Study of Transmission System with Wind Power Control and Optimal Reactive Power Flow

Abstract. This paper presents a methodology of controlling the power injected into system by wind generators and the use of Optimal Reactive Power Flow (ORPF). The methodology used two stages: in the first one scheme for Doubly-Fed Induction Generator (DFIG) is realized to control the active and reactive powers, in the second stage, the ORPF based in the Modified Barrier Lagrangian Function approach (MBLF) is used to optimize reactive power dispatch aiming to minimize active power losses system. Case studies on the modified IEEE 14 bus "modified" clearly shows the benefits of using the associated generator control whit ORPF.

Streszczenie. Artykuł przedstawia metodologię sterowania dołączaniem energii z elektrowni wiatrowych do systemu energetycznego oraz uzyskiwania algorytmu RRPF – Optimal Reactive Energy Flow. W pierwszym etapie analizowano sterowanie generatorami typu DFIG w celu kontroli mocy biernej i czynnej, w drugim etapie wykorzystano metody MBLF (Modified Barrier Lagrangian Function) do optymalizowania mocy biernej w systemie. Analiza systemu energetycznego ze sterowaną mocą elektryczną wiatrową i optymalnym przesyłem mocy

Keywords: Optimal Reactive Power Flow; Wind Power Control; Transmission System
Słowa kluczowe: optymalizacja mocy biernej, sterowanie mocą, system energetyczny.
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1. Introduction

Year after year technology and society’s “eco-thoughts” evolved together, bringing out new concepts such as sustainability and green corporative compliances. The human development and the access of technology makes energy demands continuously grows while natural sources like: oil, coal and gas becomes low. World consumption of electrical energy will increase by 84% between 2008-2035 [1], while in Brazil the increase will be 4.6% between 2010-2020 [2]. At this scenario smart grids involving hybrids power generation systems into distributed power generation created a new era for energy distribution. These systems are usually composed by parallel connection of photovoltaic solar panels and wind generators. Although the most advantageous solution for standalone use is the wind generator [3], where the energy produced by wind is considered technically usable when at a height of 50m the winds have speeds of at least 7 m/s [4].

The doubly fed induction generator (DFIG) is a generator type commonly employed in this type of application [5]. The techniques for independent control of active and reactive powers of DFIG are traditionally performed by the technique of the stator flux orientation or grid voltage by controlling the rotor currents [6]. Initially the aero generators were designed to operate with unity power factor, however, some studies [7] [8] presents techniques for controls reactive power of DFIG, that enable the operation of the generator supplying reactive to the network. With the reactive power control of the DFIG, the optimal reactive power injection can be evaluated respecting the DFIG’s constraints for wind speed, specified load/generation and constraints of the power system. This optimal can be evaluated via optimal reactive power flow (ORPF), which promotes management efficiencies, as improvements in voltage profile and lower losses in active power. The ORPF is a non convex static nonlinear programming problem; it is one of the most powerful tools to analyses static systems of electrical energy. The ORPF used has the objective of minimizing a function and, at the same time, of satisfying a set of physical and operational constraints in power systems, e.g. reactive power injection constraint. As a solution, it provides the optimal operation for the electrical network for a given load and generation configuration of the system satisfying all system constraints. It was proposed by Carpentier in the early 60's based on the economic dispatch problem [9]. Since then, many papers have been written in an attempt to solve the problem [10-13].

This work considers the reactive power injection capacity of a wind farm using DFIG to optimize the active power losses in a power system. In this way is proposed to use an ORPF for a system with DFIG. The DFIG’s power control is achieved by using stator flux orientation and proportional plus integral controller. The dynamic machine model was used to obtain the steady state output of active and reactive power to be supplied to the ORPF algorithm. The ORPF algorithm uses the Modified Barrier Lagrangian Function (MBLF) [14] in the process solution. In the section 3 the ORPF problem and MBLF is displayed. Thus, the contribution of this paper is the analyzes of the benefits of the reactive power injection, by an wind farm with reactive power control, to the power system provided by optimal reactive power injecting control performed via ORPF.

