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Adaptive Servo Control with Polynomial Approximation of Stribeck Curve

***Abstract**. We consider a servo dynamics with friction modeled by static, memoryless nonlinear function of velocity, with strong influence of Stribeck effect. We consider approximation of the Stribeck curve by a polynomial. The approximating polynomial coefficients become control parameters modified by the adaptive loop. We formulate the control algorithm and prove its stability by Lyapunov function approach, and finally we describe several numerical experiments demonstrating features of the proposed control and the influence of the parameters.*

***Streszczenie.** Przedstawiony niżej tekst opisuje problem sterowania napędowym układem nadążnym z tarciem, które może być modelowane przez statyczną, nieliniową funkcję prędkości z silnie zaznaczonym efektem Stribeck'a. Badamy możliwość aproksymacji krzywej Stribecka przez wielomian, którego współczynniki będą zmieniane w pętli sterowania adaptacyjnego. Wyprowadzamy algorytm sterowania i formułujemy warunki jego stabilności poprzez użycie odpowiedniej funkcji Lapunowa. Opisujemy szereg eksperymentów, które ilustrują charakterystyczne właściwości układu regulacji i dostarczają wniosków co do wyboru parametrów algorytmu sterowania. (**Adaptacyjne sterowanie nadążnego układu napędowego z wykorzystaniem wielomianowej aproksymacji krzywej Stribeck'a**)*

Keywords: adaptive control, friction compensation, servo control
Słowa kluczowe: sterowanie adaptacyjne, kompensacja tarcia, sterowanie napędem typu servo

doi:10.12915/pe.2014.05.04

Introduction

Friction is the most common and most destructive disturbance affecting performance of precision motion control systems, especially linear or rotational servo drives. It may cause oscillations, decrease steady state accuracy and deteriorate tracking performance. The friction phenomenon is rather complicated and not yet completely understood, so several friction models are reported in the literature [1]. Usually the experimental information about friction is corrupted by noise and outliers and inaccurate. It was reported [2], that using a fuzzy model of friction connected with adaptation of its parameters is an effective approach for friction compensation, even if dynamical friction model is/would be more adequate for the examined case.

In this contribution we consider a servo dynamics with friction modeled by static, memoryless nonlinear function of velocity, with strong influence of Stribeck effect. The aim of friction modeling is to use the model in an adaptive control scheme, so it is important to construct linearly parameterized model. We propose to approximate the Stribeck curve by a polynomial. The approximating polynomial coefficients will become control parameters modified by the adaptive loop. We check if satisfactory results may be obtained if the real friction is described by LuGre model [3] with the same steady state characteristics.

We start with describing the plant model and the control aims, next we formulate the control algorithm and prove its stability by Lyapunov function approach, and finally we describe several numerical experiments demonstrating features of the proposed control and the influence of the parameters.

Plant model and control objectives

We consider a linear servo described by

$$\frac{d}{dt}x = v \tag{1}$$

$$m\frac{d}{dt}v = c \cdot i - F_f \tag{2}$$

where v is the forcer velocity, m – the forcer mass, c represents the coefficient converting the motor current i into the thrust force, and F_f is an external load – mainly a friction force. The motor current i is supplied by a PWM inverter working in a current control mode and we assume the this loop is much faster than mechanical dynamics, so we consider i as a control input. We assume that the parameters $m, c,$ are unknown, constant or slowly varying. The friction force is modeled as nonlinear function of speed

$$F_f = \left[f_c + (f_s - f_c)g(v)\right]sign(v) + Bv, \quad g(v) = e^{-\left(\frac{|v|}{v_s}\right)^{\delta}}, \tag{3}$$

where f_s is the level of static friction , f_c is the minimum level of Coulomb friction v_s is the lubricant parameter (so called Stribeck velocity), B - viscous friction parameter and δ is an even constant. The function describing the Stribeck curve $g(v)$ proposed in (3), is only one of the possibilities – several other are reported [1].

Although we have formulated the problem using linear motion equations, the same approach may be used easily for rotational servo.

