Particle Filter Approach for Permanent Magnet Synchronous Motor State Estimation

Abstract. The paper describes an observer for a permanent magnet synchronous motor based on Particle Filter techniques. Based on introduced theory an observer of shaft position, speed and load torque is proposed. Preliminary research show good properties of proposed estimation structure.

Streszczenie. W artykule przedstawiono obserwację zmiennych stanu silnika synchronicznego o magnesach trwałych przy wykorzystaniu techniki filtracji cząsteczek. W oparciu o przedstawioną teorię zaproponowano obserwator położenia walu, prędkości i momentu obciążenia części mechanicznej napędu. Początkowe badania symulacyjne pokazują dobre właściwości zaproponowanej struktury estymacji.

(Wykorzystanie techniki filtru cząsteczkowego do estymacji stanu silnika synchronicznego o magnesach trwałych)

Keywords: permanent magnet synchronous motor, sensorless control, observers, Particle Filter

Introduction

Filtering is the problem of estimating the states (also parameters or hidden variables) of a system as a set of observations becomes online. To solve this problem necessary is modeling the system and the noises in system and measurements. The resulting models show complex nonlinearities and noise distribution, often real non-Gaussian, sometimes excluding analytical approaches. The right algorithm to solve the problem of estimation state with nonlinearities is nonlinear family of Kalman Filters, include Extended and Unscented [1, 2, 3, 4]. These solutions bases on knowing measurements models:

\[ p(x_t|z_{t-1}, u_t) \]
\[ p(y_t|x_t) \]

where \( x_t \in \mathbb{R}^n \) denotes the states (hidden variables or/and parameters) of the system in time \( t \), and \( y_t \in \mathbb{R}^m \) observations. The states follow a first order Markov chain process and the observations are assumed to be independent given the states. Based on this approach the model can be expressed as follows:

\[ \mathbf{x}_t = f(\mathbf{x}_{t-1}, y_t, u_t) \]
\[ \mathbf{z}_t = h(\mathbf{x}_t, u_t) \]

where \( y_t \in \mathbb{R}^{m_y} \) denotes the input observations, \( \mathbf{u}_t \in \mathbb{R}^{m_u} \) and \( \mathbf{v}_t \in \mathbb{R}^{m_v} \) are noises for process and measurements. In state space approach above equations can be presented as:

\[ \mathbf{x}_t = \mathbf{F}_t(\mathbf{x}_{t-1})\mathbf{x}_{t-1} + \mathbf{B}_t(\mathbf{x}_{t-1})\mathbf{u}_t + \mathbf{w}_{t-1} \]
\[ \mathbf{z}_t = \mathbf{H}_t(\mathbf{x}_t)\mathbf{x}_t + \mathbf{v}_t \]

Mathematical model of PMSM

The general mathematical model of system in statistical approach can be broken into state transition and state measurement:

\[ u_d = R_s i_d + L_d \frac{di_d}{dt} - p\omega_r L_q i_q \]
\[ u_q = R_s i_q + L_d \frac{di_q}{dt} + p\omega_r L_q i_d + p\omega_r \Psi_m \]

where: \( u_d, u_q \) are dq axis voltages, \( i_d, i_q \) are dq axis currents, \( L_d, L_q \) are dq axis inductances, \( R_s \) is stator resistance, and

Mathematical model of PMSM

The general mathematical model of PMSM includes in three main parts: electrical network, electromechanical torque production and third is mechanical subsystem[12]. The stator of PMSM and Induction Motor are similar. The rotor consists permanent magnets, there are a modern rare-earth magnets with high strength.

During investigations some simplified assumptions are made: saturation is neglected, induced electromagnetic force is sinusoidally, eddy currents and hysteresis losses are neglected, no dynamical dependencies in air-gap, no rotor cage. With these assumptions, the rotor oriented dq electrical network equations of PMSM can be described as:

\[ u_d = R_s i_d + L_d \frac{di_d}{dt} - p\omega_r L_q i_q \]
\[ u_q = R_s i_q + L_d \frac{di_q}{dt} + p\omega_r L_q i_d + p\omega_r \Psi_m \]

where: \( u_d, u_q \) are dq axis voltages, \( i_d, i_q \) are dq axis currents, \( L_d, L_q \) are dq axis inductances, \( R_s \) is stator resistance, and
\( \Psi_m \) is magnetic flux produced by permanent magnets placed on rotor.

