A plug-in direct particle swarm repetitive controller for a single-phase inverter

Abstract. The paper presents an online particle swarm optimizer (PSO) as an iterative learning controller for the single phase inverter with an output LC filter. The novelty of the solution lies in the fact that the swarm directly stores samples of the control signal. The swarm optimizes, according to a user-defined performance index, in online mode the control signal to reject the repetitive disturbance (the load current drawn, for example, by the diode rectifiers). The concept of the direct swarm controller is investigated with the help of numerical simulations.

Introduction. Control schemes that explicitly take into account the repetitiveness of the reference signal and the disturbance are very promising solutions for constant-amplitude constant-frequency (CACF) inverters. Proposed solutions originate from iterative learning control (ILC), repetitive control (RC) and run-to-run (R2R) control techniques. All mentioned techniques can be analyzed in the uniform framework discussed in [1]. The main idea behind this is to analyze a process as two-dimensional. This provides the opportunity to design a separate controller that shapes the behavior along the pass (in the $p$-direction) and a second controller that influences pass to pass dynamics (in the $k$-direction). For a repetitive process a desired control signal is constant in shape at the steady state in pass-to-pass direction. This in turn implies that the very basic P-type control law

$$u(p,k) = u(p,k-1) + K_{RCE}(p,k-1),$$

where $u$ denotes the control signal, $e$ is the control error, $K_{RCE}$ is the controller gain, $k$ is the iteration (pass, trial, cycle) index and $p$ is the time index along the pass ($1 \leq p \leq \alpha$, where $\alpha$ is the pass length), theoretically is able to force the control error to converge toward zero for a very large class of systems. It should be noticed that (1) constitutes an integral action in the $k$-direction. The situation is then equivalent to a non-repetitive process with a constant reference and a constant disturbance. However, it is widely known that (1) is impractical because of long term stability issue and additional filtering is essential to stabilize the system [2, 3, 4, 5, 6, 7, 8]. It should be noted that the long term stability problems were identified numerically and experimentally. There is no clear theoretical insight into the roots of this phenomenon. A low pass filtering prevents the controller from learning at higher frequencies. The cut-off frequency is usually determined experimentally by inspecting the control error frequency spectrum and/or using trial and error method. Moreover, it often happens that the cut-off frequency has to be relatively small, e.g. in [2] one fourth of the Nyquist frequency was necessary to stabilize the repetitive part of the controller. This significantly limits the performance of the controller. One should be also aware that due to lack of theoretical insight into the nature of this problem, it is not sure whether such an aggressive filtering action solves the problem or only shifts it in time expressed in number of passes. The problem could still hold due to lack of filters with ideal attenuation in the stop-band. This in turn implies that the learning for high frequencies is only slowed down instead of being stopped permanently. This could be not critical for an assembly line robotic arm that repeats the task at e.g. 0.1Hz and will make ca. 60 thousand passes during 7-day-long continuous operation. However, it is usually critical for power electronic converters that in the similar time horizon will make ca. 30 million passes. Practical control laws have then the form of

$$u(p,k) = Q(z^{-1})u(p,k-1) + L(z^{-1})e(p,k-1),$$

where $Q$ and $L$ are usually low-pass zero-phase-shift filters. In some designs $Q$ is assumed to be a positive scalar smaller than 1 serving as a forgetting factor. The forgetting is then equally active for all frequencies. It has been shown in [9] that such schemes with the forgetting are equivalent to standard feedback control. On the other hand, aggressive filtering compromises performance and in the case of CACF sine wave inverters results in disturbance rejection similar to multi-oscillatory (also known as multiresonant) control schemes [10, 11]. Multi-oscillatory controllers do not suffer from long term stability problems and as such are one of the best alternatives for high-performance CACF inverters.

There have been successful attempts to accomplish the repetitive control task using universal function approximators, e.g. artificial neural networks (ANN). The goal for any repetitive controller is to construct in an iterative manner a desired control signal along the pass that fulfills assumed performance requirements. This task can be put into the frame of ANN online training. The examples of such solutions for CACF inverters are described in [12, 13]. These solutions impose usually significant computational burden and are very non-straightforward in tuning.