We will assume that for the control purposes the friction curve (3) will be approximated by

$$P = sign(v)\sum_{i=0}^{N} a_i |v|^i . \tag{4}$$

In (4) we assume that the friction curve is symmetrical, but we may use different approximating polynomials for positive and negative velocities.

The control objective is that the motor position has to follow a smooth reference x_d. We denote the tracking error by

$$e_1 = x_d - x . \tag{5}$$

Control algorithm

We apply adaptive backstepping scheme [4] to design the controller. The velocity will be the 'virtual control' for position tracking. If we choose the desired velocity v_d according to

$$v_d = \dot{x}_d + k_1 \cdot e_1 . \tag{6}$$

where $k_1 > 0_1$ is a design parameter, we will be able to describe the tracking error dynamics as

(7) $$\dot{e}_1 = \dot{x}_d - \dot{x}_d - k_1 \cdot e_1 + e_2 = -k_1 \cdot e_1 + e_2 ,$$

where

(8) $$e_2 := v_d - v = \dot{x}_d + k_1 \cdot e_1 - v$$

denotes the speed tracking error. Dynamics of this error may be described using (8, 7, 2) as

(9) $$m_o \cdot \dot{e}_2 = m_o \cdot \dot{v}_d - m_o \cdot \dot{v} = m_o \cdot \dot{v}_d - i - \frac{1}{c} F_f \ , \quad m_o = \frac{m}{c} .$$

The derivative of the reference speed is given by

(10) $$\dot{v}_d = \ddot{x}_d + k_1 \cdot \left(-k_1 \cdot e_1 + e_2\right),$$

so it is available for control algorithm.

The control variable i has to compensate function

(11) $$D = m_o \cdot \dot{v}_d - \frac{1}{c} F_f$$

We will use a model $\hat{D}$ for D, incorporating friction model (4). The general structure of $\hat{D}$ will be given by

(12) $$\hat{D} = \hat{\boldsymbol{A}}^T \cdot \xi \ .$$

where $\hat{\boldsymbol{A}}^T$ is row of adaptive parameters and ξ is known (regressor) function of appropriate dimension. Because of the structure of (11) and (4) we consider model with *N*+2 parameters

(13) $$\hat{\boldsymbol{A}}^T = \left[\hat{a}_{-1}, \hat{a}_0, \cdots \hat{a}_N\right]$$

corresponding to $\frac{1}{c}\left[m \ , a_0, \cdots a_N\right]$ respectively and

(14) $$\xi = \left[\dot{v}_d , sign(v), sign(v)|v|, \cdots, sign(v)|v|^N\right]^T .$$

We assume that 'good' models exists with bounded parameters $\boldsymbol{A}^*$

(15) $$D^* = \boldsymbol{A}^{*T} \boldsymbol{\xi} \ , \quad \left\|\boldsymbol{A}^*\right\| \le a_{max} ,$$

leading to the bounded value of

(16) $$\varepsilon := D - D^* , \quad |\varepsilon| \le \varepsilon_{max} .$$

We denote the error between "good" and actual adaptive parameters by:

(17) $$\tilde{\boldsymbol{A}} = \boldsymbol{A}^* - \hat{\boldsymbol{A}} .$$

If we choose the control law according to

(18) $$i = \hat{D} + k_2 \cdot e_2 + e_1 ,$$

where k_2 is a positive design parameter, we get the tracking error dynamics

(19) $$m_o \cdot \dot{e}_2 = \varepsilon + \tilde{\mathbf{A}}^T \xi - k_2 \cdot e_2 - e_1 .$$

We verify the system stability by Lyapunov function

(20) $$V = \frac{1}{2}\left(e_1^{\,2} + m_o \cdot e_2^{\,2} + \tilde{\boldsymbol{A}}^T \Gamma^{-1} \tilde{\boldsymbol{A}}\right)$$

with positive definite matrix Γ. Calculation of the system derivative confirms, that under adaptive laws:

(21) $$\dot{\hat{\boldsymbol{A}}} = e_2 \Gamma \xi \ ,$$

or

(22) $$\dot{\hat{\boldsymbol{A}}} = e_2 \Gamma \xi - \sigma \sqrt{e_1^{\,2} + e_2^{\,2}} \, \Gamma \hat{\boldsymbol{A}}, \quad \sigma > 0$$

we are able to prove that the system derivative of (20) is negative outside a certain, bounded set, and so e_1, e_2 are uniformly ultimately bounded. For example with adaptation performed according to (21) we get

(23) $$\begin{aligned} \dot{V} &= -k_1 \cdot e_1^{\,2} + -k_2 \cdot e_2^{\,2} + e_2 \cdot \varepsilon \\ &\le -k_1 e_1^{\,2} - \left(k_2 - \frac{1}{2}\right) e_2^{\,2} + \frac{1}{2} \varepsilon^2 \end{aligned}$$

and it is negative outside the set

(24) $$e_1^{\,2} + e_2^{\,2} < \frac{1}{k} \varepsilon_{max}^{\,2} , \quad k = \min\left(k_1, k_2 - \frac{1}{2}\right),$$

so the system describing the error dynamics (7,19) is uniformly ultimately bounded (UUB) and we are able to modify the diameter of the bound by proper choice of k_1, k_2. Similarly taking adaptive law (22) we get that

(25) $$\begin{aligned} \dot{V} &= -k_1 \cdot e_1^{\,2} + -k_2 \cdot e_2^{\,2} + e_2 \cdot \varepsilon + \tilde{A}^T \sigma \|e\| \hat{A} \\ &\le -\|e\| \left[k_{min}\|e\| - \varepsilon_{max} - \sigma \left\|\tilde{A}\right\| \left(a_{max} - \left\|\tilde{A}\right\|\right)\right] \end{aligned} ,$$

where $k_{min} = min(k_1, k_2)$, so we are able to prove that both errors e and $\tilde{A}$ are UUB.

Numerical experiments

The data for friction modeling may be collected during several on-line experiments. If we can measure or estimate the external force, calculate the thrust force (from measurement of motor currents for example), we are able to estimate the friction force. Sporadically it is possible to apply a constant external force (from an another drive, or from a gravitational load), while the thrust force is zero. In this case we may try to tune the friction model parameters by curve fitting comparing measured position history with numerical solution of the motion equation. Another possibility is to use simple observers [2].

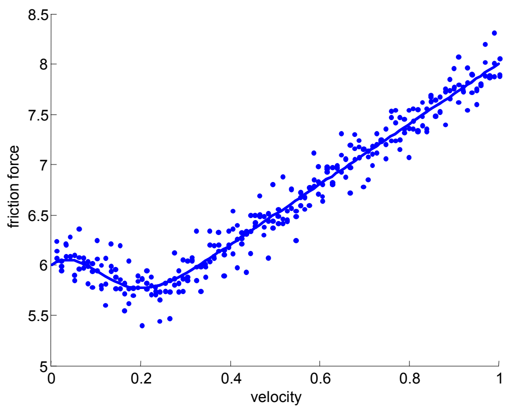

Fig.1. Stribeck curve ([N] versus [m/s]) and the data corrupted by measurement noise

Despite the method we choose, the obtained data will be corrupted by estimation error and subject to estimation/measurement noise and outliers. In such circumstances the accuracy of polynomial approximation will be limited and the usage of high order polynomials is not justified.

In figure 1 we show Stribeck curve with parameters f_c = 5 N, f_s - f_c =1 N, v_s = 0.15 m/s, B = 3 Ns/m, δ = 2 and the measured data corrupted by a measurement noise uniformly distributed in interval $[-a, a]$. Norm of residual as function of approximating polynomial degree is plotted in fig. 2 for different noise amplitudes a. As we see the choice of polynomial degree N=5 is reasonable and such polynomial P was derived.