This paper is organized as follows: Section 2 presents the machine model and rotor current vector control. Section 3 describes the ORPF approach used. The simulation results, which demonstrate the effectiveness of the analyses, are shown and discussed in Section 4. Finally, in Section 5 some concluding remarks are made.

2. Wind Generator Control

2.1. Rotor side converter:
For decoupled control of active and reactive power, it is necessary the induction machine dynamics model, also assuming stator flux, where the flux vector is aligned with the direct axis $\psi_s^d = \psi_{sd}^d$ and $v_s = v_{sd} = |v_{sd}|$ [5].

The DFIG power control is achieved by rotor current control. Hence the independent stator active $P$ and reactive $Q$ power control. In this case, $P$ and $Q$ are computed by each individual rotor current. The active and reactive power are done by [5],

$$P = -\frac{3}{2} v_s L_i^M \frac{L_s}{L_M} i_q^d$$

$$Q = \frac{3}{2} v_s \left( \frac{L_i^M}{L_s} - \frac{L_s}{L_M} i_q^d \right)$$
Thus, it is possible to control the active and reactive power of DFIG by the rotor current control. Proportional plus integral controllers (PI) can be used for this objective as shown in Fig. 1.

2.2 Grid side converter:
The grid side converter (GSC) controls the DC link voltage of the back-to-back converter, also controls the current flows through the converter and the electrical grid by using voltage [15]. Thus, it is possible to control power sent to the grid independently from the relationship:

\[
\begin{align*}
\text{P}_{g} & = \frac{3}{2}V_{gd}i_{gd} \\
\text{Q}_{g} & = \frac{3}{2}V_{gd}i_{gq}
\end{align*}
\]

PI controllers can be used in this control application again. A detailed explanation can be seen in [5,15].

The Figure 2 shows the GSC control strategy.

3. Optimal Reactive Power Flow
3.1 The Problem
The ORPF problem can be described by Eq. (5).

\[
\begin{align*}
\min f(x) \\
\text{s.t. } g_i(x) &= 0, & i = 1, \ldots, m \\
h_j^{\min} & \leq h_j(x) \leq h_j^{\max}, & j = 1, \ldots, r \\
x^{\min} & \leq x \leq x^{\max}
\end{align*}
\]

where \( x \in R^n \) is the control and state variable vector representing voltage magnitudes \( V \), voltage angles \( \theta \) and tap-changing transformer \( t \).

3.1.2 Objective Function
In this paper, the power transmission loss function \( f(x) \) is set as the objective function. The power transmission loss can be expressed by Eq. (6).

\[
f(x) = \sum_{k=1}^{N_L} g_{km} (V_k^2 + V_m^2 - 2V_kV_m \cos \theta_{km})
\]

where \( V_k \) is the voltage magnitude at bus \( k \), \( g_{km} \) is the conductance of line \( k-m \), \( \theta_{km} \) is the difference in voltage angle between the \( k \) and \( m \) bus and \( N_L \) is the total number of transmission lines.

3.1.3 Equality Constraints
The equality constraints \( g(x) \in R^n \) represent the power flow equations that provide a means for calculating the power balance that exists in the network during steady-state operation to active and reactive power and are represented by Eq. (7) and Eq. (8) respectively.

\[
\begin{align*}
P_{k} & = P_{Gk} - P_{Lk} = V_k \sum_{m=1}^{N_L} V_m (G_{km} \cos \theta_{km} + B_{km} \sin \theta_{km}) \\
Q_{k} & = Q_{Gk} - Q_{Lk} + L_{ik} = V_k \sum_{m=1}^{N_L} V_m (G_{km} \sin \theta_{km} - B_{km} \cos \theta_{km})
\end{align*}
\]

where \( P_{k} \) and \( Q_{k} \) are, respectively, the active and reactive power injections at bus \( k \); \( P_{Gk} \) and \( Q_{Gk} \) are, respectively, the scheduled active and reactive power generations at bus \( k \); \( P_{Lk} \) and \( Q_{Lk} \) are, respectively, the active and reactive power loads at bus \( k \); \( L_{ik} \) is the component reactive power injection due to the shunt element at bus \( k \); \( G_{km} \) is the real part of the element in the bus admittance matrix \( Y_{BUS} \) corresponding to the \( kth \) row and \( mth \) column, \( B_{km} \) is the imaginary part of the element in the \( Y_{BUS} \) corresponding to the \( kth \) row and \( mth \) column.