Our idea is to formulate the problem of finding a perfect shape of the control signal along the pass as an online iterative optimization problem. The behavior of a controller at the steady state (in the $p$-direction) would be optimal according to a user-defined cost function. The main goal is to develop a control scheme that does not need low-pass filtering to circumvent long term stability issues. Some inspiration came from predictive control systems. In those systems behavior is shaped solely by the user-defined cost function (the

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Model of the plant

The repetitive controller under consideration does not require modeling of the plant. However, the model of the plant has been used for testing purposes and for designing a feedback/feedforward controller that acts in the $p$-direction. The simplified model of the single-phase PWM inverter with the LC output filter is as follows

$$\frac{dx_i(t)}{dt} = A_i x_i(t) + B_i u(t) + E_i i_{\text{load}}(t),$$

with

$$A_i = \begin{bmatrix} -\frac{R_f}{L_f} & -\frac{k_1}{L_f c_f} & 0 \\ \frac{k_1}{L_f c_f} & -\frac{R_f}{L_f} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_i = \begin{bmatrix} \frac{k_2}{L_f} \\ 0 \\ 0 \end{bmatrix},$$

$$E_i = \begin{bmatrix} 0 \\ -\frac{k_3}{c_f} \\ 0 \end{bmatrix},$$

and

$$x_i = \begin{bmatrix} v_i \\ i_i \\ u_i \end{bmatrix}.$$
Therefore, it seems to be more intuitive to use radius respectively. The random numbers (inertia weight), the individuality factor and the social factor, particle and cle stores $N_c$ this paper the distributed in the unit interval. In all experiments described in problem and $D_{q}$ diversity measure has been employed [17] speed update rule (8). In this study a commonly used swarm verging to zero speed. This is done by monitoring the diver-

ing optimum. Two modifications have been implemented to enable for optimization also in a non-stationary environment, which is the case here due to anticipated variations of load current.

The first modification introduces the evaporation constant $\rho$ for the personal fitness value $P_j = J(j^{\text{pbest}})$ stored in each particle’s memory. This forces particles to gradually forget previous best solution. The formula of for-
getting is as follows [16]

$$P_j(i+1) = \begin{cases} \rho P_j(i) & \text{if } J(j, (i+1)) \leq J(j^{\text{pbest}}, i) \\ J(j, (i+1)) & \text{if } J(j, (i+1)) > J(j^{\text{pbest}}, i), \end{cases}$$

where $J(j, (i+1)) = J(J, (i) + 1)$ is the current fitness of the $j$-th particle and $\rho$ has a positive value less than 1 for any positive-definite functional $J$ and an optimization task formulated as the maximization one.

The second modification prevents the swarm from converging to zero speed. This is done by monitoring the diversity of the swarm and introducing the repelling mode into the speed update rule (8). In this study a commonly used swarm diversity measure has been employed [17]

$$D_{dist} = \frac{1}{N_p \sqrt{N_d} \sum_{j=1}^{N_p} \sum_{n=1}^{N_d} (q_{jn} - \bar{q}_n)^2},$$

where $N_p$ is the swarm size, $N_d$ is the dimensionality of the problem and $\bar{q}$ is the average point. Originally this diversity measure uses the length of the longest diagonal in the search space instead of $\sqrt{N_d}$. In the discussed solution the particle stores $N_d$ samples of the normalized into $[-1, 1]$ control signal and often the positive and negative halves of the control signal are symmetric about time axis (a glide reflection). Therefore, it seems to be more intuitive to use radius $\sqrt{N_d}$ of the search space as the normalizing coefficient in (10). If the diversity of the swarm drops below a given threshold, repulsion of particles is activated. In this study simple formula described in [17] has been adopted. The speed update rule (8) has been then modified as follows

$$v_j(i+1) = c_1 v_j(i) + c_2 r_p \delta (q_j^{\text{pbest}} - q_j(i)) +$$

$$+ c_3 r_g \delta (q_j^{\text{gbest}} - q_j(i)) \tag{11}$$

with

$$\delta = \begin{cases} 1 & \text{if } \delta < 0 \wedge D_{dist} > D_{\text{thold}} + h \\ -1 & \text{if } \delta > 0 \wedge D_{dist} < D_{\text{thold}} - h, \end{cases} \tag{12}$$

where $D_{\text{thold}}$ is the diversity threshold and $h$ is the hysteresis width, usually both problem specific.

