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Determination of reduced parameters for the impedance boundary conditions at screened surfaces

Abstract. *A method for calculating impedance boundary conditions reduced parameters at permeable screen surfaces in a quasi-stationary electromagnetic field is presented. Solving a model, spherically symmetric system allowed to obtain dependencies, determining these parameters as functions of the curvature radius of the screened surface, and material parameters of the screen and the screened area.*

Streszczenie. *W pracy zaproponowano metodę obliczania zredukowanych parametrów impedancyjnych warunków brzegowych na powierzchniach ekranów przenikalnych w quasi-stacjonarnym polu elektromagnetycznym. Na podstawie rozwiązania modelowego zagadnienia o symetrii sferycznej otrzymano zależności pozwalające określić te parametry w zależności od promienia krzywizny powierzchni ekranowanej oraz parametrów materiałowych ekranu i obszaru ekranowanego.* (***Określanie zredukowanych parametrów dla impedancyjnych warunków brzegowych na powierzchniach ekranowanych***).

Keywords: impedance boundary conditions, permeable screens, harmonic electromagnetic field.
Słowa kluczowe: impedancyjne warunki brzegowe, ekrany przenikalne, harmoniczne pole elektromagnetyczne.

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Introduction

Determining distribution of a quasi-stationary electromagnetic field in the presence of conductive bodies remains one of the most intricate problems in technical electrodynamics. Complex, three dimensional geometry of real technical systems provides a challenging task for solving such systems. It is therefore advisable to simplify the problem by developing the impedance boundary conditions (IBC) for surfaces of conductive bodies [1-4]. Then, it is no longer necessary to solve the field equations in the conductive areas, where computations require fine discretization of the areas, and the problem is reduced to seeking a scalar function of a magnetic potential [5-8].

One of the assumptions made when introducing IBC is that all the dimensions of the conductive bodies are significantly larger than equivalent depth of electromagnetic field penetration $\delta = \sqrt{2/\omega\mu\gamma}$, namely at least triple its value. Therefore, it is principally incorrect to assume IBC to be valid at penetrable screened surfaces, most frequently used to screen low frequency fields [9], as the thicknesses fail to meet this requirement. Nevertheless, it is conceivable that if all the remaining assumptions for IBC hold, then an appropriate modification of certain characteristic parameters should allow to apply this condition also at surfaces of penetrable screens.

In the presented paper a method for determining IBC characteristic parameters in relation to material parameters of the screen and the screened area, namely the screen thickness and the surface curvature radius, has been developed as based on the analysis of a model problem that can be solved analytically. It allows the relations sought to be finally expressed in an exact, though quite complex, form of algebraic expressions.

Impedance Boundary Conditions (IBC)

Impedance boundary conditions generally provide approximated relations between the electromagnetic field components or potentials at the boundary surfaces between conductive and dielectric areas. It is further assumed that the dielectric area has the permeability of a vacuum, and its conductivity equals zero.

The fundamental assumptions made for IBC are large curvature radii at the boundary surface in relation to the field penetration depth δ into the conductor, and the conductive area to be significantly bigger in size than the penetration depth. For harmonic low frequency fields the IBC take the following forms [1-8]:

(1) $$\boldsymbol{n}\times\boldsymbol{E} = Z_c\boldsymbol{n}\times\left(\boldsymbol{n}\times\boldsymbol{H}\right)$$

(2) $$\boldsymbol{E} - \boldsymbol{n}E_n = Z_c\boldsymbol{n}\times\boldsymbol{H}$$

(3) $$\boldsymbol{J} = \alpha\boldsymbol{n}\times\boldsymbol{H}$$

(4) $$\widetilde{\Delta}\varphi = -\beta\frac{\partial\varphi}{\partial n}$$

(5) $$\frac{\partial H_n}{\partial n} = \eta H_n$$

where: $\boldsymbol{E}$, $\boldsymbol{H}$, $\boldsymbol{J}$ are complex amplitudes for an electric and magnetic field, and current density, respectively, at the boundary surface, $\boldsymbol{n}$ – is a unit vector normal the boundary surface pointing outwards the conductive area, E_n, H_n – the normal components of $\boldsymbol{E}$, $\boldsymbol{H}$ vectors, φ – a magnetic potential complex amplitude ($\boldsymbol{H} = -\mathrm{grad}\varphi$),

