

# The interdigital transducer and its response in the case of the impact of the inhomogeneous surface acoustic wave

**Abstract:** In the paper, the influence of the inhomogeneous surface acoustic wave and the tilt and shift of output interdigital transducer on the transmitted power coefficient and efficiency, is analyzed.

**Streszczenie:** W artykule przedstawiono wpływ nierówności rozchodzenia się fali akustycznej i obrotu wyjściowego przetwornika na moc transmisyjną oraz sprawność. Wpływ nierówności rozchodzenia się fali akustycznej i obrotu wyjściowego przetwornika na moc transmisyjną oraz sprawność.

**Keywords:** interdigital transducers, surface acoustic wave, power transmission coefficient, the efficiency of shifted and slanted transducer.

**Słowa kluczowe:** przetwornik interdigital, powierzchniowe fale akustyczne, współczynnik transmisji mocy

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## Introduction

Normally we assume idealized conditions when designing the surface acoustic wave (SAW) filter. These are: acoustic beam radiated by the input IDT has a constant amplitude and phase across of the beam, the amplitude of SAW drops sharply at the edges of the beam and it has a zero value outside of the beam, the beam width is equal to the aperture of the transducer, the output beam exactly overlaps the beam and the electrodes of the transducer are parallel to the wave front. In reality, these conditions are not always met, because the incident SAW is generally inhomogeneous and inhomogeneity of the waves can be caused by apodization of input transducer or low number of electrodes.

Due to design inaccuracies, the output IDT can be shifted and tilted due to the input IDT and the electrodes are not parallel to the wave front. At the same time, diffraction of acoustic beam causes a change in amplitude and phase of SAW across the beam, the beam extends and causes the radiation of energy to the outside of the output IDT. Those phenomena have the effect of additional insertion loss.

This article is aimed to contribute to derive the mathematical relations and to present the experimental results for the power transmission coefficient at the impact of inhomogeneous surface acoustic wave, and at tilt and shift of the output IDT.

## 1. The power transmission coefficient and efficiency

We assume ideal conditions in deriving of the transmitted power coefficient and efficiency. Thus, we assume: homogeneous input IDT, which emits SAW with constant amplitude and phase, and the acoustic wave reaches the acoustic port of a homogeneous output IDT so that the edges of the wave coincide with the edges of the aperture of transducer and electrodes are parallel to the wave front. The given arrangement is an ideal case and it is shown in the fig1.

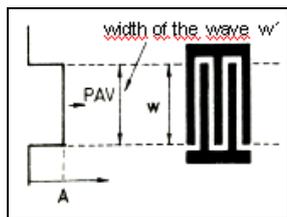


Fig.1. The incidence of SAW on IDT in an ideal case

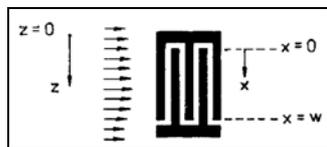


Fig.2. The incidence of inhomogeneous SAW on IDT

It results from the introduced complex transfer function  $T_{13}$  [1], that the amplitude of an electric signal at the load is  $T_{13}$  – th part of the incident SAW amplitude. Electrical matching circuit can be inserted between the output of the transducer and the load [2]. Parameters of this circuit as well as parameters of the IDT are included in the ideal transfer function  $T_{13}^0$ . Then the ideal power transmission coefficient is defined by the relation

$$(1) \quad p_{13}^0 = |T_{13}^0|^2,$$

where  $p_{13}^0$  is a real number.

Since due to the already mentioned reasons, an ideal case in practice never occurs, then in the real case is

$$(2) \quad p_{13}^0 = \eta p_{13}^0,$$

where  $\eta$  is efficiency, which is the function of the parameters describing the type and size of deviations of the front and deviations of IDT geometry from the ideal case.

As regards of the small deviations, the resulting efficiency is the product of partial efficiencies and the previous equation is changed to the following form

$$(3) \quad p_{13}^0 = \prod_i \eta_i p_{13}^0,$$

where  $\eta_i$  are the positive real numbers less than 1.