Both modifications described in this section have been implemented in the developed swarm repetitive controller.

**Swarm repetitive controller**

The main idea behind the online direct optimization is to use the real plant itself as the critic without switching the system into some sort of offline mode. If population based optimization (e.g. PSO) is to be used, there is a need for performing fair comparison of solutions proposed by agents (particles in PSO). As the performance index value $J$ strongly depends on the reference signal and the disturbance, ideally both of them should be identical when rating each agent. Otherwise, no fair comparison would be possible. This condition is easily met in batch repetitive processes that can be, and usually are, reseted after each pass. However, this condition is also roughly met in continuous repetitive processes that cannot be reseted after each pass, as in the case of the discussed inverter. The main objective here is then to verify if it is feasible to organize the control task in the following manner:

a) apply control signal proposed by the agent,

b) calculate fitness value for the agent,

c) repeat a)-b) until all agents are rated (without resetting the physical process),

d) update agents,

e) repeat a)-d) until the system is in operation (without reset-

ting the physical process).

In this study, the PSO has been chosen to perform d). Each particle is a vector of $\alpha$ consecutive samples of the control signal along the pass

$$q = [u_{PSO(1)}, u_{PSO(2)}, u_{PSO(3)}, \ldots, u_{PSO(\alpha)}].$$
Numerical experiment results

The dynamical behavior of the plant before plugging in the swarm controller is illustrated in Fig. 3. All tests presented here involve a load current imposed using the controlled current source. This makes test conditions even more challenging, e.g., a peak value of the load current does not decrease due to distorted voltage in transient states. Next the swarm repetitive controller has been plugged in and tested without any assisting controller in the $p$-direction. Such test conditions are demanding due to a very low natural damping in the discussed plant. The test scenario is as follows:

a) the swarm is initialized with near zero control signal, 

$$J (k) = \sum_{p=1}^{N_p} \left( u_C^{\text{ref}} (p) - u_C^{\text{m}} (p, k) \right)^2 + \alpha \left( u_{\text{PSO}} (p, k) - u_{\text{PSO}} (p-1, k) \right)^2,$$

(14)

penalty for control error

penalty for control signal dynamics

where $\beta$ is the subjective penalty factor. The reciprocal in (14) is introduced just to turn the optimization problem into the maximization one. The term with $\beta$ that penalizes for control signal dynamics has been introduced to make sure that no overfitting will occur. Lack of such a penalty in the classic ILC scheme allows for oscillation build-up. The classical ILC (1) tends to over fit the control signal, i.e., it tries to find the perfect control signal even if it does not exist due to physical limitations of the plant. If (14) with appropriate $\beta > 0$ is used to rate the solutions, a compromise is worked out by particles between perfect disturbance rejection and control signal dynamics.

Parameters of the swarm controller are given in Table 2 and its schematic diagram is provided in Fig. 2. The reference feedforward (RFF) has been kept from the standard open-loop control topology. The RFF introduces unity gain for zero frequency. It should be noted that the RFF is applied once again.

Table 2. Parameters of the swarm controller

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<td>Dimensionality of the problem</td>
<td>$N_d, n_s$</td>
<td>200</td>
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<td>Number of particles</td>
<td>$N_p$</td>
<td>25</td>
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<tr>
<td>Swarm update frequency</td>
<td>$\Delta t^s$</td>
<td>2Hz</td>
</tr>
<tr>
<td>Evaporation constant</td>
<td>$\rho$</td>
<td>0.97</td>
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<td>Diversity threshold</td>
<td>$D_{\text{thres}}$</td>
<td>0.1 - 325 Hz^{-1}</td>
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<td>Diversity hysteresis</td>
<td>$h$</td>
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The evolution of the root mean square error (RMSE) over the pass is demonstrated in Fig. 4. The RMSE is calculated along the pass direction $p$-direction, not pass to pass direction $\text{pass} \rightarrow \text{pass}$. Consequently, its value applies to the voltage waveform seen by the load. The evolution of the output voltage after switching on the nonlinear load is applied once again.