(6) $$\widetilde{\Delta} \equiv \frac{1}{h_1h_2}\left[\frac{\partial}{\partial s_1}\left(\frac{h_2}{h_1}\frac{\partial}{\partial s_1}\right)+\frac{\partial}{\partial s_2}\left(\frac{h_1}{h_2}\frac{\partial}{\partial s_2}\right)\right]$$ - a surface

Laplace operator, h_1, h_2 - Lamé parameters for the orthogonal coordinates s_1, s_2, describing the boundary surface,

(7) $$Z_c = \sqrt{\frac{\mathrm{j}\omega\mu}{\gamma}}$$ - the conductor wave impedance,

(8) $$\alpha = \sqrt{\mathrm{j}\omega\gamma\mu}$$ - the propagation constant in the conductive area,

(9) $$\beta = \eta = \mu_0\sqrt{\frac{\mathrm{j}\omega\gamma}{\mu}}$$

It should be noted that apart from conditions (1) and (2), the remaining conditions are not entirely equivalent to each other; hence, parameters β and η are clearly distinguished as their reduced values calculated further at the screened surface are not equal.

Formulation of a model problem

As mentioned in the introduction the assumed large sizes of conducting areas in comparison to their field penetration depth, prevent IBC to be directly applied at permeable screens surfaces. Hence, the main aim of this paper was to find a formula for calculating a modified value for characteristic coefficients Z_c, α, β, η occurring in (1) – (5) in a way that makes IBC applicable also at the surfaces of screens of any constant parameters and any thickness.

Thus, a model problem illustrated in Fig. 1 was considered. The system under analysis comprises three areas, namely: Ω_1 – a dielectric area (γ = 0, $\mu = \mu_0$), Ω_2 – a spherical conductive screen ($\gamma = \gamma_2$, $\mu = \mu_2$) of the radius R_1 and a constant thickness d, Ω_3 – a spherical screened area (interior), either dielectric or conductive ($\gamma = \gamma_3$, $\mu = \mu_3$) of the radius R_2.

A homogenous, harmonic magnetic field of a complex amplitude H_0 directed along the OZ axis of the coordinating system provides the exciting field for the system.

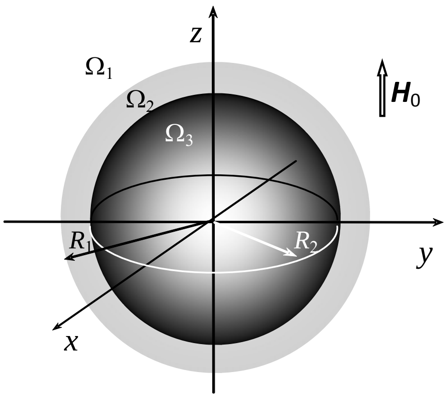

Fig.1. The system under consideration

It is assumed that
- magnetic permeability and electric conductivity are constant for all the subareas,
- all the electromagnetic field components are sinusoidal functions of time with a pulsation ω,
- displacement currents might be neglected, i.e. low frequency fields are considered,
- no unbalanced electrical charge is present in the system.

Under such assumptions the complex field functions can be introduced with the respective complex amplitudes $\boldsymbol{E}$ and $\boldsymbol{H}$ satisfying the equations

(10) $$\mathrm{rot}\boldsymbol{H} = \mu\gamma\boldsymbol{E}$$

(11) $$\mathrm{rot}\boldsymbol{E} = -\mathrm{j}\omega\mu\boldsymbol{H}$$

as well as classic electrodynamics boundary conditions on continuity of the tangential components $\boldsymbol{E}$ and $\boldsymbol{H}$ at surfaces $r = R_1$, $r = R_2$. By introducing a magnetic vector potential $\boldsymbol{A}$ ($\boldsymbol{B}$ = rot$\boldsymbol{A}$, $\boldsymbol{E} = -\mathrm{j}\omega\boldsymbol{A}$) and expressing it with a spherical coordinates r, θ, φ the problem can be reduced to seeking one function of two variables $A_\varphi(r,\theta)$ that satisfies Helmholtz equation $\Delta\boldsymbol{A}_\varphi = \alpha^2\boldsymbol{A}_\varphi$; the remaining $\boldsymbol{A}$ components are equal zero.

