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Evaluation of functional device suitability considering both random technological deviations of its parameters from their nominal values and the process of components' aging

Abstract*. One of main parameters of a device is its functional suitability. In the course of the device operation, the device components change their parameters under influence of environment; this process is called aging. Under such conditions, the problem of evaluating functional suitability of the device emerges that needs to be solved taking into account real processes of its aging.*

Streszczenie. *W pracy omówiono problem zmian funkcjonalności urządzenia ze względu na technologiczne odchylenia parametrów od wartości nominalnych i procesów starzenia. (****Ocena zmiany funkcjonalności urządzenia z uwzględnieniem odchylenia parametrów od wielkości nominalnych i procesów starzenia****)*

Keywords: functional suitability, aging process, confidence ellipsoid.
Słowa kluczowe: funkcjonalnosć urządznia, starzenia, elipsoida zaufania

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Introduction

Functional suitability of a device is a complex concept that is used for evaluation of such key characteristics of the device as performance, endurance, reliability, recoverability, and others. This is the ability of a device to perform specified functions, while maintaining its operational performance within specified limits for a required period of time. Analysis of functional suitability of a device with consideration of random deviations of its components' parameters in the course of its operation was conducted in numerous works, in particular in [1, 3]. In most cases researchers assume that random deviations of the device components' parameters from their nominal values are normally distributed, both those occurring during the manufacture process and those occurring in the course of its operation. However, while this assumption is mostly true for random deviations occurring during the manufacture process, it is not always true for random deviations of the device components' parameters occurring in the course of its operation. The point is that in the course of device operation its components age what means irreversible changes in the properties of components, units, and devices as a whole; their deterioration is manifested in deviations from initial or required engineering specifications. When the deviation reaches its critical value, "failure" occurs - a phenomenon that manifests itself in the fact that the product partially or completely ceases to perform its basic functions [3]. It is well known that aging of components of complex devices that are composed of large number of non-repairable items and fail abruptly can be described by exponential distribution [6]. Based on research [5] one can also assert that aging may be described by a function that comprises both a random component and a deterministic component; both of them must be taken into account in the analysis of functional suitability.

In view of the above, the scientific problem of evaluating functional suitability of devices that takes their actual aging into account is of vital importance.

Problem Statement

Let us consider the case of evaluation of functional suitability of a device based on the analysis of technological random deviations of the components' parameters in the process of its manufacturing, when the specified deviations are normally distributed [5]. In the deviation domain they can be described by a confidence ellipsoid [6]:

$$Q(\alpha,m)=\{\delta\vec{b}\in R \,|\, \delta\vec{b}^{T}\cdot D(\delta\vec{b})\cdot\delta\vec{b}\le\chi^{2}(\alpha,m)\} \quad (1)$$

where $D^{-1}(\delta\vec{b})$ is covariance matrix of probable technological deviations of parameters from their nominal values; $\chi^{2}(\alpha,m)$ is tabulated value of χ^{2} - distribution.

If the specified covariance matrix of probable technological deviations of the components' parameters is given, we can assess the probability of functional suitability by solving the following task [5]:

$$\chi^{2}(\alpha,m)\xrightarrow{\alpha\in[0,1]}\max\ ,\ Q(\alpha,m)\subseteq\widetilde{\Omega}_{m}\ , \quad (2)$$

where $\widetilde{\Omega}_{m}$ is the tolerance region of components' parameters.

Customarily, when setting tolerances for the device components, the limit on the output characteristics of the device in the form [1]

$$y_{i}\in[y_{i}^{-},y_{i}^{+}],i=1,...,N \quad (3)$$

is set and the relationship between the parameters' values and the discussed output characteristics

$$y_{i}=g_{i}(\vec{b}),i=1,...,N. \quad (4)$$

is specified.