The value of produced electromagnetic torque is given by equation:

\[
T_e = \frac{3}{2} \cdot p \cdot (\Psi_m - (L_q - L_d) i_d) \cdot i_q ,
\]

where \( p \) is number pole pairs, and fraction \( \frac{3}{2} \) stems from frame conversion: perpendicular stator \( \alpha \beta \) into rotor \( dq \).

Drive dynamics can be described as:

\[
d\gamma = \frac{1}{J} \left( T_e - T_l \right),
\]

where \( T_l \) is load torque and \( J \) is summary moment of inertia of kinematic chain.

Based on (9) and (10) movement equation is:

\[
d\gamma = \frac{1}{J} \left( T_e - T_l \right) ,
\]

Rotor position \( \gamma \) can be described by derivative equation of rotational speed:

\[
d\gamma \frac{dt}{dt} = p \cdot \omega_e .
\]

For this work there are true assumption that load torque \( T_l \) is invariable in a narrow interval:

\[
\frac{dT}{dt} T_l \approx 0 .
\]

Model can be described as classical discrete function model, with sample time \( T_s \):

or as state space model:

System matrix \( F_t \) is:

\[
F_t(\xi_t) = \begin{bmatrix}
1 - T_s \cdot \frac{T_e}{J} & T_s \cdot \omega_e \cdot T_s \cdot \frac{T_e}{J} & 0 & 0 & 0 \\
- T_s \cdot \omega_e & 1 - T_s \cdot \frac{T_e}{J} & - T_s \cdot \frac{T_e}{J} & 0 & 0 \\
0 & T_s & 1 & 0 & 0 \\
0 & 0 & 0 & T_s & 1 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix},
\]

where:

\( T_1 = T_s \cdot \frac{3}{2} \cdot \frac{p}{J} \cdot [\Psi_f - (L_q - L_d) i_d] \)

Output matrix \( H_t \) is a rotating matrix – Clark/Parck transformation:

\[
H_t(\xi_t) = \begin{bmatrix}
\cos \gamma & - \sin \gamma & 0 & 0 \\
\sin \gamma & \cos \gamma & 0 & 0
\end{bmatrix},
\]

and matrix \( B_t \):

\[
B_t(\xi_t) = \begin{bmatrix}
T_s \cdot \frac{1}{T_e} \cos \gamma & T_s \cdot \frac{1}{T_e} \sin \gamma \\
- T_s \cdot \frac{1}{T_e} \sin \gamma & T_s \cdot \frac{1}{T_e} \cos \gamma \\
0 & 0 & 0 & 0
\end{bmatrix}.
\]

### Particle filtering

The main assumption of Particle Filtering find sources in Monte Carlo simulation and importance sampling\[22, 23, 24, 8, 9]\. In Monte Carlo simulation, set of weighted particles, obtain from posteriori distribution, can be used to map integrals to discrete sum, the posteriori can be approximated as:

\[
\hat{p}(\xi_{t|t-1}) = \frac{1}{N} \sum_{i=1}^{N} \delta (\xi_{i|t}^{(i)} - \xi_{t|t})
\]

where the random samples \( \{\xi_{0|0}; i = 1...N\} \), are computed from posteriori distribution and \( \delta (dx) \) denotes the Dirac delta function. So expectations of the form:

\[
\mathbb{E}(g_t(\xi_{t|t})) = \int g_t(\xi_{t|t}) p(\xi_{t|t}|\xi_{t-1|t-1}) d\xi_{t|t}
\]

may be approximated by mean:

\[
\mathbb{E}(g_t(\xi_{t|t})) = \frac{1}{N} \sum_{i=1}^{N} g_t(\xi_{t|t}^{(i)})
\]

In order to compute a sequential estimate of posteriori distribution at time \( t \) without modifying the previously simulated states \( \xi_{0|0-1} \), proposal distributions of the following form can be used:

\[
q(\xi_{t|t}|\xi_{t-1|t-1}) = q(\xi_{t|t}|\xi_{t-1|t-1}) q(\xi_{t|t}|\xi_{t-1|t-1}),
\]

the ate the making the assumption that the current state is not depend on the future observations (filtering only without smoothing). Under the assumptions that the states correspond to Markov Chain and that the observations are conditionally independent given the states:

\[
p(\xi_{0|0}) = \prod_{j=1}^{t} (y_j|\xi_{j-1|j}) ,
\]

and outputs:

\[
p(\xi_{t|t}|\xi_{t-1|t-1}) = \prod_{j=1}^{t} p(\xi_j|\xi_{j-1}) .
\]