The exact solution

A general solution to the problem set as above for particular system areas can be obtained by applying variable separation method. The resulting field functions are

- for Ω^{I} area

(12) $$H_r^{\mathrm{I}} = H_0\left[1 - F^{\mathrm{I}}\left(\frac{R_1}{r}\right)^3\right]\cos\theta$$

(13) $$H_\theta^{\mathrm{I}} = -H_0\left[1 + \frac{F^{\mathrm{I}}}{2}\left(\frac{R_1}{r}\right)^3\right]\sin\theta$$

(14) $$E_\varphi^{\mathrm{I}} = -\frac{j\omega\mu_0 H_0}{2}\left[1 - F^{\mathrm{I}}\left(\frac{R_1}{r}\right)^3\right] r\sin\theta$$

- for Ω^{II} area

(15) $$H_r^{\mathrm{II}} = H_0\left(\frac{R_1}{r}\right)^3\Big[F_1^{\mathrm{II}}(1-\alpha_2 r)\exp(\alpha_n r) + \\ - F_2^{\mathrm{II}}(1+\alpha_2 r)\exp(-\alpha_2 r)\Big]\cos\theta$$

(16) $$H_\theta^{\mathrm{II}} = \frac{1}{2}H_0\left(\frac{R_n}{r}\right)^3\Big[F_1^{\mathrm{II}}(1-\alpha_2 r+\alpha_2^2 r^2)\exp(\alpha_2 r) + \\ - F_2^{\mathrm{II}}(1+\alpha_2 r+\alpha_2^2 r^2)\exp(-\alpha_2 r)\Big]\sin\theta$$

(17) $$E_\varphi^{\mathrm{II}} = -\frac{j\omega\mu_0 H_0 R_1^3}{2r^2}\Big[F_1^{\mathrm{II}}(1-\alpha_2 r)\exp(\alpha_n r) + \\ - F_2^{\mathrm{II}}(1+\alpha_2 r)\exp(-\alpha_2 r)\Big]\sin\theta$$

- for Ω^{III} area

(18) $$H_\theta^{\mathrm{III}} = \frac{1}{2}H_0 F^{\mathrm{III}}\left(\frac{R_2}{r}\right)^3\Big[(1-\alpha_3 r+\alpha_3^2 r^2)\exp(\alpha_3 r) + \\ -(1+\alpha_3 r+\alpha_3^2 r^2)\exp(-\alpha_3 r)\Big]\sin\theta$$

(19) $$H_r^{\mathrm{III}} = H_0 F^{\mathrm{III}}\left(\frac{R_2}{r}\right)^3\Big[(1-\alpha_3 r)\exp(\alpha_3 r) + \\ -(1+\alpha_3 r)\exp(-\alpha_3 r)\Big]\cos\theta$$

(20) $$E_\varphi^{\mathrm{III}} = -\frac{j\omega\mu_3 H_0 R_2^3}{2r^2}F^{\mathrm{III}}\Big[(1-\alpha_3 r)\exp(\alpha_3 r) + \\ -(1+\alpha_3 r)\exp(-\alpha_3 r)\Big]\sin\theta$$

All the other components equal zero.
Constants F^{I} - F^{III} determined from the boundary conditions

$$E_\varphi^{\mathrm{I}}(R_1,\theta) = E_\varphi^{\mathrm{II}}(R_1,\theta), \quad H_\theta^{\mathrm{I}}(R_1,\theta) = H_\theta^{\mathrm{II}}(R_1,\theta),$$
$$E_\varphi^{\mathrm{II}}(R_2,\theta) = E_\varphi^{\mathrm{III}}(R_2,\theta), \quad H_\theta^{\mathrm{II}}(R_2,\theta) = H_\theta^{\mathrm{III}}(R_2,\theta)$$

take forms of complex algebraic relationships. They can be represented as follows

