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First-Order Sensitivity Applied in Power Distribution System

***Abstract**. This paper presents a study of the power distribution system via sensitivity analysis. This study is based on first-order sensitivity analysis applied in the power flow (PF) solution. Unlike the PF algorithm, this technique does not require an iterative process, which results in a faster methodology with high accuracy and greatly reduced computational work. Starting from a known PF solution, considered as the base case, the new operating point is calculated directly after making a perturbation in the loads. The methodology analysis is applied to 34, 70 and 476-bus distribution power systems. Test results demonstrate the efficiency of the approach.*

***Streszczenie.** W artykule przedstawiono metodę kontroli dystrybucji mocy za pośrednictwem analizy czułości (analiza czułości pierwszego rzędu). Dzięki tej metodzie uzyskuje się szybsze rezultaty unikając procesu iteracji. Startując ze znanego rozwiązania rozpływu mocy PF nowy punkt pracy jest obliczany po zmianie obciążeń. (**Zastosowanie metody analizy czułości pierwszego rzędu w systemach energetycznych**)*

Keywords: Sensitivity analysis; power flow; distribution power systems.
Słowa kluczowe: in the case of foreign Authors in this line the Editor inserts Polish translation of keywords.

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1. Introduction

In the scenario of electric power, great changes happened in the last few years. New technologies are emerging allied to the concept of smart grids that will lead to deep changes and advances in the direction of new technologies enabling better system management with efficiency gains. The technologies that are currently under investigation for the smart grids will provide system control in real time enabling better control about variations of the demand and supply energy [1]. The power demand in distribution system varies during the day (load curve) as well as power generated by intermittent energy sources.

Allied to these changes the electrical power systems have experienced a large increase in demand. The global consumption of electricity will increase by 84% in the corresponding period between 2008 and 2035 [2]. In Brazil´s Decennial Plan for Expansion of Energy 2020 pro-vides for an increase of 4.6% per year of electricity consumption in the country in the period 2010-2020 [2].
To this end, electrical power systems re-quire rapid analysis tools to assist agents in making decisions. The sensitivity analysis can be used in this context to promote gains in operating systems.

Sensitivity analysis can be defined as a tool that finds out how sensitive an output is to any change in an input while keeping other inputs constant. In power systems, is very important to know the sensitivity of one electrical variable, such as the voltage *Vj* at bus *j*, in relation to another electrical variable, such as active production P_k at bus *k*. The calculation this sensitivity relation is objective this work.

In power systems, much research has been done in the field of sensitivity analysis aided by computer. The sensibility of the solution for parameter variations, that is, perturbation, has been largely studied since the late 70's. Peschon et al. [3] analyzed power system performance by means of first and second-order sensitivities and give an interesting interpretation of their meaning. Belati et al. [4] described a parametric Optimal Power Flow formulation to perform sensitivity analysis. In recent decades several other algorithms have been proposed for sensitivity analysis in power systems [5, 6, 7, 8, 9].

This work presents a study in distribution power systems by first-order sensitivity analysis applied to Power Flow (PF) solution. This methodology allows analyzing the effects of perturbations in the variables or functions of the network whenever perturbations in the loads and generations occur. Thus, the contribution of this work is presents a new methodology of sensitivity analysis in distribution power systems and analyse the accuracy of the results and CPU time.

This paper has the following organization: Section 2 describes the sensitivity analysis methodology. Section 3 the results obtained from tests on three systems (34, 70, 476-bus) are reported. Finally, some concluding remarks are made in Section 4.

2. Sensitivity analysis methodology description

The sensitivity analysis methodology consists in the apply a perturbation in the operation point obtained by PF problem, considered as the base case solution, and compute a new operation point directly after making a perturbation in the loads or generators. This way is possible to calculate the sensitivity of one function, such as active power losses in respect of load variation in the system, the voltage at node *i*, with respect to another electrical variables, nodal losses evaluation and other analysis.

The methodology can be divided into two parts: the first consists of solving the PF using the Newton-Raphson method [10] to the base case; the second parts in the application of first-order sensitivity analysis on the PF solution obtained in the first part.

2.1 Power Flow

The goal of power flow study is to obtain complete voltage angle (θs) and magnitude (Vs) information for each bus in the network. Once this information is known, real and reactive power flow on each branch as well active power losses in the lines can be analytically determined. The methods have evolved following the technical advances that allow, through the use of computer; prepare sophisticated algorithms for solving the problem. One of the most widely used methods for solving the problem of PF for transmission systems was proposed by [10], which uses the method of Newton-Raphson (NR). Although the NR method is not the most used in distribution systems, it can be applied when sparsity techniques are used [11].