3.1.4. Inequality Constraints
All variables have upper and lower bounds that must be satisfied in the optimal solution. In this paper the functional constraints \( h(x) \in R^r \), with lower bound \( h_j^{\min} \) and upper bound \( h_j^{\max} \) represent the limits of reactive power injections and the inequality \( x^{\min} \leq x \leq x^{\max} \) the variables bounded \( V \) and \( \theta \), presented as:

\[
\begin{align*}
Q_{k}^{\min} & \leq Q_{k}(x) \leq Q_{k}^{\max}, & (9) \\
V_{k}^{\min} & \leq V_{k} \leq V_{k}^{\max}, & (10) \\
\theta_{k}^{\min} & \leq \theta_{k} \leq \theta_{k}^{\max}, & (11)
\end{align*}
\]
where $Q_i^{\text{min}}$ and $Q_i^{\text{max}}$ are, respectively, the lower and upper bounds of $Q_i$, $V_i^{\text{min}}$ and $V_i^{\text{max}}$ are, respectively, the lower and upper bounds of $V_i$ and $\theta_i^{\text{min}}$ and $\theta_i^{\text{max}}$ are, respectively, the lower and upper bounds of $\theta_i$. These inequality constraints must be satisfied in the optimal solution.

This is a typical nonlinear and no convex problem. The ORPF used employs the formulation presented in Sousa et al. [14]. This formulation considers the application of logarithmic barrier method to voltage magnitude, voltage angles, tap-changing transformer variables and augmented Lagrangian method to other constraints. In this work wind turbines are treated as reactive control buses.

### 3.2 Modified Barrier Lagrangian Function Method

In this work, the MBLF method is used to solve the ORPF problem, Eq (5), which can be represented by Eq. (12).

\[
\text{Minimize } f(x) \\
\text{subject to } g(x) = 0 \\
\begin{align*}
\mathbf{h}^{\text{min}} &\leq h(x) \leq \mathbf{h}^{\text{max}} \\
\end{align*}
\]

In this method, the bounded constraints are transformed into two inequalities and slack variables are introduced, transforming these inequalities into equalities.

\[
\begin{align*}
\text{Minimize } f(x) \\
\text{subject to } g(x) = 0 \\
h(x) + s_1 &= \mathbf{h}^{\text{max}} \\
h(x) - s_2 &= \mathbf{h}^{\text{min}} \\
s_1 \geq 0 \\
s_2 \geq 0 \\
\end{align*}
\]

where the slack vectors $\mathbf{s_1} \in \mathbb{R}^r$ and $\mathbf{s_2} \in \mathbb{R}^r$.

The slack variables of problem (13) are relaxed and treated by the Modified Barrier Functions. The non-negative conditions of problem (13) are relaxed by the barrier parameter.

\[
\begin{align*}
\text{Minimize } f(x) \\
\text{subject to } g(x) = 0 \\
h(x) + s_1 &= \mathbf{h}^{\text{max}} \\
h(x) - s_2 &= \mathbf{h}^{\text{min}} \\
s_1 \geq -\mu \\
s_2 \geq -\mu \\
\end{align*}
\]

where $\mu > 0$ is the barrier parameter. This represents an expansion of the feasible region of the original problem.