Given that in general case the characteristics of the device are nonlinear, it is reasonable to apply the linearization of those characteristics to pass on to the system of linear inequalities in such form:

$$\delta\vec{Y}^{-}\le S\cdot\delta\vec{b}\le\delta\vec{Y}^{+}, \quad (5)$$

where $\delta\vec{Y}^{-}=\{\delta y_{i}^{-},i=1,...,N\},\delta\vec{Y}^{+}=\{\delta y_{i}^{+},i=1,...,N\}$ are vectors that consist of lower and upper limits of output characteristics' deviations from their nominal values; $S=\{S_{ij},i=\overline{1,N},j=\overline{1,m}\}$ is a given matrix comprising the derivatives of $g_{i}(\vec{b})$ functions obtained in the procedure of their linearization at the point $\vec{b}_{0}$ (i.e. for nominal values of the parameters); $\delta\vec{b}=(\delta b_{1},...,\delta b_{m})^{T}$ are vectors of relative deviations of the device parameters from their nominal values.

In case of the system (4) compatibility its solution region is the area Ω of the device parameters determined as [2]:

$$\Omega=\left\{\delta\vec{b}\in R^{m}\,\middle|\,\delta\vec{Y}^{-}\le S^{T}\cdot\delta\vec{b}\le\delta\vec{Y}^{+}\right\}. \quad (6)$$

For the most part, the number of characteristics' constraints is less than the number of parameters. In such case we have to extend the system of linear inequalities (6) by the constraints set on the values of certain parameters in the form of inequalities: $\delta Y_j^- \le \delta b_j \le \delta Y_j^+$. The tolerance region (6) in the parameters' space looks like a parallelotope [1]:

$$\tilde{\Omega}_m = \left\{ \vec{\delta b} \in R^m \middle| \vec{\delta Y}_m^- \le S_m^T \cdot \vec{\delta b} \le \vec{\delta Y}_m^+ \right\}, \tag{7}$$

where $\vec{\delta Y}^- = \{\delta y_i^-, i = 1,...,m\}, \vec{\delta Y}^+ = \{\delta y_i^+, i = 1,...,m\}$; $S_m = \{S_{ij}, i = \overline{1,m}, j = \overline{1,m}\}$.

Under such conditions, the task (2) is the task of finding such value of probability α, that provides the maximal volume of a confidence ellipsoid inscribed into the tolerance region $\tilde{\Omega}_m$. If the center of the confidence ellipsoid (1) coincides with the zero point, it coincides concurrently with the center of the tolerance region $\tilde{\Omega}_m$. Then for the solution of the task (2) we may use a lemma [1], about the possibility of inscribing m- dimensional ellipsoid of maximum volume into the tolerance region $\tilde{\Omega}_m$ in such a way that it touches the parallelotope (tolerance region) edges at their centers.

Based on the above lemma it was shown [3] that the solution of the task (2) has the following form:

$$\chi^2(\alpha, m) = 1 \Big/ \max_{i=1,..,m} \{\Lambda'_{ii}\} \tag{8}$$

where Λ'_{ii} are diagonal elements of the matrix:

$$\Lambda' = E^{-1} \cdot S_m \cdot D^{-1}(\vec{\delta b}) \cdot S_m^T \cdot E^{-1}, \tag{9}$$

where $E = diag\{\Delta_i = 0{,}5 \cdot (\delta y_i^+ - \delta y_i^-), i = 1,...,m\}$ is a diagonal matrix comprising tolerances for device characteristics.

Therefore, this approach provides the possibility to obtain the assessment of functional suitability probability in the form of confidential probability α.

Owing to the fact that components' aging reduces the probability of functional suitability, the obtained relations (3) and (4) give an inflated estimate of the probability of functional suitability. Therefore, it is necessary to develop a method of evaluating functional suitability of a device considering the process of components' aging.

Method of evaluation of functional suitability of a device with consideration of its components' aging

By analogy with the above method of evaluating functional suitability of a device based on the analysis of random technological deviations of the components' parameters, we can assess functional suitability, related to random deviations of components' parameters from their nominal values caused by integrated long-term influence of environmental factors. For this purpose, the covariance matrix $D^{-1}(\vec{\delta b})$ of random technological deviations of parameters from their nominal values has be replaced by a covariance matrix of random deviations of parameters caused by influence of environmental factors. However, this approach allows us to assess the functional device suitability based on consideration of components' aging and ignores the random technological deviations of parameters from their nominal values. Moreover, the proposed method is based on the assumption that the random parameters' deviations of the device components' related to aging are normally distributed. As it was shown in numerous publications, such assumption is very often untrue to facts. That is why we propose other approach based on determination of a dependence describing components' aging that is obtained through the analysis of experimental interval data.

