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Description of the manipulator robot's workspaces with three mobility degrees in the form of the logical expressions

***Abstract**. This work considers problems of the manipulating robot working spaces description, which have three degrees of mobility. Depending on the limits of the generalized coordinates value changes by mobility degrees, there were achieved the descriptions of the manipulating robots working spaces in the form of logical expressions.*

***Streszczenie.** W pracy rozważany jest problem opisu przestrzeni roboczej manipulatora robota o trzech stopniach swobody. W zależności od zakresu zmian wartości stopni mobilności we współrzędnych uogólnionych zostały uzyskane opisy przestrzeni roboczej manipulatora robota w postaci wyrażeń logicznych. (**Opis przestrzeni roboczej manipulatora robota o trzech stopniach swobody w formie wyrażeń logicznych**).*

Keywords: Manipulating robot, kinematic chain, mobility degree, working space.
Słowa kluczowe: Manipulator robota, łańcuch kinematyczny, stopień swobody, przestrzeń robocza.

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Introduction

Handling the robot is an open kinematic chain. The kinematic chain consists of series-connected different parts of joints. The gripper is attached to the last link of a robot manipulator. Handling the robot can move things along a predetermined path of motion or from a given point to the next set point. These operations can be performed only within the working space of a robot manipulator. Therefore, one of the important characteristics of manipulation robots is their workspace.

In general, the working space is a closed spatial figure. The workspace is determined by the kinematic structure, the size of units and limits of changing the parameters of the joints. The mathematical description of the robot workspace will solve the problem of coverage of all the defined workspace paths to manipulate objects. This problem is also important in case of modern model-based control systems [1,2] basically for the sake of model simplicity resulting in better robustness and speed.

Let there be given a kinematic structure, the size of units, types of joints of a robot manipulator. Consider the task of changing the form of the working space of a robot manipulator based on the limits of variation of the parameters of joints.

Description of robot manipulator work space with three rotation joints

Let the manipulation robot has three mobility degrees (fig. 1), the first degree of mobility – is the rotation around the axis OZ to the angle α, the second – is the rotation around the straight line of the presented angle α and point O to the angle β, the third – the rotation around the straight line of the presented angle α and point A to the angle γ. Moreover, the links' lengths are performed by the condition $l_1 > l_2$. The angles of rotation α, β, γ, the generalized coordinates, determine the robot position in the space $OXYZ$.

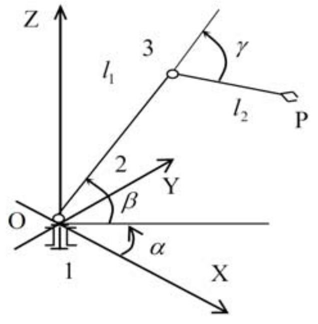

Fig.1. Structure of the manipulating robot

The result of the above stated condition is a workspace (fig. 2), limited in the plane OXZ by the radius spheres R and r, cones of rotation received by rotations of straight lines OA, OD around the axis OZ, torus obtained by rotations of the radius circle l_2 with centers at the points A and D. In the plane OXY (fig. 3) the robot's workspace is limited by radius spheres R and r, planes passing through the straight lines MP and NQ, which are perpendicular to the plane OXY.