(21) $$F^{\mathrm{I}} = 1 - 3\frac{a_{12}c_{22} - a_{13}c_{21}}{D}, \quad F_1^{\mathrm{II}} = \frac{3c_{22}}{D},$$
$$F_2^{\mathrm{II}} = \frac{-3c_{21}}{D}, \quad F^{\mathrm{III}} = -3\frac{a_{32}c_{22} - a_{33}c_{21}}{a_{34}D}$$

where

$$c_{11}=a_{12}-a_{22}, \quad c_{12}=a_{13}-a_{23}, \quad c_{21}=a_{32}a_{44}-a_{42}a_{34},$$
$$c_{22}=a_{33}a_{44}-a_{43}a_{34}, \; D=c_{11}c_{22}-c_{12}c_{21},$$
$$a_{12}=\frac{\mu_2}{\mu_1}(1-\alpha_2R_1)\exp(\alpha_2R_1),$$
$$a_{13}=\frac{\mu_2}{\mu_1}(1+\alpha_2R_1)\exp(-\alpha_2R_1),$$
$$a_{22}=\left(1-\alpha_2R_1+\alpha_2^2R_1^2\right)\exp(\alpha_2R_1),$$
$$a_{23}=\left(1+\alpha_2R_1+\alpha_2^2R_1^2\right)\exp(-\alpha_2R_1),$$
$$a_{32}=\left(\frac{R_1}{R_2}\right)^3(1-\alpha_2R_2)\exp(\alpha_2R_2),$$
$$a_{33}=\left(\frac{R_1}{R_2}\right)^3(1+\alpha_2R_2)\exp(-\alpha_2R_2),$$
$$a_{34}=-\frac{\mu_3}{\mu_2}\left[(1-\alpha_3R_2)\exp(\alpha_3R_2)-(1+\alpha_3R_2)\exp(-\alpha_3R_2)\right],$$
$$a_{42}=\left(\frac{R_1}{R_2}\right)^3\left(1-\alpha_2R_2+\alpha_2^2R_2^2\right)\exp(\alpha_2R_2),$$
$$a_{43}=\left(\frac{R_1}{R_2}\right)^3\left(1+\alpha_2R_2+\alpha_2^2R_2^2\right)\exp(-\alpha_2R_2),$$
$$a_{44}=-\left(1-\alpha_3R_2+\alpha_3^2R_2^2\right)\exp(\alpha_3R_2)+$$
$$+\left(1+\alpha_3R_2+\alpha_3^2R_2^2\right)\exp(-\alpha_3R_2)$$

Solution with IBC applied

The solution where IBC are applied is considered valid solely for the internal area Ω^{I}, and parameters Z_c, α, β, η occurring in (1) – (5) are considered free, i.e. unrelated by (7) – (9). To make them different from the parameters described therein, we denote them as $Z_c', \alpha', \beta', \eta'$ respectively, and term them as reduced parameters for IBC. The solution in its general form is identical to the one presented in section 4, by equations (12) – (14), with the exception of the constant value F^{I}. For specific conditions (1) - (5) we then obtain

(22) $$F^{\mathrm{I}}(Z_c')=-\frac{2Z_c'-j\omega\mu_0R_1}{Z_c'+j\omega\mu_0R_1} \quad \text{for (1) and (2)}$$

(23) $$F^{\mathrm{I}}(\alpha')=-\frac{2\alpha'-j\omega\gamma_2\mu_0R_1}{\alpha'+j\omega\gamma_2\mu_0R_1} \quad \text{for (3)}$$

(24) $$F^{\mathrm{I}}(\beta')=\frac{\beta'R_1-2}{\beta'R_1+1} \quad \text{for (4)}$$

(25) $$F^{\mathrm{I}}(\eta')=\frac{\eta'R_1}{\eta'R_1+3} \quad \text{for (5)}$$

By inversing these relations we arrive at

(26) $$Z_c'=j\omega\mu_0R_1\frac{1-F^{\mathrm{I}}}{2+F^{\mathrm{I}}}$$

(27) $$\alpha'=j\omega\mu_0\gamma_2R_1\frac{1-F^{\mathrm{I}}}{2+F^{\mathrm{I}}}$$

(28) $$\beta'=\frac{1}{R_1}\frac{2+F^{\mathrm{I}}}{1-F^{\mathrm{I}}}$$

(29) $$\eta'=\frac{1}{R_1}\frac{3F^{\mathrm{I}}}{1-F^{\mathrm{I}}}$$

For constant F^{I} defined in (21) we obtain a solution identical to the exact solution.