As in the case of analysis of functional suitability of a device taking into account random technological deviations of components' parameters occurring during the manufacture process, random deviations of components' parameters due to their aging is described by a confidence ellipsoid [1]. Additionally, the aging of components' parameters occurring in the course of their operation is represented by time-dependent functions of the components' parameters deviations from their nominal values within certain time of the device operation. In such case, functional suitability of the device is evaluated by the analysis of a confidence ellipsoid in the form:

$$Q_s(\alpha, m) = \{\vec{\delta b} \in R | (\vec{\delta b} - \vec{\delta b}(t))^T \times \times D(\vec{\delta b}) \cdot (\vec{\delta b} - \vec{\delta b}(t)) \le \chi^2(\alpha, m)\} \tag{10}$$

where $\vec{\delta b}(t)$ is a vector that comprises time-dependent functions of the deviations of components' parameters from their nominal values, i.e. from the ellipsoid center, as a result their change with time, related to the influence of the environment (aging of the components, temperature drift, etc.).

If the covariance matrix of probable technological deviations of the components' parameters is given, the probability of functional suitability can be obtained from the solution of the task [4]:

$$\chi^2(\alpha, m) \xrightarrow{\alpha \in [0,1]} \max, \; Q_s(\alpha, m) \subseteq \tilde{\Omega}_m, \tag{11}$$

In contrast to the task (2), where the symmetry centers of tolerance region and confidence region coincide, the position of the confidence ellipsoid (1) center is some function that takes into account the deviations of components' parameters due to their aging.

Let us find the solution of the task (11) in general. We convert the confidence ellipsoid (10) to the following form:

$$Q_s(\alpha, m) = \{\vec{\delta b} \in R | (\vec{\delta b} - \vec{\delta b}(t))^T \cdot \frac{1}{\chi^2(\alpha, m)} \times \times D(\vec{\delta b}) \cdot (\vec{\delta b} - \vec{\delta b}(t)) \le 1\} \tag{12}$$

Solution of the task (11) has a comprehensive graphical interpretation. Namely, it is necessary to find such value of confidence probability α, that provides the maximal volume of a confidence ellipsoid inscribed into the tolerance region $\tilde{\Omega}_m$. Since the symmetry centers of tolerance region and confidence region of parameters' scattering do not coincide, the ellipsoid of maximum volume (12) touches at least one (or more) of the nearest edges of the parallelotope $\tilde{\Omega}_m$.

To obtain the solution to the task (11) we use the following algorithm. Firstly, we determine the conditions of touching each of the *m*-parallelotope $\tilde{\Omega}_m$ edges by the ellipsoid (12), and then from the condition $Q_s(\alpha, m) \subseteq \tilde{\Omega}_m$ we compute $\chi^2(\alpha, m)$. It is obvious that the above condition is true for the point of contact that is the nearest to the center of the ellipsoid.

Using the expression (12), we write down expressions of vectors that are normal to the planes tangent to the ellipsoid at the points of contact $\vec{\delta b}^i, i = 1,...,2m$ situated on the parallelotope edges:

(13) $$\vec{n}_i = \frac{1}{\chi^2(\alpha, m)} D(\delta(\vec{b})) \cdot (\delta\vec{b}^i - \delta\vec{b}(t)), i = 1,...,2m.$$

On the other hand, using the system (7), we write down expressions of vectors that are normal to the edges of the ellipsoid that specifies the tolerance region $\widetilde{\Omega}_m$, with concurrent normalization of the distance between the parallelotope center and its edges to a unit distance:

(14) $$\vec{n}_i = \vec{S}_i^T / 0{,}5 \cdot (\delta y_i^+ - \delta y_i^-) = \vec{S}_i^T / \widetilde{\Delta}_i, i = 1,...,2m \text{ ,}$$

where $\vec{S}_i$ is the i-th row of the matrix $S_m = \{S_{ij}, i = \overline{1,m}, j = \overline{1,m}\}$

When we equate both expressions, we obtain the equation for touching each of the parallelotope edges by the ellipsoid (12):