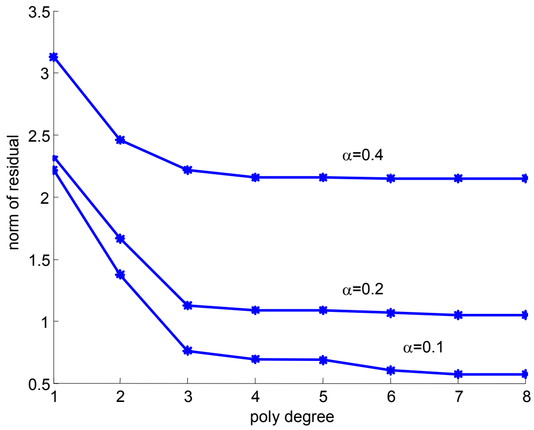

Fig.2. Norm of residual versus approximating polynomial degree for noise amplitude a=0.1,0.2,0.4

The proposed approach was verified by numerous numerical simulations conducted with the motor model parameters m=7.04 kg, c=39N/A. The system was to track sinusoidal position trajectory $x_d(t) = 0.4\,sin(0.8t + 0.25\pi)$ with initial condition $x(0)$=0.

First we demonstrate the system performance under linear control. It is well known that if we apply linear controllers, overestimation of friction results in oscillations while underestimation produces steady state tracking error [5]. In fig. 3 we illustrate the tracking error with PD controller in case of 'almost accurate' compensation, i.e. compensation by polynomial P, overcompensation (by polynomial 1.3P) and undercompensation (by polynomial 0.7P).

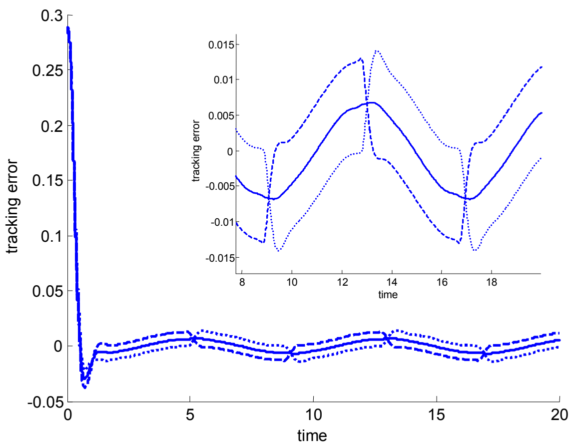

Fig.3. Position tracking error [m]: friction compensated (*solid*), overcompensated (*dashed*), undercompensated (*dotted*)

In fig. 4-6 we illustrate the performance of adaptive backstepping controller (18) with two adaptive parameters and adaptation law (22). Starting values of adaptive parameters correspond to approximation of m and c with 20% error. Adaptation was blocked while the current saturation (the saturation level was 1A) was active – as it was recommended in [6]. As we notice the tracking accuracy is excellent. The adaptive gains are bounded and approach the desired values. The control input is bounded.

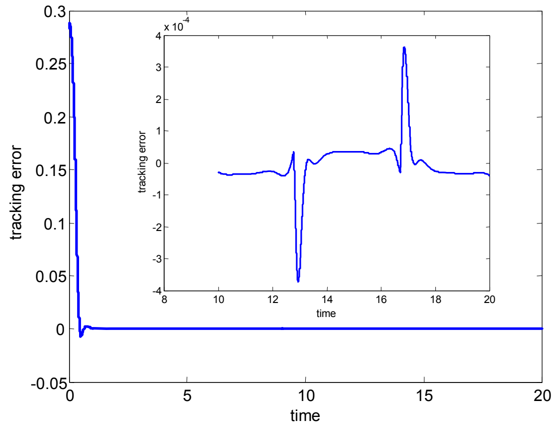

Fig.4. Position tracking error [m]