The total power transmission coefficient from the input electrical port to the output one, is given by the following equation (4), and if  $T_{31}$  and  $T_{13}$  are the transfer functions of the input and output IDT, and the dissipative losses in IDT and in substrate, on which the SAW travels are not considered, this is defined by the relation

$$(4) \quad p_{13}^0 = \eta |T_{31} T_{13}|^2.$$

In the case of an ideal adaptation of IDT applies that,  $T_{31} = T_{13} = 1/\sqrt{2}$  is the power transmission coefficient  $p_{13} = \eta/4$ .

The adaptation and efficiency are independent of one another, the adapted IDT ( $T_{13} = 1/\sqrt{2}$ ) may be ineffective and effective transducer may be not adapted. The low efficiency due to the large inserted loss can not be improved by changing of the electrical circuit, because by this change can be achieved only the improvement of the adaptation.

Let us consider the case shown in fig.2 if an inhomogeneous SAW is incident on the input IDT, where aperture of transducer is labelled  $w$ ,  $x$  is the distance

measured from one edge of the transducer and  $z$  denotes the distance from the zero point optionally positioned, measured parallel to the aperture of IDM. We assume the complex distribution of the amplitude of SAW, given by the relation  $Ae^{-ja}$ , where  $A$  and  $a$  are the real functions of  $z$ . Distribution of power will be  $P = A^2$  and power in the element  $dz$  in the point of  $z$  will be  $P(z)dz = A^2(z)dz$ . The complex amplitude of the wave hitting the element  $dx$  of the IDT aperture will be  $Ae^{-ja}\sqrt{dx}$  in this case.

To calculate the efficiency it is necessary to know the coefficient of transmitted amplitude from the element  $dx$  to the electric port, it is denoted as  $t_{13}$ . Ideally,  $A$  is a constant,  $a$  can be considered zero, and the transfer of amplitude from the element  $dx$  to the electric port is  $A\sqrt{dx}t_{13}$ , the total amplitude of the wave incident on aperture is  $A\sqrt{w}$  and the total transmitted amplitude is  $A\sqrt{w}t_{13}$ . Number of elements  $dx$  across the aperture is  $w/dx$  and total transmitted amplitude is  $At_{13}\sqrt{dx}w/dx = At_{13}w/\sqrt{dx}$  and must be equal to the  $AT_{13}\sqrt{w}$ . Thus, the following applies:

$$(5) \quad t_{13} = T_{13}\sqrt{dx/w}.$$

When the SAW with amplitude  $Ae^{-ja}\sqrt{dx}$  is hitting an element  $dx$ , then in the real case, the amplitude  $t_{13}Ae^{-ja}\sqrt{dx} = T_{13}Ae^{-ja}dx/\sqrt{w}$  is transmitted to the electric port. Then the transmitted amplitude from the whole aperture will be equal to the integral from  $x = 0$  to  $x = w$ , i.e.:

$$(6) \quad \frac{T_{13}}{\sqrt{w}} = \int_0^w Ae^{-ja} dx,$$

transmitted power is

$$(7) \quad P_t = \frac{|T_{13}|^2}{w} \left| \int_0^w Ae^{-ja} dx \right|^2,$$

and total power transmitted by SAW is expressed as:

$$(8) \quad P_0 = \int_{-\infty}^{\infty} A^2 dz.$$

The power transmission coefficient  $p_{13} = P_t/P_0$  will be defined by the following relation:

$$(9) \quad p_{13} = \frac{P_t}{P_0}$$

Basic relationship for the efficiency, which will be subsequently used, we obtain by comparing the equation (2), and (9):

$$(10) \quad p_{13} = \frac{|T_{13}|^2 \left| \int_0^w Ae^{-ja} dx \right|^2}{w \int_{-\infty}^{\infty} A^2 dz},$$

## 2. The efficiency of the transducer at the incidence of surface acoustic wave generated by an apodized transducer

The front of surface acoustic wave that spreads from the apodized transducer with a large number of electrodes

is deformed. The central part of the wavefront, which spreads under multiple electrodes, is delayed regard to the edges, because it spreads more slowly. Between the centre and the edge, we can achieve the phase difference to  $1^\circ$  for each electrode [3]. We will assume that the wave deformed in this way reaches the transducer with several electrodes, and we calculate the power transmission coefficient and we determine the transfer efficiency. To simplify the analysis we choose V-shaped wave front (Fig.3).