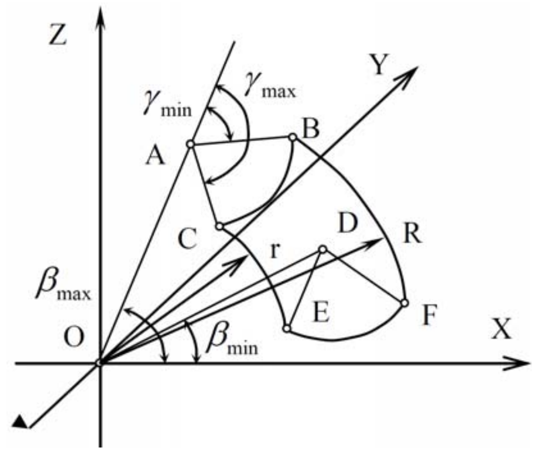

Fig.2. Working space of the manipulating robot

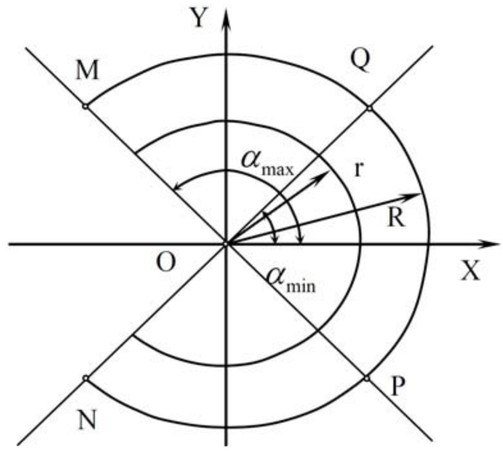

Fig.3. Projection of working space in the plane OXY

In order to describe the robot's workspace, let us apply the R-functioned mathematical apparatus [3,4,5]. To define the surfaces limiting the working space we determine their logical variables:

D_1 – the logical variable that determines the internal subspace of the radius sphere $R=\sqrt{(l_1+l_2 Cos Cos\gamma_{\min})^2+(l_2 Sin\gamma_{\min})^2}$:

(1) $$D_1=\begin{cases}1, \text{ if } R^2-x^2-y^2-z^2\geq 0,\\ 0, \text{ otherwise.}\end{cases}$$

D_2 – the logical variable that determines the external subspace of the radius sphere $r=\sqrt{(l_1-l_2 Cos(180^\circ-\gamma_{\max}))^2+(l_2 Sin(180^\circ-\gamma_{\max}))^2}$:

(2) $$D_2=\begin{cases}1, \text{ if } x^2+y^2+z^2-r^2\geq 0,\\ 0, \text{ otherwise.}\end{cases}$$

D_3 – the logical variable that determines the external subspace of the cone obtained by rotation the straight OA, $z=\tan\beta_{\max}\times x$, around the axis OZ:

(3) $$D_3=\begin{cases}1, \text{ if } z^2-\tan^2\beta_{\max}(x^2+y^2)\geq 0,\\ 0, \text{ otherwise.}\end{cases}$$

D_4 – the logical variable that determines the internal subspace of the cone obtained by rotating the straight OD, $z=\tan\beta_{\min}\times x$, around an axis OZ:

(4) $$D_4=\begin{cases}1, \text{ if } z^2-\tan^2\beta_{\min}(x^2+y^2)\leq 0,\\ 0, \text{ otherwise.}\end{cases}$$

D_5 – logical variable that determines the external subspace of the torus obtained by rotation the circle centered at the point A, around the axis OZ, $(x-l_1\cos\beta_{\max})^2+(z-l_1\sin\beta_{\max})^2=l_2^2$:

(5) $$D_5=\begin{cases}1, \text{if} & ((z-l_1\sin\beta_{\max})^2x^2+y^2-l_1^2\cos^2\beta_{\max}-l_2^2)^2-\\ & -4l_1^2\cos^2\beta_{\max}(x^2+y^2)\geq 0,\\ 0, & \text{otherwise.}\end{cases}$$

D_6 – the logical variable that determines the internal subspace of the torus obtained by rotation the circle centered at the point D, around the axis OZ $(x-l_1\cos\beta_{\min})^2+(z-l_1\sin\beta_{\min})^2=l_2^2$,:

(6) $$D_6=\begin{cases}1, \text{ if} & ((z-l_1\sin\beta_{\min})^2x^2+y^2-l_1^2\cos^2\beta_{\min}-l_2^2)^2-\\ & -4l_1^2\cos^2\beta_{\min}(x^2+y^2)\leq 0,\\ 0, & \text{otherwise.}\end{cases}$$

D_7 – the logical variable that determines the half plane, located below the plane, defined by the line NQ, $y=\tan\alpha_{\min}\times x$, perpendicular to the plane OXY:

(7) $$D_7=\begin{cases}1, \text{ if } y-\tan\alpha_{\min}\times x\leq 0,\\ 0, \text{ otherwise.}\end{cases}$$

D_8 – the logical variable that determines the half plane, located above the plane defined by the line MP, $y=\tan\alpha_{\max}\times x$, perpendicular to the plane OXY:

(8) $$D_8=\begin{cases}1, \text{ if } y-\tan\alpha_{\max}\times x\geq 0,\\ 0, \text{ otherwise.}\end{cases}$$

Let the robot working space is located in the sector limited by the angle $\angle QOP$ (fig. 3).