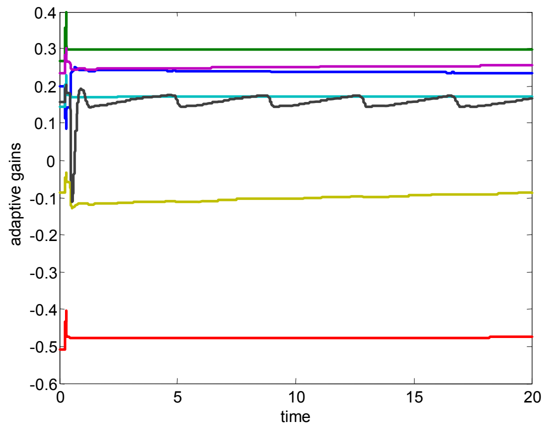

Fig.5. Adaptive gains

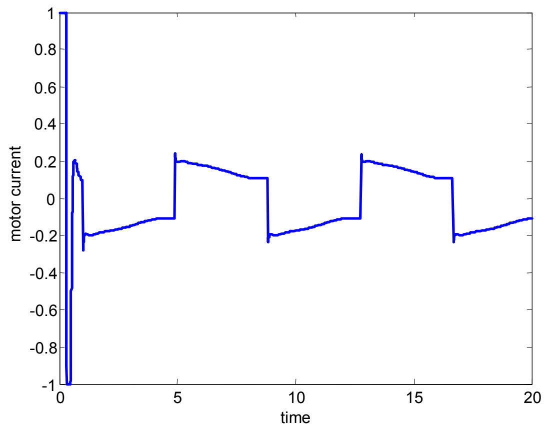

Fig.6. Control input – motor current [A]

Experiments were repeated for a linear servo with friction described by LuGre model [3]:

$$(26)\quad \dot{z}(t) = v - \frac{|v|z}{g(v)}\,,\quad g(v) = \frac{1}{\sigma_0}\left[f_c + (f_s - f_c)e^{-\left(\frac{|v|}{v_s}\right)^{\delta}} \right]$$

(27) $$F_{friction} = \sigma_0 z + \tau \dot{z} + Bv ,$$

where σ_0 is the equivalent stiffness coefficient and τ is the equivalent damping coefficient of bristles.

In the case of high bristle stiffness the control system performance was quite similar to the previous case, as it is illustrated by fig. 7. Lower stiffness results in slight deterioration of tracking accuracy as it is visible in fig.8, and may be improved by parameter tuning.

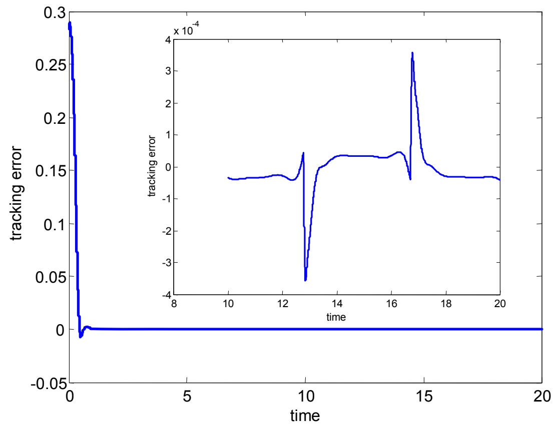

Fig.7. Position tracking error. LuGre friction, high bristle stiffness $\sigma_0=10^4$, $\tau=400$

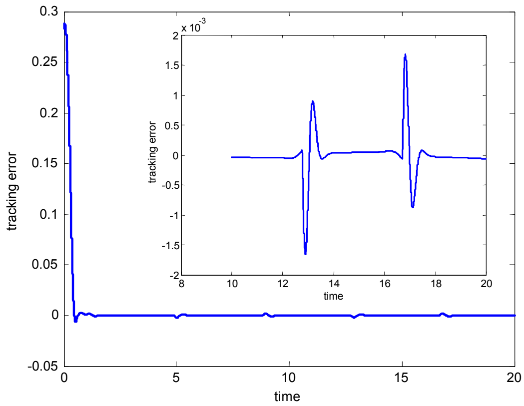

Fig.8. Position tracking error [m]. LuGre friction, low bristle stiffness $\sigma_0=10^3$, $\tau=100$