Injections of active and reactive power are obtained by imposing the Kirchhoff's Currents Law at each bus and can be calculated in polar form by Eq (1) and Eq (2), respectively.

$$P_k^{cal}(\boldsymbol{V},\boldsymbol{\theta}) = {V_k}^2 G_{kk} + V_k \sum_{m \in k} V_m (G_{km} cos\theta_{km} + B_{km} sin\theta_{km}) \quad (1)$$

$$Q_k^{cal}(\boldsymbol{V},\boldsymbol{\theta}) = -{V_k}^2 B_{kk} + V_k \sum_{m \in k} V_m (G_{km} sin\theta_{km} - B_{km} cos\theta_{km}) \quad (2)$$

where: G_{km} – real element of the matrix Y_{BUS} associated with the bus *k* and *m*; B_{km} – imaginary element of the matrix Y_{BUS} associated with the bus *k* and *m*; $m \in k$ – set of all *m* bus connected with the bus *k*; θ_{km} – difference in voltage angle between the k and m bus.

The solution of the problem of PF is obtained with solution of the active and reactive power balance equations for each Load Bus and the active power balance equation for each Generator Bus, by Eq. (3) and Eq. (4).

(3) $$\Delta P_k = P_k^{esp} - P_k^{calc}(V,\theta) = 0$$

(4) $$\Delta Q_k = Q_k^{esp} - Q_k^{calc}(V,\theta) = 0$$

Where ***esp*** superscript represents the values specified of power injections at load bus that are considered constant (constant power load model) and ***calc*** superscript is the calculated values of power injections obtained from the vector of state variables ($\boldsymbol{V},\boldsymbol{\theta}$) and system parameters.

2.2 Sensitivity analysis

The sensitivity analysis used in this work can directly estimate the θs and Vs for each bus in the network when perturbations (changes) occur in the control variables of the system.

The PF equations (Eq. 3 and Eq. 4) can be expressed in vector notation as:

(5) $$\boldsymbol{g}(\boldsymbol{x},\boldsymbol{u}) = 0$$

where $\boldsymbol{x}$ is the dependent vector (θs and Vs) and $\boldsymbol{u}$ is independent or controllable vector (power injections *Ps* and *Qs*).

It is assumed that $\boldsymbol{x^*}$ and $\boldsymbol{u^*}$ is the solution of Eq. 3, obtaining:

(6) $$\boldsymbol{g}(\boldsymbol{x}^*,\boldsymbol{u}^*) = 0$$

It is now desired to determine the changes $\Delta\boldsymbol{x}$ which would result from changes $\Delta\boldsymbol{u}$ away from the nominal $\boldsymbol{x^*}$ and $\boldsymbol{u^*}$. A Taylor-series expansion in Eq. 6 provides this relation:

(7) $$\boldsymbol{g}(\boldsymbol{x}^* + \Delta\boldsymbol{x},\boldsymbol{u}^* + \Delta\boldsymbol{u}) \cong \boldsymbol{g}(\boldsymbol{x}^*,\boldsymbol{u}^*) + \boldsymbol{S_x}\Delta\boldsymbol{x} + \boldsymbol{S_u}\Delta\boldsymbol{u}$$

where $\boldsymbol{S_x}$ and $\boldsymbol{S_u}$is given as follows,

(8) $$\boldsymbol{S_x} = \frac{\partial(g_1,g_2,\cdots g_{2N})}{\partial(x_1,g_2,\cdots x_{2N})}$$

(9) $$\boldsymbol{S_x} = \frac{\partial(g_1,g_2,\cdots g_{2N})}{\partial(u_1,u_2,\cdots x_{2M})}$$

the $x_1, x_2, \cdots x_{2N}$ are elements of the vector x and $u_1, u_2, \cdots u_{2M}$ are the elements of the vector u. The N is the number of buses of the systems, and M is the number of load buses of the systems.

Combining equations (5) and (7) gives:

(10) $$\boldsymbol{S_x}\Delta\boldsymbol{x} + \boldsymbol{S_u}\Delta\boldsymbol{u} = 0$$

By rearranging we have:

(11) $$\Delta\boldsymbol{x} = -\boldsymbol{S_x^{-1}}\boldsymbol{S_u}\Delta\boldsymbol{u} = \mathbf{S}\Delta\boldsymbol{u} = 0$$

where $\boldsymbol{S}$ is the sensibility matrix. With it, the variation of $\Delta\boldsymbol{x}$ with respect to the variation $\Delta\boldsymbol{u}$ can be determined.