The Modified Barrier Function (MBF) $\ln(\mu^{-1}s + 1)$, proposed by [16], is used to transform problem (14) into the following modified problem.

\[
\begin{align*}
\text{Minimize } f(x) \\
\text{subject to } g(x) = 0 \\
h(x) + s_1 &= \mathbf{h}^{\text{max}} \\
h(x) - s_2 &= \mathbf{h}^{\text{min}} \\
\mu \ln(\mu^{-1}s_1 + 1) &\geq 0 \\
\mu \ln(\mu^{-1}s_2 + 1) &\geq 0 \\
\end{align*}
\]

The following Lagrangian function is associated to problem (15). It is called the modified barrier Lagrangian function.

\[
L = f(x) - \mu \sum_{j=1}^{p} \mathbf{u}_j \ln(\mu^{-1}s_j + 1) - \mu \sum_{j=1}^{p} \mathbf{u}_j \ln(\mu^{-1}s_j + 1) - \lambda g(x) - \pi_1 h_1(x) - \pi_2 h_2(x)
\]

where $\mathbf{u}_j \in \mathbb{R}^r$, $\mathbf{u}_j \in \mathbb{R}^r$, $\lambda \in \mathbb{R}^r$, $\pi_1 \in \mathbb{R}^r$ and $\pi_2 \in \mathbb{R}^r$ are the vectors of the Lagrange multipliers.

The first-order necessary conditions are applied to Eq (16), generating nonlinear system equations. Then Newton’s method is applied to the nonlinear system equations to find the search direction vector $\Delta \mathbf{d}$ resulting in linear system equations represented by Eq (17).

\[
\begin{align*}
W = \nabla^2 L
\end{align*}
\]

The direction vector is given by

\[
\Delta \mathbf{d} = -\nabla^2 L
\]

where:

\[
\begin{align*}
V_x^L &= \sum_{j=1}^{p} \lambda_j V_{s_j}^L g_j(x) - \sum_{j=1}^{p} \pi_1^j V_{h_1}^L h_1(x) - \sum_{j=1}^{p} \pi_2^j V_{h_2}^L h_2(x)
\end{align*}
\]

and the sub matrices $\mathbf{S}_1$ and $\mathbf{S}_2$ given by

\[
\mathbf{S}_1 = \begin{pmatrix}
\frac{\mu}{(\mu x + 1)} & \cdots & 0 \\
0 & \ddots & \vdots \\
0 & \cdots & \frac{\mu}{(\mu x + 1)}
\end{pmatrix}
\]

\[
\mathbf{S}_2 = \begin{pmatrix}
\frac{\mu}{(\mu x + 1)} & \cdots & 0 \\
0 & \ddots & \vdots \\
0 & \cdots & \frac{\mu}{(\mu x + 1)}
\end{pmatrix}
\]

The Hessian matrix is sparse and symmetric and its structure is constant through iterations.

Using the search directions obtained from (17), the vectors of variables $\mathbf{x}$, $\mathbf{s}$, $\lambda$ and $\pi$ are updated as follows:

\[
\begin{align*}
\mathbf{x}^{k+1} &= \mathbf{x}^k + \alpha_p \mathbf{d}^k \\
\mathbf{d}^{k+1} &= \mathbf{d}^k + \alpha_p \mathbf{d}^k \\
\mathbf{s}^{k+1} &= \mathbf{s}^k + \alpha_s \mathbf{s}^k \\
\mathbf{s}^{k+1} &= \mathbf{s}^k + \alpha_s \mathbf{s}^k \\
\mathbf{\pi}_1^{k+1} &= \mathbf{\pi}_1^k + \alpha_s \mathbf{\pi}_1^k \\
\mathbf{\pi}_2^{k+1} &= \mathbf{\pi}_2^k + \alpha_s \mathbf{\pi}_2^k \\
\end{align*}
\]

where $\alpha_p$ and $\alpha_s$ are the scalar step sizes used to update the primal and dual variables, respectively. The step sizes are calculated according to [10].