Computations results

For a model system described in section 3 (Fig. 1) a few cases, typical for technical applications, were analysed. The induction field frequency of 50 Hz was adopted. Two types of the screened areas Ω^{III} were taken into account, namely a dielectric area of a vacuum permeability, and a conductive area with parameters typical for a structural steel. Parameters for screens were those of copper/ aluminium, and transformer steel, for electromagnetic and magnetic screens, respectively.

Detailed data:

The screened area: dielectric $\mu_r = 1$, $\gamma = 0$; steel $\mu_r = 500$, $\gamma = 8$ MS/m, radius $R_2 = 0 – 100$ mm

Screens: Al $\mu_r = 1$, $\gamma = 35$ MS/m; Cu $\mu_r = 1$, $\gamma = 58$ MS/m; steel, $\mu_r = 2000$, $\gamma = 8$ MS/m; thickness $d = 0$ - 30 mm

Internal area: $\mu_r = 1$, $\gamma = 0$

Figures 2 – 3 present the computed distribution for magnetic flux density spherical components for the systems under analysis.

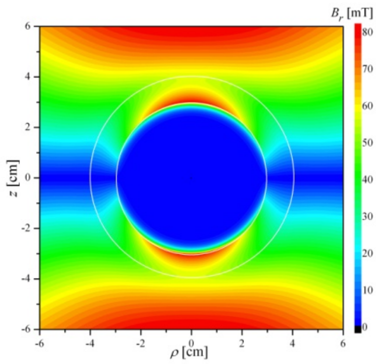

Fig. 2. Distribution for B_r component of the magnetic flux density for a steel sphere and an Al screen

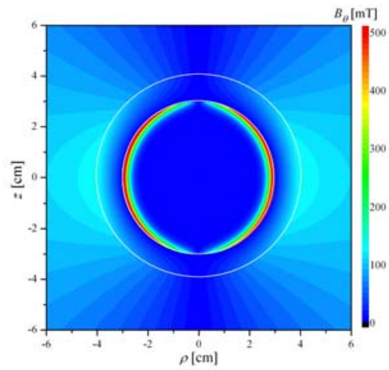

Fig. 3. Distribution for B_θ component of the magnetic flux density for a steel sphere and an Al screen

The dependence of the reduced coefficient β' on the sphere radius R_2 and screen thickness d was studied in details as the remaining IBC reduced parameters can be expressed as dependent on β' by substituting (24) to (26), (27) and (29). The computed results are presented in Figures 4 – 6.

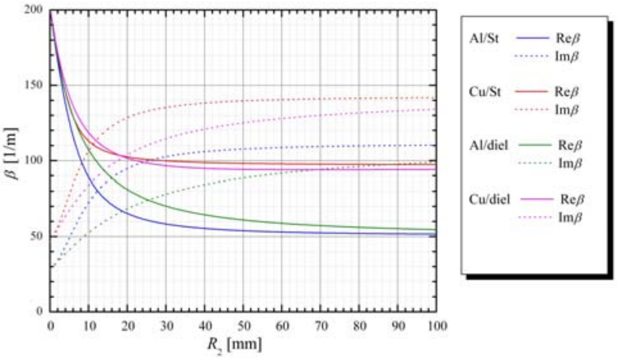

Fig. 4. The relationship between β and the radius R_2 for various configurations of sphere-screen systems

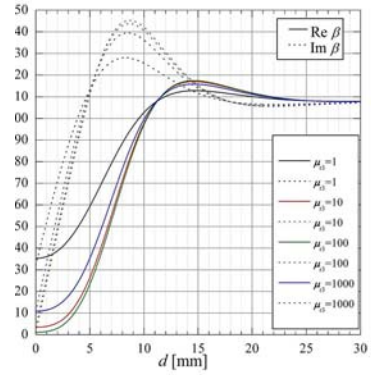

Fig. 5. The relationship between β and the screen thickness d (Cu screen) for the conductive area, $\gamma = 8$ Ms/m