(15) $$\frac{1}{\chi^2(\alpha, m)} D(\delta(\vec{b})) \cdot (\delta\vec{b}^i - \delta\vec{b}(t)) = \vec{S}_i / \widetilde{\Delta}_i, \; i = 1,...,2m.$$

Let us denote by $\chi^2(\alpha_i, m)$ the value of distribution quantile that ensures the fulfillment of the condition (15) for i-th edge. Then the equation (15) looks like:

(16) $$\frac{1}{\chi^2(\alpha_i, m)} D(\delta(\vec{b})) \cdot (\delta\vec{b}^i - \delta\vec{b}(t)) = \vec{S}_i^T / \widetilde{\Delta}_i, \; i = 1,...,2m.$$

and the solution of the task (11):

(17) $$\chi^2(\alpha, m) = \min_{i=1,...,2m} \chi^2(\alpha_i, m)$$

Let rewrite the equation (16) by using the expression of an edge point $\delta\vec{b}^i$ of the parallelotope in the form of a linear combination of the edge vertices:

(18) $$\frac{1}{\chi^2(\alpha_i, m)} D(\delta(\vec{b})) \cdot (\sum_{s=1}^{2^{m-1}} \lambda_s^i \cdot \delta\vec{b}_s^i - \delta\vec{b}(t)) = \vec{S}_i^T / \widetilde{\Delta}_i,$$
$$i = 1,...,2m.$$

The coordinates of the vertices $\delta\vec{b}_s$ may be given in the form[1]:

(19) $$\delta\vec{b}_s = S^{-1} \cdot \delta\vec{Y}_s, s = 1,...,2^m.$$

where $\delta\vec{Y}_s$ are vectors comprising positive $\widetilde{\Delta}_i$ or negative $-\widetilde{\Delta}_i$ values of tolerances for the device characteristics.

Substituting for $\delta\vec{b}_s$ in the equation (18) its expression (19), we obtain

(20) $$\frac{1}{\chi^2(\alpha_i, m)} D(\delta(\vec{b})) \times$$
$$\times (S_m^{-1} \cdot (\sum_{s=1}^{2^{m-1}} \lambda_s^i \cdot \delta\vec{Y}_s^i) - \delta\vec{b}(t)) = \vec{S}_i^T / \widetilde{\Delta}_i, \quad i = 1,...,2m.$$

Since the vectors $\delta\vec{Y}_s$ determine the combinations of tolerances for device characteristics, e.g. $\delta\vec{Y}_s = (\widetilde{\Delta}_1, -\widetilde{\Delta}_2, ..., \widetilde{\Delta}_i, ..., -\widetilde{\Delta}_m)^T$, and for fixed $i = 1,...,2m$ they include a common component $\widetilde{\Delta}_i$ or $-\widetilde{\Delta}_i$, the expression (20) can be written in the following form:

(21) $$\frac{1}{\chi^2(\alpha_i, m)} D(\delta(\vec{b})) \times$$
$$\times (S_m^{-1} \cdot \begin{pmatrix} \sum_{s=1}^{2^{m-1}} \lambda_s^i \cdot \pm \widetilde{\Delta}_1 \\ \vdots \\ \widetilde{\Delta}_i \\ \vdots \\ \sum_{s=1}^{2^{m-1}} \lambda_s^i \cdot \pm \widetilde{\Delta}_m \end{pmatrix} - \delta\vec{b}(t)) = \vec{S}_i^T / \widetilde{\Delta}_i, \; i = 1,...,m$$

(22) $$\frac{1}{\chi^2(\alpha_i, m)} D(\delta(\vec{b})) \times$$
$$\times (S_m^{-1} \cdot \begin{pmatrix} \sum_{s=1}^{2^{m-1}} \lambda_s^i \cdot \mp \widetilde{\Delta}_1 \\ \vdots \\ -\widetilde{\Delta}_i \\ \vdots \\ \sum_{s=1}^{2^{m-1}} \lambda_s^i \cdot \mp \widetilde{\Delta}_m \end{pmatrix} - \delta\vec{b}(t)) = \vec{S}_i^T / \widetilde{\Delta}_i, \; i = m+1,...,2m.$$