Equation (11) can be written in matrix form:

(12) $$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_{(2N)} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1(2M)} \\ S_{21} & S_{22} & \cdots & S_{2(2M)} \\ \vdots & \vdots & \vdots & \vdots \\ S_{(2N)1} & S_{(2N)2} & \cdots & S_{(2N)(2M)} \end{bmatrix} \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \\ \vdots \\ \Delta u_{(2M)} \end{bmatrix}$$

As mentioned the vector $\boldsymbol{u}$ consists of independent variables that are the real and reactive power injection at buses and the vector $\boldsymbol{x}$ consists of the controlled variables that are voltages magnitudes and phase angles, then the matrix system (12) can be rewrite as follows:

(13) $$\begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \vdots \\ \Delta\theta_N \\ \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1(2M)} \\ S_{21} & S_{22} & \cdots & S_{2(2M)} \\ \vdots & \vdots & \vdots & \vdots \\ S_{(2N)1} & S_{(2N)2} & \cdots & S_{(2N)(2M)} \end{bmatrix} \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_M \\ \Delta Q_1 \\ \Delta Q_2 \\ \vdots \\ \Delta Q_M \end{bmatrix}$$

With the system represented by Eq. (13) can be obtained new solutions, vector $\boldsymbol{x_{new}}$, when perturbations occur in active and reactive power injected in certain buses.

The vectors$\boldsymbol{x_{new}}$, is calculate as follows:

(14) $$\boldsymbol{x_{new}} = \boldsymbol{x}^* + \Delta\boldsymbol{x}$$

This methodology will be identified by the word SENS in this work.

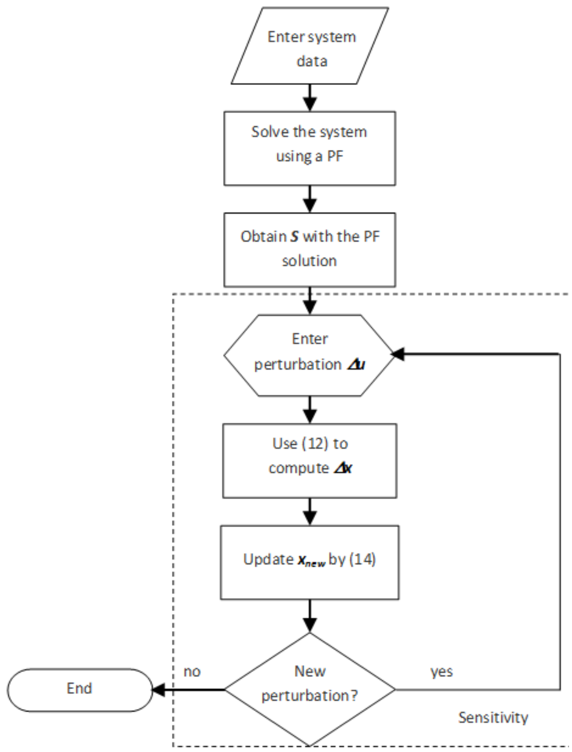

Fig. 1. SENS methodology flowchart

2.3 Algorithm

An algorithm proposed to solve the problem consists of the following steps represented in the flowchart of Fig. 1:

i. Enter system data;
ii. Obtain operational point by PF;

iii. Obtain $\boldsymbol{S}$ with the PF solution of (ii);
iv. Perturb the power injections with $\Delta\boldsymbol{u}$;
v. Use (12) to compute $\Delta\boldsymbol{x}$;
vi. Use (14) to update $\boldsymbol{x_{new}}$;
vii. If a new perturbation is desired go to item (iv);
viii. Otherwise - End.