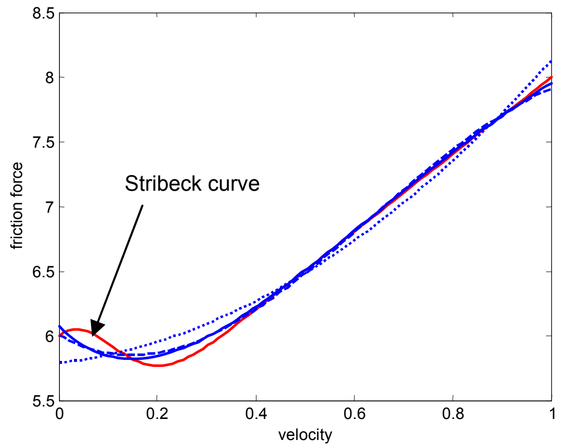

Fig.9. Stribeck curve ([N] versus [m/s]) and approximating polynomials: degree=5 (solid), degree=3 (dashed), degree=2 (dotted)

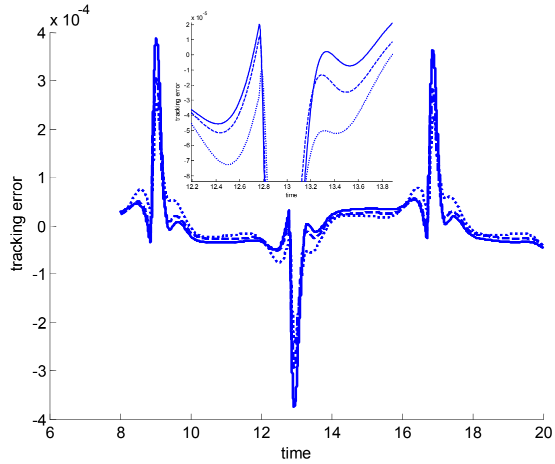

Fig.10. Position tracking error [m] for different polynomials approximating the friction: degree=5 (solid), degree=3 (dashed), degree=2 (dotted)

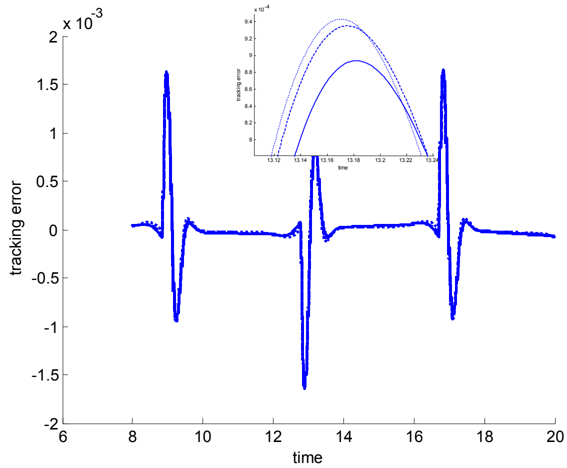

Fig.11. Position tracking error [m] for different polynomials approximating the friction: degree=5 (solid), degree=3 (dashed), degree=2 (dotted), LuGre friction, low bristle stiffness $\sigma_0=10^3$, $\tau=100$

Finally we investigate the influence of the approximating polynomial degree on the control system performance. In fig. 9 we plot the original Stribeck curve and some approximating polynomials. As we may notice none of them exactly approximates the friction, especially in a speed range [0, 0.4] in which the system operates under the proposed excitation. In fig. 10 we see the tracking error history if the friction is modelled be approximating polynomials demonstrated in fig. 9. The tracking error decreases with the polynomial degree increase, but in all cases the adaptive loop is able to compensate the model inaccuracy and to provide satisfactory tracking. So, under the desired trajectory considered, proper tuning of adaptive control parameters is more important than the friction model accuracy. The same thesis may be repeated for a linear servo with friction described by LuGre model. In fig. 11 we demonstrate the tracking error in case of low bristle stiffness and unmodelled current control dynamics with time constant 1 ms.

Conclusions

Numerical experiments have proven that polynomial approximation of friction may be utilized together with adaptive backstepping control to improve electric linear servo performance. The proper choice of control system parameters was not difficult and the derived system stability and convergence was confirmed by simulations. All signals (especially current) remain bounded and acceptable.

The proposed approach is also promising in case when plant friction demonstrates dynamical behavior corresponding to LuGre model features.

The controller was also tested against unmodeled dynamics in the current control loop. It was robust in presence of current control time constant up to several ms.

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