Now we rewrite the equations (21) and (22) in the matrix form:

(23) $$D(\delta(\vec{b})) \times$$
$$\times \left(S_m^{-1} \cdot \begin{pmatrix} \widetilde{\Delta}_1 & \cdots & \Lambda_{1i} \cdot \widetilde{\Delta}_1 & \cdots & \Lambda_{1m} \cdot \widetilde{\Delta}_1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \Lambda_{i1} \cdot \widetilde{\Delta}_i & \cdots & \widetilde{\Delta}_i & \cdots & \Lambda_{im} \cdot \widetilde{\Delta}_i \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \Lambda_{m1} \cdot \widetilde{\Delta}_m & \cdots & \Lambda_{mi} \cdot \widetilde{\Delta}_m & \cdots & \widetilde{\Delta}_m \end{pmatrix} - B_\Delta(t) \right) \times$$
$$\times X^{-1} = S_m \cdot E^{-1}, i = 1,...,m$$

(24) $$D(\delta(\vec{b})) \times$$
$$\times \left(S_m^{-1} \cdot \begin{pmatrix} -\widetilde{\Delta}_1 & \cdots & -\Lambda_{1i} \cdot \widetilde{\Delta}_1 & \cdots & -\Lambda_{1m} \cdot \widetilde{\Delta}_1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ -\Lambda_{i1} \cdot \widetilde{\Delta}_i & \cdots & -\widetilde{\Delta}_i & \cdots & -\Lambda_{im} \cdot \widetilde{\Delta}_i \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ -\Lambda_{m1} \cdot \widetilde{\Delta}_m & \cdots & -\Lambda_{mi} \cdot \widetilde{\Delta}_m & \cdots & -\widetilde{\Delta}_m \end{pmatrix} - B_\Delta(t) \right) \times,$$
$$\times X'^{-1} = S_m \cdot E^{-1}, i = m+1,...2m,$$

where $\Lambda_{ji} = \sum_{s=1}^{2^{m-1}} \lambda_s^i \cdot \frac{\Delta_j^s}{\widetilde{\Delta}_j}$, $\Delta_j^s \in \{-\widetilde{\Delta}_j, \widetilde{\Delta}_j\}$, $j = 1,...,m$;

$X = diag\{\chi^2(\alpha_1, m),...,\chi^2(\alpha_i, m),...,\chi^2(\alpha_m, m), i = 1,...,m\}$ is a diagonal matrix comprising values $\chi^2(\alpha_i, m)$ of distribution quantile that ensures the fulfillment of the condition (15) for i-th edge ($i = 1,...,m$);

$X' = diag\{\chi^2(\alpha_{m+1}, m),..,\chi^2(\alpha_{2m}, m), i = m+1,..,2m\}$ is a diagonal matrix of values $\chi^2(\alpha_i, m)$ of distribution quantile that ensures the fulfillment of the condition (15) for i-th edge ($i = m+1,...,2m$);

$B_\Delta(t) = (\delta\vec{b}(t) \;\; \cdots \;\; \delta\vec{b}(t))$ is a $m \times m$-matrix; its columns consist of time-dependent functions of deviations of components' parameters from their nominal values.

Using the denotation

$$\Lambda = \begin{pmatrix} 1 & \cdots & \Lambda_{1i} & \cdots & \Lambda_{1m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \Lambda_{i1} & \cdots & 1 & \cdots & \Lambda_{im} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \Lambda_{m1} & \cdots & \Lambda_{mi} & \cdots & 1 \end{pmatrix}.$$

We can rewrite the system (23, 24) in the following form:

(25) $D(\delta(\vec{b}))\cdot\left(S_m^{-1}\cdot E\cdot\Lambda - B_\Delta(t)\right)\cdot X^{-1} = S_m\cdot E^{-1}$, $i = 1,...,m$

(26) $D(\delta(\vec{b}))\cdot\left(-S_m^{-1}\cdot E\cdot\Lambda - B_\Delta(t)\right)\cdot X'^{-1} = S_m\cdot E^{-1}$, $i = m+1,...,2m$

If we solve the system (25) and (26) with respect to the diagonal matrices X and X'