A wave progressing from the point  $P$ , reaches the point  $Q$  with certain phase delay  $\varphi$  regard to the junction  $\overline{RS}$ .

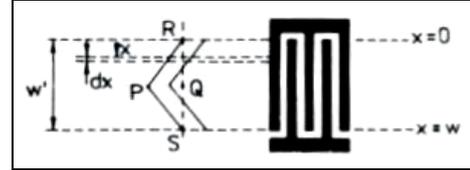


Fig.3. An example of the incidence of SAW radiated from an apodized transducer

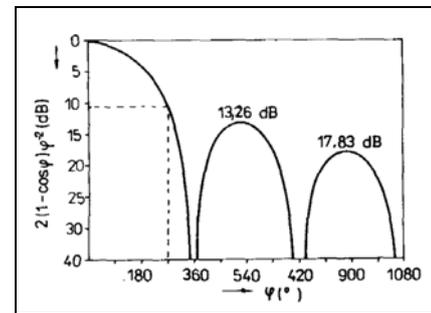


Fig.4. Graphical dependence of  $\cos \varphi$  from  $\varphi$

The complex amplitude at the junction  $\overline{RS}$  in the distance

$x$  from  $R$  is  $Ae^{-ja}$ , where  $A$  is a constant

$$(11) \quad \begin{aligned} a &= 2\varphi x/w & \text{for } 0 < x < w/2, \\ a &= 2\varphi(w-x)/w & \text{for } w/2 < x < w. \end{aligned}$$

By substituting to the equation (9) we get

$$(12) \quad p_{13} = \frac{|T_{13}|^2}{w} \frac{\left| A \int_0^w e^{-ja} dx \right|^2}{A^2 \int_0^w dx},$$

and after modification:

$$(13) \quad p_{13} = \frac{2|T_{13}|^2}{\varphi^2} (1 - \cos \varphi).$$

where the function  $\eta = (1 - \cos \varphi)/\varphi^2$  is shown in fig.4.

### 2.1 The results arising from the experimental partial solution

In the case, that instead of  $w$  aperture we consider the aperture from  $a/2$  to  $-a/2$  (i.e., if the input transducer is narrower than the incident beam), then the power transmission coefficient is reduced by the factor  $(1-a/w)$ . Let us assume, for example,  $a/w = 1/3$ . Then  $1-a/w = 2/3$  and  $p_{13}$  is reduced by factor of  $2/3$  (or 1,76 dB).

Let us assume further that for the transducer with the full aperture the phase  $\varphi = 3\pi/2$  can be realized by the converter with 250 to 300 electrodes. If the width of the transducer is reduced by the factor  $1/3$ , then  $\varphi$  is reduced by the factor  $1/3$ , and fig.4 shows that the attenuation caused by the phase-change will decrease from 10,45 dB to 3,92 dB, with the gain of 6,53 dB.

It follows from the foregoing that the power transmission coefficient can be increased (in the case of wave with variable phase) by reducing of the aperture of the transducer. By reducing of the aperture in the case of SAW with constant phase, understandably, the increase of  $p_{13}$  cannot be reached. To remove of the wavefront deformation caused by apodization, the passive electrodes are used to align the wavefront [4].

### 3. The influence of the ripple of amplitude and phase of an incident wave at the efficiency of transducer

In the case of the delay lines or filters with two IDT, having a small number of electrode pairs (i.e. 5-10), the beam incident on the output transducer is distinguished from the beam in the case of an ideal impact so that the ripple of amplitude and phase occurs also across the beam, and the amplitude at the edges of the beam does not fall sharply to zero, but rather linearly within the large beam width. Some energy is located outside the main beam, in the lateral lobes [6], [7].