3. Case studies

Tests were done in distribution systems to verify the efficiency of the proposed approach. The computational program was implemented in MATLAB on an Intel Core i5 CPU with a 5.5 GHz microprocessor. The studies were carried out on the 34-bus [12], 70-bus [13] distribution systems, as well as a large-scale Brazilian distribution system 476-bus [14]. For each system, the base case solution was obtained via PF with a Newton-Raphson method [10], with a precision of 10^{-5} pu for the power balance equations. In the studies performed, demand variations were supplied by the slack bus (substation). Table 1 lists the specifications of these test cases.

of the main characteristics of the studied distribution systems

Power systems	Lines	Feeders	Load buses with load	System Load (MW)	System Load (Mvar)
34 bus	33	1	29	4.637	2.886
70 bus	69	1	48	3.802	2.695
476 bus	475	2	93	9.010	3.500

Starting with the base values, the active and reactive power on the load bus were raised by steps of 2, 4, 6%, …, up to a maximum increase of 50% of the base loads, which could be considered a very large perturbation. During the tests, the slack bus supplied all the variation of the load. After each 2% perturbation, the sensitivity technique was applied, to estimate the new system state. The solution by SENS was compared with the exact solution for the same perturbed state, calculated as the PF algorithm [10] in same tests to verify the efficiency of the methodology.

The table 2 shows the time computational to calculate the system state using the PF and SENS for the distribution systems studied. The CPU time to SENS was obtained considering the base solution known.

Table 2. Time computational to calculate the system state using the PF and SENS

Power systems	PF - CPU time (s)	SENS - CPU time(s)	Gain (%) SENS/PF
34 bus	0.093	0.0008	99.04
70 bus	0.53	0.0056	98.94
476 bus	10.52	0.26	97.52

3.1. Using SENS to determine the active power losses

In Fig. 2, Fig. 3 and Fig. 4 the variations of both the exact real power losses by PF and the sensitivity analysis estimate by SENS are plotted against percent perturbation for the respective 34, 70 and 476 bus systems. Note that, for perturbations up to 20%, the active losses remain very near the exact values. Above 20%, small differences occur. These may be considered a large change in the load. The real power loss (P_{loss}) can be expressed by Eq. (15).

$$P_{loss} = \sum_{k=1}^{NL} g_{km}\left(V_k^2 + V_m^2 - 2V_k V_m \cos\theta_{km}\right) \quad (15)$$

where g_{km} is the conductance of line k-m, and NL is the total number of distribution lines. It is important to note that g_{km} is different from G_{km} shown in Eq. 1 and Eq. 2.

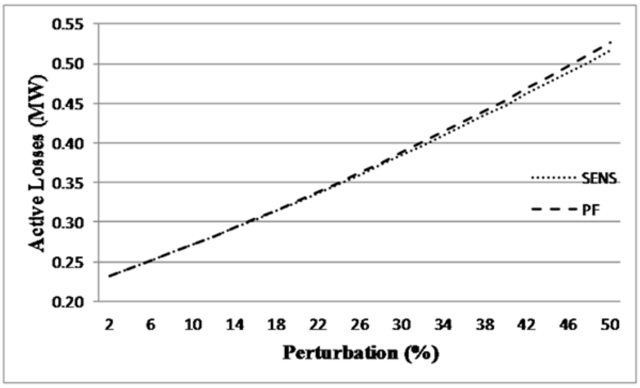

Fig.2. Active power losses to 34 bus systems using the SENS and PF.

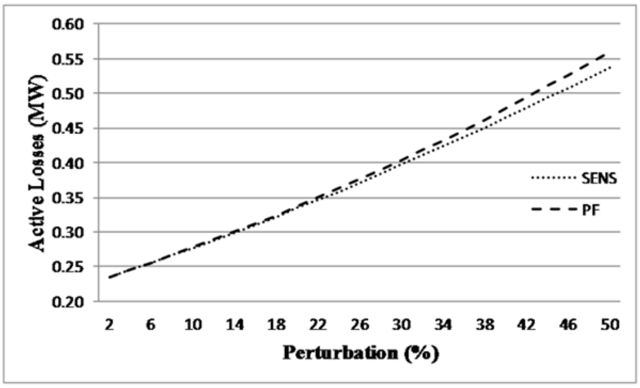

Fig. 3. Active power losses to 70 bus systems using the SENS and PF.

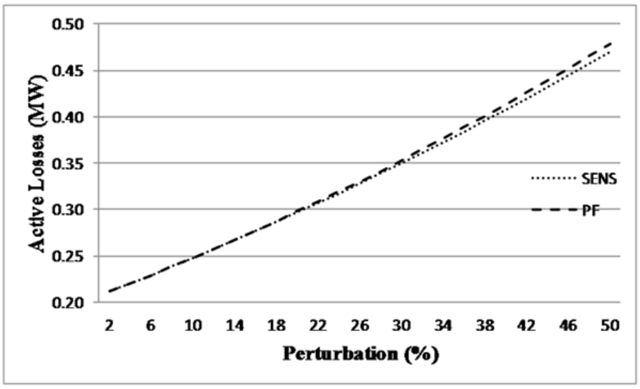

Fig. 4. Active power losses to 476 bus systems using the SENS and PF.