(27) $X = E\cdot S_m^{-1}\cdot D(\delta(\vec{b}))\cdot\left(S_m^{-1}\cdot E\cdot\Lambda - B_\Delta(t)\right)$, $i = 1,...,m$,

(28) $X' = E\cdot S_m^{-1}\cdot D(\delta(\vec{b}))\cdot\left(-S_m^{-1}\cdot E\cdot\Lambda - B_\Delta(t)\right)$, $i = m+1,...,2m$,

We can obtain the solution of the task (11) applying the formula (17), where $\chi^2(\alpha_i, m)$ are diagonal elements of the matrices X and X'.

Application example

For better comprehension of the proposed approach let us demonstrate the steps of our algorithm for functional suitability evaluation using a simple example.

We consider a low-pass filter (Fig.1) with elements connected in series. Functional suitability of the filter is satisfactory if its most important characteristic - voltage gain module – deviates no more than ±15% from its nominal value calculated for the nominal value of its capacitance $C_0 = 0{,}5$μF, for the nominal value of its resistance $R_0 = 0{,}5$kOhm and for two nominal values of frequency: $f_1 = 1000$ Hz, $f_2 = 2000$ Hz [2].

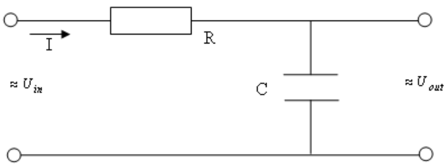

Fig.1. A circuit of the low-pass filter.

We use two parameters (m =2) - the logarithmic values of capacitance and resistance, i.e. $\vec{b} = (b_1, b_2) = (\ln R, \ln C)$; $\delta\vec{b} = (\delta b_1, \delta b_2) = (\ln R - \ln R_0, \ln C - \ln C_0) \approx$. $\approx (\delta R / R_0, \delta C / C_0)$. The covariance matrix of probable technological deviations of the parameters from their nominal values is defined as:

(29) $$D^{-1}(\delta\vec{b}) = 10^{-2}\cdot\begin{pmatrix} 0{,}0025 & 0{,}55 \\ 0{,}01 & 0{,}0025 \end{pmatrix}.$$

Let us calculate the amplitude-frequency characteristic of the filter. Input voltage:

(30) $\underline{U}_{in} = \underline{I}\cdot\underline{Z} = \underline{I}(R - jX_c) = \underline{I}(R + 1/(j\omega C))$,

where $\omega = 2\pi f$ is angular frequency.

Output voltage:

(31) $$\underline{U}_{out} = \underline{I}/(j\omega C)$$

Voltage gain module

(32) $$K = \frac{U_{out}}{U_{in}} = \left|\frac{1}{R + 1/(j\omega C)}\right| = \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}}$$

We consider two cases (for two given frequencies):

(33) $$K_1 = 1/\sqrt{1 + \omega_1{}^2 C^2 R^2}; \quad K_2 = 1/\sqrt{1 + \omega_2{}^2 C^2 R^2}.$$

For nominal values of resistance and capacitance:

$$K_{01} = 1/\sqrt{1 + \omega_1{}^2 C_0^2 R_0^2}; \quad K_{02} = 1/\sqrt{1 + \omega_2{}^2 C_0^2 R_0^2}.$$

Since the characteristic of voltage gain module is a non-linear function with respect to the filter parameters, we apply its linearization in the neighborhood of the nominal values of the parameters and determine the sensitivity of the filter characteristic to the change of its parameters. Sensitivity functions for $i = 1, 2$.

(34) $$S_{iR} = \frac{\partial K_i}{\partial(\ln R)} = \frac{\partial K_i}{\partial R}\cdot\frac{dR}{d(\ln R)} = \frac{\partial K_i}{\partial R} : \frac{d(\ln R)}{dR} = R\frac{\partial K_i}{\partial R};$$

(35) $$S_{iC} = \frac{\partial K_i}{\partial(\ln C)} = \frac{\partial K_i}{\partial C}\cdot\frac{dC}{d(\ln C)} = \frac{\partial K_i}{\partial C} : \frac{d(\ln C)}{dC} = C\frac{\partial K_i}{\partial C}.$$