For the calculation of a typical value of efficiency in this case we will assume the idealized shape of distribution of amplitude and phase as shown in fig.5.

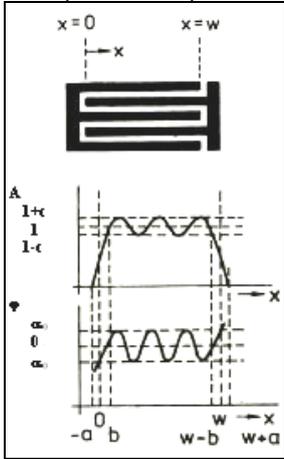


Fig. 5. The graph of amplitude and phase of SAW that is incident on an IDT with the aperture  $w$

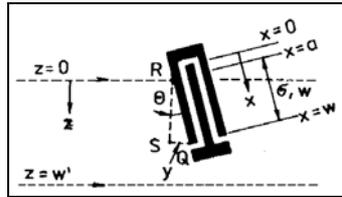


Fig. 6. The graph of amplitude and phase of SAW that is incident on an IDT with the aperture  $w$

It is evident from the picture that the transducer aperture is within the limits of  $x = 0$  and  $x = w$  and the amplitude of the beam increases linearly from the value of 0 at  $x = -a$  to the value of 1 at  $x = b$  and on the other side, it symmetrically decreases from the value of 1 at  $w - b$  to the value of 0 at  $w + a$ . In the section from  $x = b$  to  $x = w - b$  the amplitude has the sinusoidal ripple with the half periods of  $p$  and amplitudes of  $c$ . The course of the phase is a cosine wave with the amplitude  $a_0$  in the range from  $x = 0$  to  $x = w$ . Outside the area  $x = -a$  and  $x = w + a$  there are the side lobes, which contain from 2 to 3% of the total energy, and this contributes to the attenuation by the value of about 0,1 dB.

The complex amplitude of an incident beam can be expressed by the following relations:

$$(14) A e^{-ja} = \left( \frac{x+a}{b+a} \right) e^{-ja_0 \cos \left[ \frac{p\pi(x-b)}{w-2b} \right]}, \quad -a < x < b$$

$$(15) A e^{-ja} = \left\{ 1 + c \sin \left[ \frac{p\pi(x-b)}{w-2b} \right] \right\} e^{-ja_0 \cos \left[ \frac{p\pi(x-b)}{w-2b} \right]}, \quad b < x < (w-b)$$

$$(16) A e^{-ja} = \left( \frac{w+a-x}{b+a} \right) e^{-ja_0 \cos \left[ \frac{p\pi(x-b)}{w-2b} \right]}, \quad (w-b) < x < (w+a)$$

For the values  $a_0 < 0,25$  rad, the following approximation can be used:

$$(17) e^{-ja_0 \cos \left[ \frac{p\pi(x-b)}{w-2b} \right]} = 1 - ja_0 \cos \left[ \frac{p\pi(x-b)}{w-2b} \right].$$

By the application of that approximation and by neglecting of members, in which occurs the product  $ja_0$  and after substituting into equation (14), (15), (16) into the equation (10) we obtain the following relationship.

$$(18) \eta = \frac{\left( (a+b)w - b^2 - \frac{2ja_0}{p\pi}(w-2b)\sin\psi - \frac{2ja_0}{p^2\pi^2}(w-2b)^2(1-\cos\psi) \right)^2}{w(a+b)^2 \left[ w + \frac{2}{3}a - \frac{4}{3}b \right]}$$

which is valid for the even  $p$ , where  $\psi = p\pi b/(w-2b)$  and the relation for the odd  $p$ :

$$(19) \eta = \frac{\left( (a+b)w - b^2 + \frac{2c}{p\pi}(a+b)(w-2b) \right)^2}{w(a+b)^2 \left[ w + \frac{2}{3}a - \frac{4}{3}b + \frac{4c}{p\pi}(w-2b) \right]}$$