\[
\alpha_p = \min \left\{ \frac{\mathbf{s}}{\Delta \mathbf{s}} : \Delta \mathbf{s} < 0 \text{ and } \mathbf{s} > 0, 1 \right\}
\]

\[
\alpha_s = \min \left\{ \frac{-\mathbf{\pi}_2}{\Delta \mathbf{\pi}_2} : \Delta \mathbf{\pi}_2 < 0 \text{ and } \mathbf{\pi}_2 > 0, \frac{\mathbf{\pi}_2}{\Delta \mathbf{\pi}_2} : \Delta \mathbf{\pi}_2 > 0 \text{ and } \mathbf{\pi}_2 > 0, 1 \right\}
\]

The barrier parameter is smoothly decreased according to [17], as follows:

\[
\mu^{k+1} = \mu^k \left( 1 - \left( \frac{\sigma}{\sqrt{2r}} \right) \right)
\]

where:

\[
\sigma = \max \left( 1 / \left( \mu^{-1} s_j + 1 \right), 1 \right), \quad j = 1, \ldots, 2r \text{ for } s_j > 0
\]

The Lagrange multiplier vector, $\mathbf{u}$, is updated according to rule [16], which has a very low computational complexity, as follows:
3.2.1 Simplified algorithm

Initialization Step

- Given problem (12), construct the MBLF (16);
- Let \( k = 0 \);
- Choose initial values for the problem variables:
  \( d^1 = (x^1, s^1, \lambda^1, \pi^1) \), \( u^1 > 0 \) and \( \mu^1 > 0 \).
- \( x \) can be the same as the initial values for a power flow, \( \Delta x \geq 0 \), \( \pi_1 \geq 0 \) and \( \pi_2 \leq 0 \) or any other reasonable guess.
- Go to Main Steps.

Main Steps

1. Evaluate \( V \lambda \) as a function of \( d \).
2. Evaluate matrix \( W \) as a function of \( d \) and solve the system (17).
3. Compute the step length \( \alpha_x \) and \( \alpha_y \). Update \( d \) by \( \Delta d \) and the step lengths.
4. If \( d^{k+1} \) satisfies the convergence criteria, then STOP. If not, then set \( k = k + 1 \), update \( u \) and the Lagrange multipliers, \( u \), using (19) and (20) respectively, and then return to Step 1; In the solution the Karush-Kuhn-Tucker conditions, \( s \geq 0 \), \( \pi_1 \geq 0 \) and \( \pi_2 \leq 0 \), must be satisfied.

In [14] is shown that computationally this formulation is more attractive.

4. Simulation Results

The reactive power control strategy was simulated using MATLAB/SimPowerSystems package. The DFIG parameters are shown in Appendix (Tables A.1 – A.3). Figure B.1 shows the schematic of the implemented system. The inverter was modeled as controlled voltage source. It was simulated the wind energy system making the FP = 0.95 and several wind operation as shown in Table A.1 for all wind speed.

4.2. System performance considering the DFIG

Considering the wind farm connected at bus 8 was carried out simulations for the wind conditions of 6 m/s to 14 m/s considering the data generator according to the Table A.1 for all wind speed.

Figure 4 shows in a clear way that reactive power injection contributes to improving voltage profile. From 9 m/s the generator provides reactive power. From this speed, the voltage profile improves resulting in an active power losses reduction.
generation of the bus 2. The slack bus contributed with the reactive power balance.

The Figure 6 shows the optimal active power injection in the system. The wind farm (bus 8) injected active power in accordance with Table A.2. The bus 2 remained constant in 18.3 MW the active injection. The slack bus contributed with the active power balance.

5. Conclusions

This paper presented an approach to optimal operation of power system with reactive power control in wind turbines. In this work is used a reactive power control for DFIG-based wind turbine using stator field orientation for high control performance. The steady state simulations results are used in ORPF algorithms. An ORPF based in the Modified Barrier Lagrangian Function approach to optimize reactive power dispatch aiming to minimize active power losses system was utilized. The ORPF was able to optimize the reactive power dispatch of the system considering the operational constraints. In the tests performed with the modified IEEE 14 bus system was observed a better voltage profile and power loss which shows the importance of injecting reactive power provided from wind generators. Therefore it is evident the benefits of using wind generators with reactive power control for optimize the system.