Fig. 3. Open-loop behavior of the plant (only the RFF is implemented)

Particles are rated according to the following performance index

$$J (k) = \sum_{p=1}^{N_p} \left( u_C^{\text{ref}} (p) - u_C^{\text{m}} (p, k) \right)^2 + \alpha \left( u_{\text{PSO}} (p, k) - u_{\text{PSO}} (p-1, k) \right)^2,$$

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The proposed direct swarm controller plugged into the system with the FSF is illustrated in Fig. 8. Quality of the voltage for the swarm near its equilibrium is depicted in Fig. 6. This illustrates the swarm’s ability to cope with a significantly underdamped system. Since the PDPSRC acts only in the $k$-direction, which is typical for any plug-in type repetitive controller, transients in the $p$-direction have to be shaped by a separate controller assumed as being out of the scope of this paper. However, it will be demonstrated that an additional controller acting along the pass can appear to be beneficial also for the performance of the PDPSRC. It has been observed that the swarm is able to find better solution in terms of the RMSE ripples near the equilibrium if the damping is increased. To illustrate this, the full-state feedback (FSF) is added and poles of the plant seen by the PDPSRC are shifted to the left. All design relevant formulas are given, e.g., in [18]. In this particular experiment the closed-loop conjugate poles are designed to increase damping 5 times. The dynamics of the closed-loop system in the $p$-direction is shown in Fig. 7. This of course deteriorates response of the system along the pass in terms of the voltage drop (in comparison to the situation from Fig. 3). The increased damping comes from the increased virtual resistance of the closed-loop system. The swarm controller plugged into such a system is depicted in Fig. 8. The better effectiveness of the control strategy in terms of the RMSE is presented in Fig. 9. It is noticeable that the consecutive passes are more repeatable from its RMSE point of view — the ripples are much lower than the ones in Fig. 4. The tran-
sient behavior of the system after connecting the nonlinear load is shown in Fig. 10. The quality of the voltage at the steady state is given in Fig. 11. Unsurprisingly, the online optimization problem in the $k$-direction is better conditioned if controllers are shaping the control signal in the $y$-direction in such a way that the landscape of the functional (14) is less rugged in $U_{\text{PNO}}[x,\alpha]$ space.

### Stability, exploitation and responsiveness of the swarm

No stability proof for the proposed control system has been elaborated yet. Taking into account the no free lunch theorem for optimization [19] one can conclude that such a proof, if ever derived, would be always objective function specific. By the objective function it is meant here (14) along with the plant and the disturbance which are essential to determine the cost. At the moment, numerical simulations are the only way to verify the stability. Nevertheless, such an approach to stability verification is quite common in online optimization problems, e.g. most predictive control schemes elaborated for power electronic converters are not accompanied by any relevant proof of stability. There is a belief among control practitioners that performance index online optimization based control methods are strong candidates for future control schemes [20].

In the case of the discussed controller the evaporation constant $\rho$ and the diversity threshold $D_{\text{hold}}$ are main parameters that are used to tailor the responsiveness of the swarm to load current shape variations. For comparison purposes the dynamics of the system with faster knowledge evaporation and higher diversity is illustrated in Fig. 12. This clearly sacrifices quality of the output voltage (Fig. 13). It is not uncommon for stand alone power generation units to operate under a moderate load variations for hours. A UPS for an office building can serve as an example. In such an application the fine voltage shaping that requires several minutes to reach new optimum seems to be acceptable.

### Conclusions

The novel swarm based repetitive controller has been developed and tested numerically. Extensive tests indicate that the proposed algorithm does not suffer from long term stability issues — the common phenomenon encountered in the classic iterative learning controllers. An online optimization concept has been employed to iteratively shape the control signal along the pass. The particle swarm optimizer modified for non-stationary environments has been used. The cost function explicitly takes into account the dynamics of the control signal. This intrinsically prevents control signal build-up in the presence of a repetitive disturbance. No low-pass filtering is needed to robustify the control scheme. Thus, the available controller bandwidth can be fully exploited. Only minor modifications, from computation burden point of view, to the standard PSO have been identified as necessary for tracking the moving optimum. It has been illustrated that the PSO technique can be used in online mode to directly shape the control signal for the repetitive process. Obtained results indicate that the computational intelligence techniques of gradientless population-based optimization can be helpful in overcoming limiting issues of the traditional ILC schemes. Experimental verification is planned.

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