### 3.1 The results arising from the experimental partial solution

Typical values of  $a$  and  $b$  are from  $0,05w$  to  $0,1w$ ,  $a_0$  can be from  $10^\circ$  to  $15^\circ$  and  $c$  is between 0,1 to 0,2. With these values the contributions from the members of  $a_0$  (for even) and from members of the  $c$  (for the odd) are very small. The deviation of  $\eta$  from 1 is mainly due to non-zero values of  $a$  and  $b$ . For example, if  $a_0 = 0,2$ ,  $c = 0,2$ ,  $a = b = w/10$ ,  $p = 9$  then  $\eta = 0,967$  (0,147 dB), while with the same values of  $a_0$ ,  $c$ ,  $b$  and  $a$ , for  $p = 10$  is  $\eta = 0,968$  (0,141 dB).

### 4. The efficiency of shifted and slanted transducer

Previous results can be used to determine the effect of deviations of the geometry of IDT on its characteristics. There are known various cases of incorrect interaction of ideal transducer and incident wave - the aperture is not equal to the width of the acoustic wave [8]. The transducer is actually outside the wave, although its aperture equals the width of wave (e.g. the transducer is tilted, thus the acoustic wave does not fall perpendicularly to the aperture, etc.). In the next part, the expression for power transmission coefficient and efficiency will be derived, where the abovementioned conditions are under consideration. The case of IDT with aperture  $w$ , in which only a part of this aperture is receiving SAW, is shown in fig.6., where  $\theta$  is the angle of the tilt of the transducer and

along the line  $\overline{RS}$  the transducer has a wave phase  $a = a(z)$ . In the point Q there is a delay caused by the length of

$y$ , equal to  $2\pi y/\lambda$  or  $\frac{2\pi}{\lambda}(w-a)\sin\theta$ . In the points lying

between points R and Q the delay phase is  $\frac{2\pi}{\lambda}(x-a)\sin\theta$ . The phase of the wave incoming on the transducer in these points can be expressed by the formula:

$$(20) a(z) + \frac{2\pi}{\lambda}(x-a)\sin\theta.$$

In the element  $dz$  at the point of  $z$ , the power of the wave is  $P(z)dz$ , and because  $z = (x-a)\cos\theta$ , then it can be expressed by the relation  $P[(x-a)\cos\theta]dz$  and the complex amplitude of the wave incident on the transducer is:

$$(21) \sqrt{P[(x-a)\cos\Theta]} dx \cos\Theta e^{-j\left[a+\frac{2\pi}{\lambda}(x-a)\cos\Theta\right]}$$

The amplitude of the wave transmitted on the electrical port is  $T_{13}\sqrt{dx/w}$  - th part of this amplitude and the total amplitude is the integral from  $a$  to  $w$ . The following applies:

$$(22) T_{13} \sqrt{\frac{\cos\Theta}{w}} \int_a^w \sqrt{P[(x-a)\cos\Theta]} e^{-j\left[a+\frac{2\pi}{\lambda}(x-a)\cos\Theta\right]} dx.$$

The total power of the beam is

$$(23) \int_0^w P(z) dz$$

and then the power transmission coefficient is.

$$(24) p_{13} = \frac{|T_{13}|^2 \cos\Theta \int_a^w \sqrt{P[(x-a)\cos\Theta]} e^{-j\left[a+\frac{2\pi}{\lambda}(x-a)\cos\Theta\right]} dx}{\int_0^w P(z) dz}.$$