3.2. Active and reactive power mismatches

The maximum active and reactive power mismatches occurred in the buses 27, 66 and 214 for the respective 34, 70 and 476 bus systems. The curves are displayed in Fig. 5, Fig. 6 and Fig. 7 for the respective bus systems. In both systems the reactive power mismatches remained less than 0.01 pu to perturbation lower than 10%. For active power mismatches, the performance was better, as shown in Figs. 5, 6 and 7.

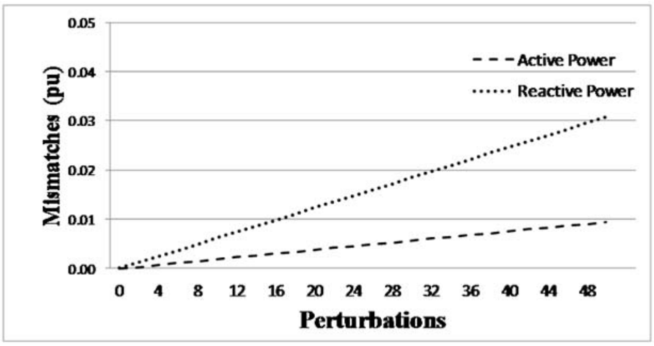

Fig. 5. Maximum active/reactive power mismatches to 34 bus system.

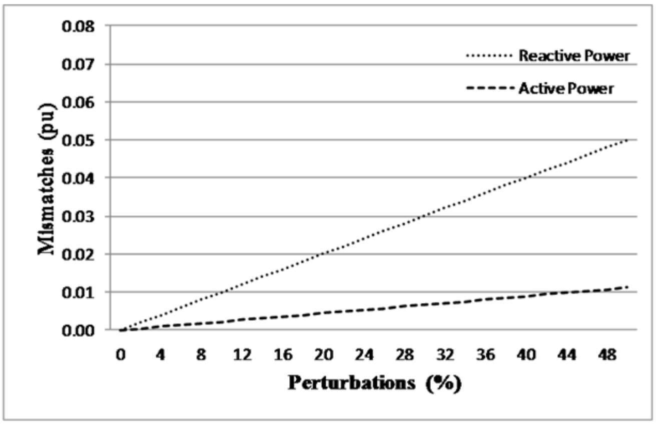

Fig. 6. Maximum active/reactive power mismatches to 74 bus system.

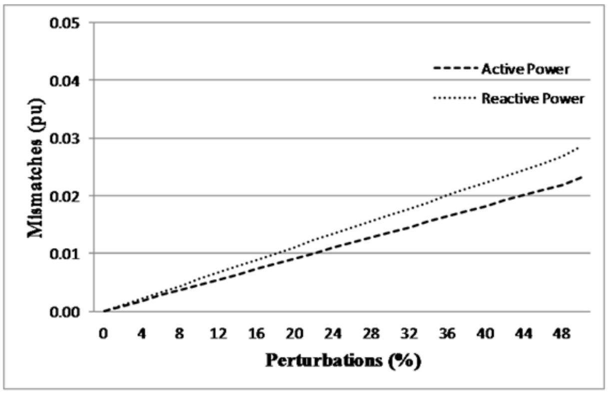

Fig. 7. Maximum active/reactive power mismatches to 476 bus systems.

3.3. Variation of magnitude and phase angle voltage

The variation of voltage angle (θ) and magnitude (V) in bus 55, of the 70 bus system, is shown in Fig. 8 and Fig 9 respectively, demonstrating the excellent results obtained by technique. The bus 55 was chosen randomly.

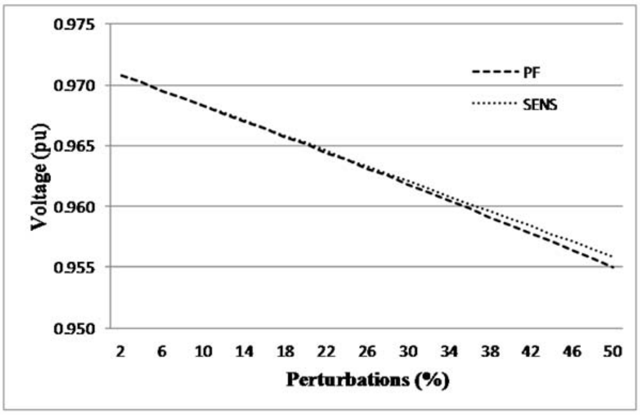

Fig. 8. Voltage bus 55.