Substituting equations 21 and 22, and Bayesian importance weights\[9, 24\]:

\[
w_j = \frac{p(\xi_{t-1}^j)p(\xi_{t|t}|\xi_{t-1}^j)}{q(\xi_{t|t}|\xi_{t-1}^j)}
\]

can obtain:

\[
w_t = w_{t-1} \cdot \frac{p(\xi_{t|t}|\xi_{t-1}^j)}{q(\xi_{t|t}|\xi_{t-1}^j)}
\]

Proposed in equation 24 mechanism to sequentially update the importance weights give an appropriate choice of proposal distributions \( q(\xi_{t|t-1}|\xi_{t-1}^j) \). This form of distribution is critical design issue. This procedure, known as sequential importance sampling (SIS), allows to obtain estimates importance weights \( w_t \) simpler for every particle as:

\[
w_{t,i} = p(\xi_{t|t-1}^i) w_{t-1,i}
\]
Recombining complementary pair 29 into:

\[ A(x_t \rightarrow x'_t) = p(x'_t) \frac{g(x'_t \rightarrow x_t)}{p(x_t) g(x_t \rightarrow x'_t)}, \]

it is possible to define Metropolis choice by:

\[ A(x_t \rightarrow x'_t) = \min \left( 1, \frac{p(x'_t) g(x'_t \rightarrow x_t)}{p(x_t) g(x_t \rightarrow x'_t)} \right). \]

If above minimal is fulfilled new acceptance probability is valid, otherwise remains.

**Validation of proposed estimation scheme**

Open-loop analysis of Particle Filter for PMSM drive was performed on simulated drive system of rated power equal 1.35 kW working with vector control sensor strategy. An overview scheme of the proposed PMSM sensorless speed control is shown in figure 1, with the classic subordinate speed and torque control [11]. The complete drive control system, which is used in this approach, includes: PWM IGBT inverter, current sensors and converters, and elements of processor code. There are PI controllers of current and speed, frame reference transformations modules \((dq/ab, ab/dq, abc/ab)\) and Unscented Kalman Filter are fully implemented in DSP.

The PI speed controller feeds current \(i_q^*\) in \(q\) axis in order to keep Field Oriented Control. The demanded current is computed by using the difference between requested speed \(\omega_r^*\) and speed \(\dot{\omega}_r\) estimated in Kalman filter. Motor working does not require the field weakening, as assumed. Therefore desired current in \(d\) axis is maintained to zero \(i_d^* = 0\). These signals are inputs of PI current controller, which provides desired voltages in \(dq\) reference frame. Basing on the shaft position \(\hat{\gamma}\), voltages are converted into the stationary two axis frame \(\alpha/\beta\), and control PWM inverter. The main component is Particle Filter that provides estimated signals of angular position \(\hat{\gamma}\), speed \(\dot{\omega}_r\), and load torque \(\hat{T}_l\).

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**Fig. 1. Drive control scheme with Particle Filter**

with necessity of normalize to \(\sum w_{t,i} = 1\) by:

\[ w_{t,i} = \frac{w_{t,i}}{\sum_{i=1}^{N} w_{t,i}}. \]

In case that weight \(w_{t,i}\) is bigger then assumed there is replaced by neighboring \(w_{t,i-1} - \) resampled.

Other important way to obtain importance weights is Residual Resampling [8, 9, 24] as an efficient means to decrease the variance due to resampling. In this approach for every particle \(i\) in every time \(t\) is decomposed:

\[ N_{t,i} = \lfloor nw_{t,i} \rfloor + N_{t,i}, \]

where \(\lfloor \rfloor\) denotes integer part and set of \(N_{t,1:N}\) according to cumulative distribution function (CDF) up into \(N\) components. Important weights can be determined:

\[ w_{t,i} = \frac{nw_{t,i} - \lfloor nw_{t,i} \rfloor}{n - \sum_{i=1}^{N} \lfloor nw_{t,i} \rfloor}. \]

During resampling every weights \(w_{t,i}\) are maintained.

The purpose of the Metropolis–Hastings algorithm [22, 21], also used in proposed solution, is to generate a collection of states according to a desired distribution \(p(x_t)\). To accomplish this, the algorithm uses a Markov process which, under certain conditions, asymptotically reaches a unique stationary distribution \(\pi(x_t)\).