Since in the homogeneous wave are  $P$  and  $a$  independent from  $x$  or  $z$ , we can adjust the equation (24) to the following form:

$$(25) p_{13} = \frac{|T_{13}|^2 \cos\Theta \int_a^w e^{-j\frac{2\pi}{\lambda}(x-a)\cos\Theta} dx}{w'}$$

i.e.:

$$(26) p_{13} = 2|T_{13}|^2 \cos\Theta \frac{\sigma^2 w}{w'} \left\{ \frac{\lambda^2/\sigma^2 w^2}{4\pi^2 \sin^2\Theta} \left[ 1 - \cos\left(\frac{2\pi \sin\Theta}{\lambda/\sigma w}\right) \right] \right\},$$

where  $kde \sigma = 1 - a/w$ . If we denote  $\varphi = 2\pi \sin\Theta \cdot \sigma w/\lambda$ , then:

$$(27) p_{13} = 2|T_{13}|^2 \cos\Theta \frac{\sigma^2 w}{w'} \left\{ (1 - \cos\varphi/\varphi^2) \right\}.$$

The efficiency can be expressed on the base of the equation (2) by the following relation:

$$(28) \eta = 2 \cos\Theta \frac{\sigma^2 w}{w'} \left\{ 1 - \cos\varphi/\varphi^2 \right\},$$

where individual coefficients can be interpreted as follows:  $w/w'$  expresses part of the power of the wave, which is transmitted to the transducer, if the electrodes are perpendicular to the direction of energy flow,  $\cos\Theta$  is the factor which expresses the tilt of the transducer, the active power is not  $w$  but  $w\cos\Theta$ . The factor  $\sigma^2$  reflects the fact that only part of the aperture receives SAW, amplitude of the output electrical signal is proportional to  $\sigma$ , then power is proportional to  $\sigma^2$  and a factor of 2 expresses the change in phase along the connecting line  $\overline{RQ}$ .

#### 4.1 The results arising from the experimental partial solutions

At a small tilt of a transducer is  $\cos\Theta = 1$ , while  $\sigma w/\lambda$  is large, so  $\varphi$  can be considerable even if  $\Theta$  is small. Let us consider, e.g., that the tilt can be  $\pm 0,1^\circ$ , i.e.  $\Theta = 0,1^\circ$  and  $\sigma w/\lambda = 100$ , then  $\varphi = 1,097$  and  $2(1 - \cos\varphi)/\varphi^2 = 0,9037$ , which corresponds to 0,44 dB.

Because the direction from the output IDT to the input IDT need not be the same as the direction of the wave, the output transducer may capture an incident wave only partially. If there is the difference in the directions of  $0,1^\circ$ , then for the typical values of the distance between the IDT,

i.e. 25mm, at a frequency of 70MHz and at a speed of 3500  $\text{ms}^{-1}$  is the displacement of the output IDT with regard to the incident wave  $44,3\mu\text{m}$ . If the wavelength  $\lambda_0 = 50\mu\text{m}$  (aperture of transducer is  $50\lambda_0$ ) i.e.  $2500\mu\text{m}$ , then part of transducer receiving the SAW is equal to  $\sigma = (1 - 44,3/250) = 0,982$  (or  $\sigma^2 = 0,965$ ), which corresponds to 0,16 dB.

#### Conclusion

In this paper, we examined the effect of inhomogeneous SAW impact, and the impact of the tilt and shift of the output IDT on the power transmission factor and efficiency. It can be concluded that the phase change caused by the apodization can generate the considerable insertion loss, which depends on the aperture of the output IDT and it depends in a complicated way on the nature of phase changes. Based on the analysis it can be concluded that when designing IDT are as a rule used passive electrode to suppress the effect of apodization.

Furthermore, we investigated the effect of ripple amplitude and phase of the incident wave. We have shown that in such cases, the insertion loss caused by inhomogeneity of SAW can take value from 0,1 to 0,2 dB. The additional loss also causes a tilt and shift of the output IDT. We found that:

- in the typical cases it receive the value of 0,44 dB,
- the mentioned losses do not affect the transfer function  $T_{13}$ , but the efficiency  $\eta$ .

We have also shown that the electrical circuits attached between the IDT and the load can affect the transfer function but these losses can not be reduced by changing of the parameters of these circuits.

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