After the substitution of the expressions (33) of voltage gain module into (34), (35) we receive

$$S_{iR} = R\frac{\partial K_i}{\partial R} = -\frac{R^2 C^2 \omega_i{}^2}{\sqrt{\left(1 + R^2 C^2 \omega_i{}^2\right)^3}}; S_{iC} = C\frac{\partial K_i}{\partial C} = -\frac{R^2 C^2 \omega_i{}^2}{\sqrt{\left(1 + R^2 C^2 \omega_i{}^2\right)^3}}$$

For nominal values of resistance and capacitance:

$$S_{1R} = S_{1C} = -\frac{R_0^2 C_0^2 \omega_1{}^2}{\sqrt{\left(1 + R_0^2 C_0^2 \omega_1{}^2\right)^3}}; S_{2R} = S_{2C} = -\frac{R_0^2 C_0^2 \omega_2{}^2}{\sqrt{\left(1 + R_0^2 C_0^2 \omega_2{}^2\right)^3}}$$

Upper and lower limits of voltage gain module deviations:

$$\delta K_1^+ = -\delta K_1^- = 0{,}15 K_{10} = 0{,}0806;$$

$$\delta K_2^+ = -\delta K_2^- = 0{,}15 K_{20} = 0{,}0455.$$

The condition of satisfactory functional suitability may be written in the form:

(36) $$\delta K_i^- \le \sum S_{ij}\cdot\delta b_j \le \delta K_i^+, \quad i = 1,2; j = R, C.$$

In the matrix form, the sensitivity function takes the form:

(37) $$S = \begin{pmatrix} S_{1R} & S_{1C} \\ S_{2R} & S_{2C} \end{pmatrix} = \begin{pmatrix} -0{,}3822 & -0{,}3822 \\ -0{,}2755 & -0{,}2755 \end{pmatrix}.$$

Let us calculate the diagonal matrix E by the formula:

(38) $$E = \begin{pmatrix} 0{,}0806 & 0 \\ 0 & 0{,}0455 \end{pmatrix}.$$

When we assess the functional suitability of the filter taking into account the technological deviations of its parameters occurring during the manufacture process, we assume that they are normally distributed. As a result, the center of a corresponding confidence ellipsoid (in our case it converts into an ellipse) represented by (1) coincides with the center of the tolerance region (Fig. 2).

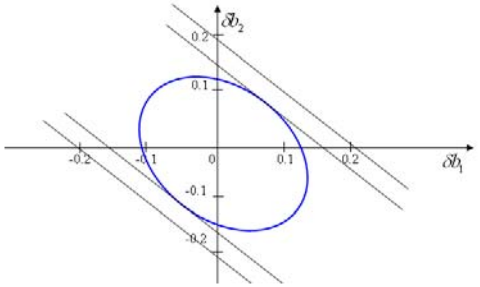

Fig.2. The confidence ellipse for deviations occurring during the manufacture process.

To implement the method of confidence ellipsoids we apply the formulae (8-9) to find the quantile of χ^2-distribution:

(40) $$\Lambda' = \begin{pmatrix} 12{,}409 & 0 \\ 0 & 21{,}969 \end{pmatrix} \cdot \begin{pmatrix} -0{,}3822 & -0{,}3822 \\ -0{,}2755 & -0{,}2755 \end{pmatrix} \times 10^{-4} \times$$
$$\times \begin{pmatrix} 0{,}25 & 55 \\ 1 & 0{,}25 \end{pmatrix} \cdot \begin{pmatrix} -0{,}3822 & -0{,}3822 \\ -0{,}2755 & -0{,}2755 \end{pmatrix} \cdot \begin{pmatrix} 12{,}409 & 0 \\ 0 & 21{,}969 \end{pmatrix}$$

As a result we receive the value of the quantile $\chi^2(\alpha,m) = 4{,}83$ for $m = 2$. Using standard tables for χ^2- distribution, we find the probability of functional suitability $P = 0{,}93$.

As noted above, the formulae (8-9) give an inflated estimate of the probability of functional suitability, because of ignoring the components' aging that reduces the probability of functional suitability. It is, therefore, advisable to apply the evaluation of functional device suitability with regard to components' aging.