A Markov process is uniquely defined by its transition probabilities, the probability \(P(x \rightarrow x')\) of transitioning between any two of its states \(x\) to new \(x'\). It is possible to say that:

\[ \pi(x_t)p(x_t \rightarrow x') = \pi(x')p(x' \rightarrow x_t). \]

Introducing the concept of acceptance distribution \(A(x_t \rightarrow x'_t)\), and proposal distributions \(g(x_t \rightarrow x'_t)\) the conditional probability to accept the proposed state \(x'_t\) given \(x_t\):

\[ P(x_t \rightarrow x'_t) = g(x_t \rightarrow x'_t)A(x_t \rightarrow x'_t). \]
The research proof was performed on simulated setup with PMSM on parameters:

- nominal speed 3000 rpm,
- nominal still load torque 4.3 Nm,
- max load torque 11.7 Nm,
- moment of inertia 2.5 kg · cm²
- still current 2.45 A,
- electromechanical constant 1.6 Nm / V,
- EMF constant 98 V / 1000rpm.

As a result of the combination of two motor the moment of inertia reduced to the motor shaft has increased the value of \( J = 35,36 \) kg · cm².

During particles generation assume noise covariances \( Q \) and \( R \) in accordance with the rules for the expected normal Gaussian distribution for \( v_{1,t} \) and \( n_{1,t}^i \):

\[
Q = \text{diag} \{ q_1, q_2, q_3, q_4, q_5 \},
\]

\[
R = \text{diag} \{ r_1, r_2 \},
\]

where \( q_1 \) and \( q_2 \) are covariances of motor currents (\( d \) and \( q \) axis) and \( q_3 \) - speed \( \omega_r \), \( q_4 \) - angular position \( \gamma \), and \( q_5 \) means in as maximum possible change of of \( T_l \) during one computation period \( T_c \). Coefficients \( r_1 \) and \( r_2 \) are variances of measurements motor currents \( i_d \) and \( i_q \).

Presented results bases on 100 particles.

The investigations presented below are in some sense introduction confirming the use of Particle Filter to observe the electrical and mechanical variables of the drive. There can provide the possibility of using these variables to the full sensorless control.

The entire experiment presented on figure 2 during which the observer was tested for 100 particles, sequential importance sampling, in motor vector sensor control strategy. Main stages of experiment consist reference speed \( \omega_r^* \) changing:

- freely start up to desired speed \( \omega_r = 1000 \text{ obr/min} \), what is \( \frac{1}{10} \) nominal speed,
- reverse of established speed \( \omega_r = 1000 \text{ obr/min} \) into \( \omega_r = -1000 \text{ obr/min} \),
- breaking to steady of shaft.

During preliminary investigation examined a few types of resamplig methods, there are sequential importance sampling and residual resampling. Details are presented in the theoretical part above. Other examined algorithm was with additional Metropolis-Hasting step.

Table 1 presented RMS errors for state variable estimation with same condition for presented simulated system and proposed model of particles. For examined cases the better solution is Particle Filter with sequential importance sampling and supported by Metropolis-Hasting step.

**Summary**

This paper presents the design and simulation verification observer based on Particle Filter for state estimation of non-linear object, which is a Permanent Magnet Synchronous Motor. The estimated variables can be used in a subordi-
Table 1. Average RMS error for state variable estimation

<table>
<thead>
<tr>
<th>Variable</th>
<th>RMS (PF)</th>
<th>SIS (PF)</th>
<th>RMS (PF-MH)</th>
<th>SIS (PF-MH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_d )</td>
<td>0.376</td>
<td>0.375</td>
<td>0.376</td>
<td>0.341</td>
</tr>
<tr>
<td>( I_q )</td>
<td>0.1097</td>
<td>0.105</td>
<td>0.115</td>
<td>0.085</td>
</tr>
<tr>
<td>( \omega_c )</td>
<td>0.847</td>
<td>0.718</td>
<td>0.8616</td>
<td>0.636</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.057</td>
<td>0.057</td>
<td>0.058</td>
<td>0.054</td>
</tr>
<tr>
<td>( f_i )</td>
<td>0.357</td>
<td>0.3263</td>
<td>0.370</td>
<td>0.285</td>
</tr>
</tbody>
</table>

Worth noting on the basis of preliminary simulation tests that errors play strictly depend on the tuning filter. Such a system is an interesting object of research, and also can find its use in industrial applications.

REFERENCES


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