QRS complex morphologies identification by the Lipschitz exponents of Wavelet Transform

**Abstract.** Wavelet transform is the effective tool for detailed description of the analysed signal. The electrocardiography (ECG) signal is not the exception in this term. Lipschitz exponents represent an additional tool that can be used to define the measure of the local signal shape/regularity. There are results of the wavelet transform Lipschitz exponents analysis presented in the paper. The main goal was to find their usability in the task of different QRS complex types discrimination.

**Keywords:** Wavelet transform, Lipschitz exponents, ECG signal analysis, Morphological analysis of QRS complex.

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**Introduction.** Automated ECG analysis modules are very popular even mandatory components in the contemporary ECG recorders [1,2,3,5,8]. Nowadays one would rather say that DSP module is the natural component of the ECG medical device. The DSP module performs several functions starting from often adaptive noise rejection, ending with complete ECG signal automated analysis. The latter stage is of the crucial importance because the final computer aided diagnosis depends on the DSP stage reliability.

The paper does not cover the entire area of the ECG signal automated analysis. It is concentrated on the QRS detection and also its description as these stages play essential role in the computer aided diagnosis.

**Background.** Lipschitz exponents (LE) are widely used in computer based signal analysis, especially in the pattern recognition problems [4,6,8,9]. This tool is very helpful in the characterisation of signal regularity. Combined with a wavelet transform LE can be used to parameterise local scale (an also frequency) properties of the analysed signal. Generally by the use of LE one can describe the decay of wavelet transform across the scale axis. This preserve the detailed information on the local signal regularity. LE are used to distinguish QRS complex from among other ECG waves [8]. As it is performed in the defined neighbourhood, namely QRS complex in Author’s research, the properties of this wave (QRS morphologies) are expected to be extracted. Author uses differential formula (1), derived from [4,6] during the LE computations.

\[
\alpha = \frac{\log_2 \left( \frac{\text{Wf}(s_{j+1}, x)}{\text{Wf}(s_j, x)} \right)}{\log_2 \left( \frac{s_{j+1}}{s_j} \right)}
\]

(1)

Lipschitz exponents are calculated based on the Continuous Wavelet Transform (CWT) that in turn is defined by formula (2). Author intentionally uses CWT commonly regarded as redundant. Particular advantage of CWT over fast DWT is that CWT ensures better and more comfortable possibility of analysing and viewing the results. Any time it can be substituted for DWT if necessary or justified.

\[
W_{\psi,f}(u, s) \equiv \left\langle f(t), \psi_{u,s}(t) \right\rangle = \int_{-\infty}^{\infty} f(t) \cdot \frac{1}{s} \psi^* \left( \frac{t-u}{s} \right) dt
\]

where \(\psi\) is a wavelet function; \(f\) is an analysed function; \(t\) is a time variable, \(s\) and \(u\) are dilation and translation coefficients respectively.

The main goal of the research is to define Lipschitz exponent usefulness in the problem of different QRS complex type (morphology) discrimination.

**Proposed solution.** There is MIT-BIH Arrhythmia Database [7] used during the experiments. It is characterised by many different, described QRS complex morphologies recorded and included into the signal set. It serves as a golden standard during the research. At the beginning stage of the research Author performed Lipschitz exponents verification against the standard QRS complex (N in fig. 1) and one of the most common and characteristic, arrhythmic QRS complex depicting premature ventricular contraction (V in fig. 1).
All CWT computations were performed for three different wavelets: original Quadratic Spline (QS), averaged MITBIH normal sinus rhythm QRS complex wavelet (QRSN) and averaged MITBIH premature ventricular contraction QRS wavelet (QRSV). The first one widely proved its usability in the area of ECG signal analysis. Latter two wavelets were already prepared earlier during Author’s research in order to introduce some adopting techniques into the ECG signal analysis process. The results of the analysis of the N type QRS complexes were additionally compared: the ones obtained with Quadratic Spline (QS) wavelet against the ones obtained with averaged MITBIH normal sinus rhythm QRS complex wavelet based (QRSN). Results of the analysis of the QRS complexes type V were investigated in the same manner (this time QRSV wavelet was used respectively).