An important condition for the evaluation of functional device suitability by the method of confidence ellipsoids taking into account the components' aging is fulfillment of the requirement (16) that stipulates the existence of at least one point of contact between the ellipsoid and the edges (in our case – between the ellipse and the sides of a parallelogram) of the tolerance region.

(41) $$\frac{1}{\chi^2(\alpha,m)} \cdot \begin{pmatrix} -45{,}51 & 10011{,}38 \\ 182{,}03 & -45{,}51 \end{pmatrix} \cdot (\vec{\delta b}^{\,i} - \vec{\delta b}(t)) =$$
$$= \begin{pmatrix} -4{,}74 & -8{,}4 \\ -3{,}41 & -6{,}05 \end{pmatrix}, \; i = 1,\dots,2m.$$

Let the deviations of parameters' values from their nominal values be of 5% for both elements, i.e. $B_\Delta(t) = \begin{pmatrix} 0{,}05 & 0{,}05 \\ 0{,}05 & 0{,}05 \end{pmatrix}$. Applying the formulae (27-28) we can calculate the quantile of χ^2- distribution by the formula (17); as a result we receive $\chi^2(\alpha,m) = 2{,}13$. Using standard tables for χ^2- distribution, we find the probability of functional suitability: $P = 0{,}73$.

In Fig. 3 the confidence ellipse calculated by the formula (1) is shown in red and the confidence ellipse calculated by the formula (10) taking into consideration the components' aging is shown in blue.

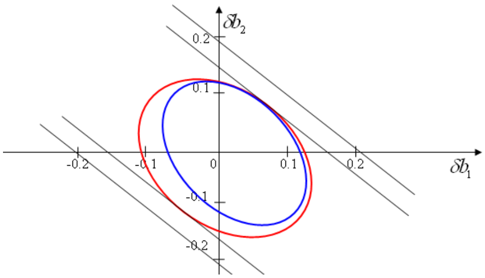

Fig.3. The confidence ellipse for deviations occurring during the manufacture process and the confidence ellipse for additional deviations of 5% occurring due to aging.

Let us consider the case when the deviation (due to aging) of one parameter from its nominal value equals to 5%, and the deviation of the second parameter increases from 2% to 8%. We can see in Fig.4 that the probability of functional filter suitability evaluated by the confidence ellipses decreases with every next period of time of the device operation. In Fig. 4 the confidence ellipse shown in red corresponds to the probability $P = 0{,}86$ of functional suitability of the filter for the parameters' deviations of 5% and 2%; the confidence ellipse shown in green does to the probability $P = 0{,}81$ for the parameters' deviations of 5% and 4%; that shown in purple does to the probability $P = 0{,}65$ for the parameters' deviations of 5% and 6%, that shown in yellow does to the probability $P = 0{,}54$ for the parameters' deviations of 5% and 8%. With increase in parameters' deviations from their nominal values the probability of functional device suitability dramatically decreases what causes the failure of the device.

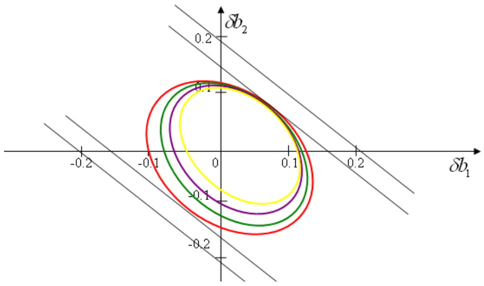

Fig.4. Confidence ellipses corresponding to different deviations of the second parameter due to aging.

Conclusion

The method of evaluation of functional device suitability based on the analysis of random technological deviations of components' parameters occurring during their manufacture process has been discussed for the case of their normal distribution.

It has been pointed out that aging reduces the functional suitability, so it is necessary to develop a method for evaluating functional device suitability with consideration of components' aging.

The authors have proposed a new approach to the evaluation of functional suitability based on determination of a dependence describing components' aging that is obtained through the analysis of experimental interval data.

A specific example of assessment of functional suitability of the low-pass filter has been provided that shows that in the course of the device operation the probability of its functional suitability decreases due to its deterioration (aging).

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