Appendix

<table>
<thead>
<tr>
<th>Buses</th>
<th>lines</th>
<th>Generation buses</th>
<th>Load buses</th>
<th>DFIG buses</th>
</tr>
</thead>
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<tr>
<td>14</td>
<td>20</td>
<td>2</td>
<td>11</td>
<td>1</td>
</tr>
</tbody>
</table>

A. Wind farm electrical systems parameters:

<table>
<thead>
<tr>
<th>Wind (m/s)</th>
<th>P (MW)</th>
<th>Q (Mvar)</th>
<th>S (MVA)</th>
<th>FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>16.3</td>
<td>0</td>
<td>16.3</td>
<td>1</td>
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<td>7</td>
<td>23.75</td>
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<td>23.75</td>
<td>1</td>
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</tr>
<tr>
<td>9</td>
<td>64.36</td>
<td>21.33</td>
<td>65.69</td>
<td>0.95</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
<td>26.34</td>
<td>86.64</td>
<td>0.95</td>
</tr>
<tr>
<td>11</td>
<td>98.55</td>
<td>32.5</td>
<td>131.03</td>
<td>0.95</td>
</tr>
<tr>
<td>12</td>
<td>124.24</td>
<td>40.93</td>
<td>165.17</td>
<td>0.95</td>
</tr>
<tr>
<td>13</td>
<td>157.32</td>
<td>51.76</td>
<td>166.51</td>
<td>0.95</td>
</tr>
<tr>
<td>14</td>
<td>164.64</td>
<td>54.11</td>
<td>173.30</td>
<td>0.95</td>
</tr>
</tbody>
</table>

The reactive power Q for each wind speed (more than 9m/s) can be adjusted from FP=1 so Q = 0 Mvar till FP=0.95 lead.

| Min. Rotor Speed - variable speed (rpm) | 9       |
| Nom. Rotor Speed – variable speed (rpm) | 14      |
| Rotor diameter (m) | 75 |
| Area covered by rotor (m²) | 4418 |
| Nom. Power (MW) | 2 |
| Nom. Wind Speed – variable speed (rpm) | 14 |
| Gear box ratio - variable speed | 1:100 |
| Inertia constant (s) | 2.5 |
| Shaft stiffness – fixed speed (pu/el rad) | 0.3 |

B. Modified IEEE 14 bus system:

<table>
<thead>
<tr>
<th>from</th>
<th>to</th>
<th>r (pu)</th>
<th>x (pu)</th>
<th>bpu (pu)</th>
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<tbody>
<tr>
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<td>2</td>
<td>1.94</td>
<td>5.92</td>
<td>5.28</td>
</tr>
<tr>
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<td>5</td>
<td>5.4</td>
<td>22.3</td>
<td>5.28</td>
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<td>5.7</td>
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<td>3.40</td>
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<tr>
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<td>4</td>
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<td>3.46</td>
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<td>4</td>
<td>5</td>
<td>1.34</td>
<td>4.21</td>
<td>1.28</td>
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<tr>
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<td>6</td>
<td>0.01</td>
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</tr>
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<td>8</td>
<td>0.01</td>
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<td>0.01</td>
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</tr>
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</tr>
<tr>
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<td>11</td>
<td>8.2</td>
<td>19.21</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>22.09</td>
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<tr>
<td>13</td>
<td>14</td>
<td>17.09</td>
<td>34.80</td>
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Table B.2 - Data buses with reactive control.

<table>
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<tr>
<th>bus</th>
<th>Q (Mvar)</th>
<th>Qmin (Mvar)</th>
<th>Qmax (Mvar)</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>0</td>
<td>-200</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>12.7</td>
<td>-40</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
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Fig. B.1. Modified IEEE 14 bus system configuration.

REFERENCES


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