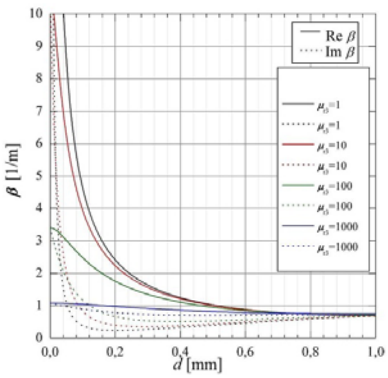

Fig. 6. The relationship between β and the screen thickness d (transformer steel screen) for the conductive area, $\gamma = 8$ Ms/m

Conclusions

A method for calculating IBC reduced parameters at permeable screen surfaces in a quasi-stationary electromagnetic field is presented. Solving a model, spherically symmetric system allowed to obtain dependencies, i.e. equations (21) and (26) – (29), determining these parameters as functions of the curvature radius of the screened surface, and material parameters of the screen and the screened area. Such dependencies for typical technical systems applied were subject to analysis, which allowed to draw the following conclusions.

- The curvature radius R of the screened surface significantly influences the values of IBC reduced parameters for $R < 3\delta$ (see Fig. 5 for comparison). For $R > 6\delta$ the influence proved to be negligible.
- IBC reduced parameters differ significantly from non-reduced parameters for magnetic screens of thickness $d < 1.5\delta$, while for thicknesses $d > 2\delta$ the discrepancies are practically negligible (see Figs. 15, 16).
- The dependencies of real and imaginary parts of the IBC reduced parameters upon the screen thickness were found to be principally different from that of the unreduced parameters where real and imaginary parts are identical – see (7)-(9) for comparison. It means that the phase displacement between the electric and magnetic field vectors at the surface topped with a penetrable screen is greatly impacted by the screen thickness (see (1), (2)).

REFERENCES

[1] Leontovich M. A.: On the approximate boundary conditions for the electromagnetic field on the surface of well conducting bodies, *Investigations of Radio Waves*, (1948), 5-12.
[2] Jackson J. D.: *Classical Electrodynamics*, PWN, Warszawa (1982).
[3] Gramz M., Ziółkowski M.: A.: Trójwymiarowa analiza pola magnetycznego z wykorzystaniem warunku brzegowego typu impedancyjnego, *15-th Seminar on fundamentals of electrotechnics and circuit theory, SPETO,* Gliwice-Wisła (1992).
[4] Apanasewicz S., Pawłowski S.: An improvement of the boundary conditions of impedance type. *COMPEL: The International journal for Computation and Mathematics in Electrical and Electronic Engineering,* 19, (2000), No. 2, 211-216.
[5] Apanasewicz S.: Zastosowanie równań całkowych do obliczania rozkładu pola elektromagnetycznego w transformatorach dużych mocy. Rozprawy Elektrotechniczne, 1, (1986), No. 32.
[6] Pawłowski S.: Analysis of Leakage Field in Power Transformers with Use of Boundary-Iterative Method, *WSEAS Transaction on Circuits and Systems*, 11, (2005) No. 4, 1620-1627.
[7] Apanasewicz S., Pawłowski S., Plewako J.: Zastosowanie iteracyjnej metody rozwiązań fundamentalnych do analizy quasistacjonarnego pola elektromagnetycznego w obecności ciał o nieliniowych właściwościach magnetycznych, *Przegląd Elektrotechniczny*, 11, (2013), 304-307.
[8] Apanasewicz S., Pawłowski S., Plewako J.: Analysis for quasi-stationary electromagnetic field with ferromagnetic objects present within, *Przegląd Elektrotechniczny*, 12, (2013), 169-172.
[9] Turowski J.: *Obliczenia elektromagnetyczne elementów maszyn i urządzeń elektrycznych*, (1982), WNT, Warszawa.

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