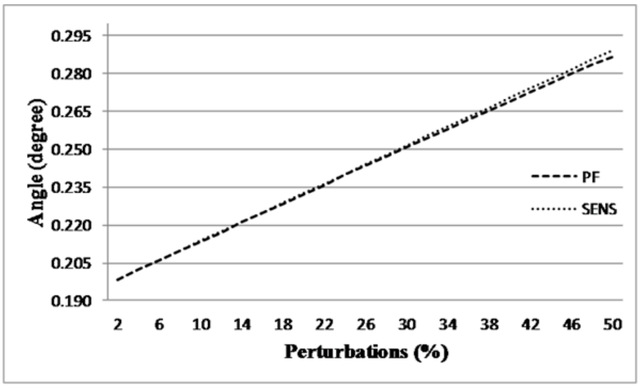

Fig. 9. Angle bus 55.

3.4 Estimation of active power losses nodal variation for 70 bus systems

With the solution of the base case at hand, the SENS methodology was applied to estimate the system losses variation after power consumption changes at the load buses. In this test, an increase single of 100 kW was performed in all load buses (with load) maintaining a constant power factor. The Fig. 10 presents the results obtained by SENS methodology and shows the buses most sensitive to load changes. Regarding the CPU time, the SENS routine spent approximately 0.27 s to perform the calculation. The same calculation using the PF would take approximately 25 s.

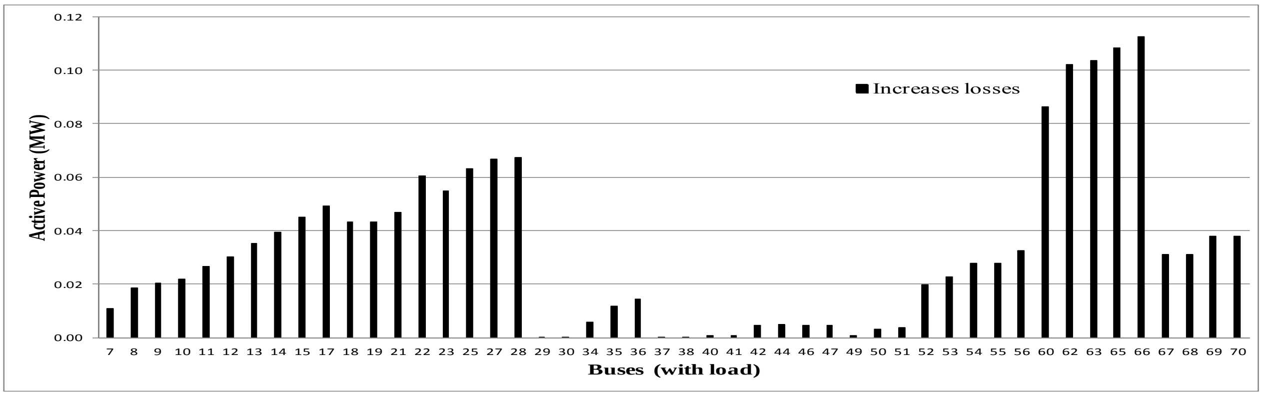

Fig. 10. Active power system losses variation obtained by SENS for an increase singly of 100 kW at load buses in the 70 bus system.

This test shows that buses 60, 62, 63, 65 and 66 cause higher losses in the system for this load situation studied. Buses 29, 30, 37, 38, 40, 41 and 49 cause fewer losses in the system.

4. Conclusions

This paper reports an approach to estimate operating point in distribution power system networks using sensitivity analysis oriented by PF. Perturbations were introduced up to an increase of 50% of the base loads and were observed that the active power losses provided by the SENS and PF are very similar. In most of the cases studied, it was shown that a solution could be safely estimated for perturbations in the order of 10%, by the sensitivity technique. The gain in time for processing methodology is approximately 99% compared to the PF. It was also shown that is possible to estimate indicators in few seconds to helping in decision-making and compose MWh tariff values over time for each load bus of the system. The tests performed with the 34, 70 and 476 bus systems confirm these statements.

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