Fig. 2. Wavelet transform of a QRS complex, type N computed independently with QRSN and QS wavelets. Darker areas represent higher values of CWT coefficients.

Fig. 3. Effective frequency spectra calculated at the DWT scales used during computations. Spectra calculations were repeated for different wavelets:
- original Quadratic Spline wavelet (QS);
- averaged MITBIH normal sinus rhythm QRS complex wavelet (QRSN);
- averaged MITBIH premature ventricular contraction QRS wavelet (QRSV);

Results calculated during experiments confirmed Quadratic Spline (QS) wavelet adequateness in the ECG signal wavelet analysis, as its frequency spectra (plotted with symbol “o” in the fig. 3) are almost the same as the frequency spectra of the averaged MITBIH normal sinus rhythm QRS complex wavelet (QRSN plotted with symbol “+” in the fig. 3). The latter wavelet was prepared as the mean of the set of normal sinus rhythm QRS complexes extracted from the reference database [7]. From the figure 1 one can deduce the differences of the frequency spectra for the averaged MITBIH premature ventricular contraction QRS wavelet (QRSV plotted with symbol “+” in the fig. 3), as compared to the normal QRS complex wavelet. Spectra of the premature ventricular contraction are all moved toward lower frequency bands. Author uses different wavelets (also differently decimated) in order to obtain better DWT coefficients dynamic for different QRS complex morphologies and to narrow down the necessary scale set used in DWT calculations. These two aspects are important when reliability of the QRS detection algorithm to computational effort ratio is concerned. This is a very valuable property if the application is to run in a real-time or constrained computing power environment.

All computational procedure can be described as follow:

- signals coming from [7] with significant amount of N type (common case) and V type QRS complexes were selected to the experiments.
- selected areas containing N or V type QRS complexes were transformed (wavelet transform) with the use of QS and QRSN/QRSV wavelets respectively. An example of QRS type N is presented in the figure 2.

Fig. 3. Successive steps of ECG signal wavelet transform extraction. The wavelet transform was computed upon the signal presented in the figure 2.

Fig. 4. Scale and time projection of wavelet transform coefficients of a sample QRS complex. Arrow in the upper picture points the scale slope of wavelet transform local extreme decay. This property is used directly to compute Lipschitz exponents.
coefficient extreme lines were detected based on the QRS complex wavelet transform and sample results were presented in the bottom picture of the figure 3, and in the figure 4. In fact there are two lines associated with a single QRS complex: a single minimum line and a single maximum line. Plots presented in all figures apply to the absolute values of the wavelet transform coefficients. That is the reason, that they are named extreme lines instead of maximum (or minimum) lines.

Extreme lines obtained at the last stage of the ECG signal processing comprise input data of the Lipschitz exponents computations (regarding to the equation 1)

Results

Contrary to the figures plots, outcomes were analysed independently in terms of: minimum or maximum line membership. Of course, they were also analysed in parallel dependent on the QRS type and respective wavelet used as well.

Regardless to the type of QRS, type of extreme line and the type of wavelet used there is one important and essential deduction coming straight from the obtained results and plots. Analysing the exponents presented in the figure 5, it can be seen that exponents computed for QRS type V present in the signal (plots labelled with: \(\nabla, \times, \diamond, +\)) meaningfully differ from exponents computed for QRS type N (plots labelled with: \(\Box, \circ, \triangle, \ast\)) in the range of scales between 10 and 20.

Conclusions

Obtained outcomes are promising and show clearly that Lipschitz exponents can be used in applications that are used to differentiate main morphologies of QRS complexes. Different wavelets used during the computations confirm this affirmation. Observed properties can also be used during Dyadic Wavelet Transform (DyWT) computations. The region of differences in Lipschitz exponent distribution is quite wide so it can be utilised by DyWT coefficients. It must be mentioned here, that research should be extended to the whole set of N and V type QRS complexes present in the database in order to make presented results confirmed in complete referenced manner. Continued research carried upon the entire reference database can potentially prove Lipschitz exponents useful in discrimination of an extant types of QRS complex morphologies.

REFERENCES

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