view the total inductance should fall in bounds 0.25H ≤ L_{tot} ≤ 0.45H.

The number of winding turns W, the core depth D_{Fe} and the copper wire diameter d_{Cu} influence copper mass, and implicitly, the copper loss. From the other hand, magnetic flux density of the inductor core mostly contributes to the iron mass, and implicitly to the iron loss. The air-gap length and saturation level are prevailing parameters when computing the inductance. Thus, N = 5 independent design variables, shown in Table 2, are selected; in the last column are presented their respective values of the generic model.

Table 2 Independent design variables of optimisation

#	Value	Description	Lower bound	Upper bound	Generic model
x_1	W (turns)	Number of coil turns	500	800	608
x_2	n (/)	Number of core sheets	50	80	70
x_3	B_{mg} (T)	Air-gap flux density	0.7	1.3	1.136
x_4	g^*(mm)	Equivalent air-gap length g^*=2g	1.0	2.2	1.63
x_5	d_{cu} (mm)	Wire diameter	0.56	0.83	0.71

The L=4 levels of change were selected; eq. (3) shows that the full number of experiments, would be 4^5 = *1024*. The standard Taguchi orthogonal array OA_{16} is chosen, and hence, considering all aspects of the problem, the number of experiments is reduced to only 16. The 16 combinations of 5 design variables with 4 levels, using Taguchi pattern are assigned to each experiment [5]. In addition to m_{cu}, the five constraints with their prescribed values, m_{fe}, m_{tot}, G_{cu}, L_{tot} and r^*, are computed during the optimisation procedure.

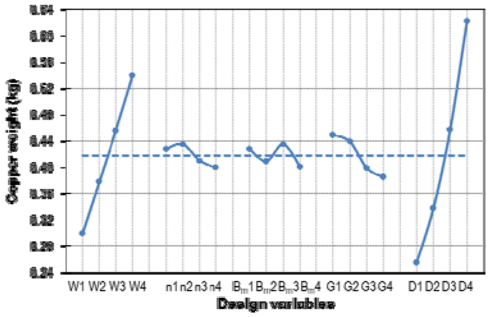
The objective function and 5 constraints are calculated by using the same analytical mathematical model, as for the generic model of the inductor. The summary of experiments is given in Table 3, showing the orthogonal matrix of OA_{16}.

Table 3 Design of Experiments output summary

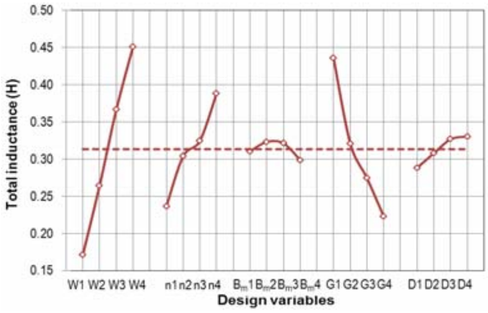
m_{cu} (kg)	m_{fe} (kg)	m_{tot} (kg)	G_{cu} (A/mm^2)	L_{tot} (H)	r^* (mm)
0.1865	0.9055	1.0920	3.9990	0.1893	7
0.2472	1.0866	1.3338	5.3629	0.1737	9
0.3281	1.2677	1.5959	6.3944	0.1648	11
0.4346	1.4488	1.8834	7.1276	0.1583	13
0.5643	0.9055	1.4698	3.4431	0.1755	17
0.4104	1.0866	1.4971	4.0627	0.1747	13
0.3030	1.2677	1.5708	4.8849	0.3780	10
0.2341	1.4488	1.6830	6.8904	0.3299	9
0.3690	0.9055	1.2745	6.9356	0.2016	12
0.2730	1.0866	1.3596	8.6931	0.2781	9
0.6832	1.2677	1.9510	1.8474	0.3994	19
0.4984	1.4488	1.9472	2.2852	0.5871	15
0.5921	0.9055	1.4976	3.8082	0.3805	17
0.8093	1.0866	1.8960	1.9166	0.5890	21
0.3260	1.2677	1.5937	6.2981	0.3565	11
0.4304	1.4488	1.8792	3.2383	0.4786	13

Optimisation Procedure and Optimal Solution

Once the Taguchi design of experiments is performed and OA_{16} matrix is completed, the optimisation procedure continues with *ANOM* and *ANOVA*. The details of these analyses the authors presented in [5]. Besides the objective function, i.e. the copper mass m_{Cu} the total inductance of the inductor L_{tot}=L_s+L_m (where L_s is a leakage inductance) is analysed. The charts are presented in Fig. 4 (a) and (b); they show the main design variable effects on the functions, illustrating qualitatively and quantitatively how each function is sensitive to the variations of each design variable and disclosing the contribution of each main factor to the change of selected function.



(a) objective function m_{Cu}



(b) total inductance $L=L_S+L_m$

Fig. 4 Effect of design variable values on the functions

The charts show that function may vary linearly – with different slope, or with arbitrary shape around its respective mean value, presented with dashed line. ANOVA provides the *measure of confidence* and was used to extract the relative contribution of each factor to the total variation of each analysed function [5]. The results are used for estimating the effect of each design parameter on each function, and for determining the relative importance of each design parameter in (%). Targeting to the reduction of copper mass and keeping the inductance in prescribed bounds, the calculated results are summarised in Table 4.

Table 4 ANOVA results for effect of design variables

	Copper mass m_{Cu} (kg)		Total inductance L_{tot} (H)	
	Sum of squares	Variable Effect (%)	Sum of squares	Variable Effect (%)
W	0.12771074	28.2613	0.177269	**53.8135**
n	0.00318923	0.7057	0.046933	14.2473
B_m	0.00306989	0.6793	0.001530	0.4646
$g^*=2g$	0.0115155	2.5483	0.099182	30.1086
d_{cu}	0.30640806	**67.8054**	0.004500	1.3660
Total:	0.451893	100	0.329414	100

*) Cells marked in bold show factors with highest effect on functions

Coming up to the end of Taguchi methodology and the analysis of all design of experiments, the best combination of design variables and their optimal values are determined; the results are shown in Table 5, for the generic and optimal model of the studied E-I core inductor.

Table 5 Optimal values of the design variables

Design variable	W (turns)	n (/)	B_m (T)	$g^*=2g$ (mm)	d_{cu} mm^2
Generic model	608	70	1.136	1.63	0,71
Optimal model	720	60	0.9	1.8	0.63

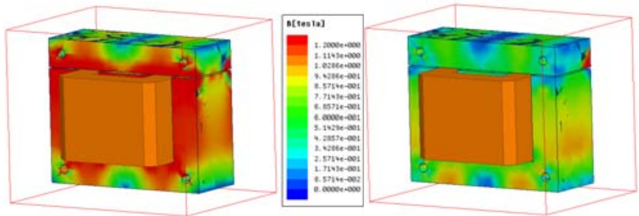
Analysis of Results

Optimisation results, the value of objective function and values of auxiliary functions are presented comparatively in Table 6, for the original generic model and optimal model. The numbers prove that optimisation targets are achieved.

The 3D FEM is used to visualise some of the important electromagnetic quantities in both models that can complete the optimisation results. As an example, the 3D magnetic flux density distribution of the magnetic core, for the generic and optimal model, is shown in Fig. 5 (a) and (b), respectively. From the presented figures, the significant reduction of the saturation level of the core is evident, which obviously resulted in the substantial iron loss reduction.

Table 6 Optimal values of the analysed design functions

Design function	**m_{cu} (kg)**	m_{tot} (kg)	P_{cu} (W)	P_{fe} (W)
Generic model	**0.416**	1.684	24.386	3.854
Optimal model	**0.379**	1.466	19.138	2.073
Design function	G_{cu} A/mm^2	L_{tot} (H)	S=UI (VA)	r* (mm)
Generic model	4.978	0.2651	315.7	13
Optimal model	4.627	0.2957	188.6	12



(a) generic model (b) optimal model

Fig. 5 Magnetic flux density distribution of the E-I core inductor

Conclusion

In the present paper an innovative technique for optimisation, using the design of experiment based Taguchi method, was proposed. Apart from the other optimisation methods that are widely in use, the approach is based on the simple calculations and shows much more flexibility and easy application. As a demonstration of the proposed optimization approach, E-I core inductor design example was presented. Taguchi method is proved as fast and efficient tool for redesigning the existing E-I core inductor.

The presented results show that optimisation objectives are accomplished, and the copper mass is significantly reduced for 8.9%. Although not directly targeted, the iron mass is reduced even more, for 12.95%. At the same time, significant reduction is gained in the losses: the optimised inductor design is with 21.52% smaller copper loss, while the iron loss is smaller for impressive 46.21%. The thermal conditions are not deteriorated, as the current density is also reduced for 7.05%. The winding width is 1 mm smaller, thus fulfilling the geometrical requirements. The value of stotal inductance is kept in the prescribed bounds.

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