On the basis of the logical variables (1) - (8), let us form the description of the robot workspace in the following logical expression [2,3]:

$$((D_1\cap D_2\cap D_3\cap D_4\cap D_5)\cup \\ \cup(D_1\cap D_2\cap \overline{D_4}\cap D_6))\cap(D_7\cap D_8)=1.$$

Now, let suppose that the working space of the robot is limited in the space OXY in the angle $\angle MON$. At that moment, the logical expression takes the following form:

$$((D_1\cap D_2\cap D_3\cap D_4\cap D_5)\cup \\ \cup(D_1\cap D_2\cap \overline{D_4}\cap D_6))\cap(D_7\cup D_8)=1.$$

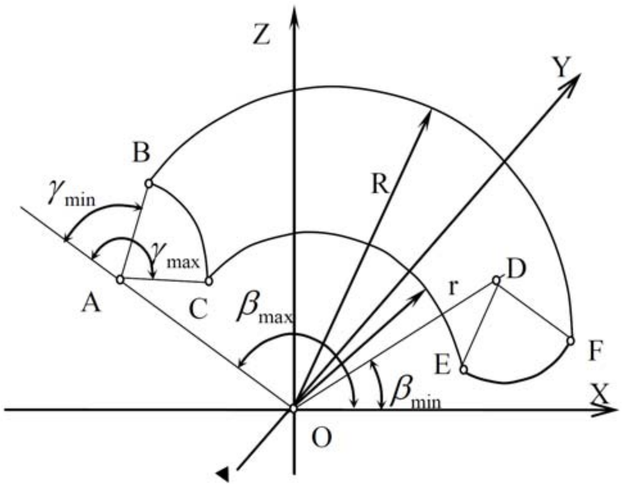

Fig.4. Projection of working space in the plane OXZ

Let us change the value of the angle $\beta_{\max}$, in this case we get the robot working space which is presented in Fig. 4.

Let the robot workspace is located in the sector limited by the angle $\angle QOP$ (Fig. 3) and also performed by the following condition:

(9) $$180^\circ-\beta_{\max}>\beta_{\min}$$

Then, to describe the robot's working space, in addition, we introduce the logical variable D_9 that determines the external subspace of the torus obtained by the rotation of the circle centered at point *A*, $(x+l_1 Cos(180^\circ-\beta_{\max})^2+(z-l_1 Sin(180^\circ-\beta_{\max})^2=l_2^2$, around the axis OZ, takes the form:

(10) $$D_9=\begin{cases}1, \text{if} & ((z-l_1\sin(180^\circ-\beta_{\max})^2x^2+y^2-l_1^2\cos^2(180^\circ-\beta_{\max})-l_2^2)^2-\\ & -4l_1^2\cos^2\beta_{\max}(x^2+y^2)\geq 0\\ 0, & \text{otherwise.}\end{cases}$$

On the basis of the logical variables (1) - (4), (6) - (8), (10) let us form the description of the working space of the given robot as follows:

$$(D_1\cap D_2\cap D_4\cap D_9\cap \overline{D_7}\cap \overline{D_8})\cup((D_1\cap D_2\cap \overline{D_3})\cup \\ \cup(D_1\cap D_2\cap D_3\cap D_6))\cap(D_7\cap D_8)=1.$$

Let the robot working space is located in the sector limited by the angle $\angle MON$ (fig. 3) and also performed by the condition (9). Then, the description of the robot working space takes the form:

$$(D_1\cap D_2\cap D_4\cap D_9\cap \overline{D_7}\cap \overline{D_8})\cup((D_1\cap D_2\cap \overline{D_3})\cup \\ \cup(D_1\cap D_2\cap D_3\cap D_6))\cap(D_7\cup D_8)=1$$

Let the robot working space is located in the sector limited by the angle $\angle QOP$ (fig. 3) and also performed the following condition:

(11) $$180^\circ-\beta_{\max}\leq\beta_{\min}.$$

On the basis of the logical variables (1) - (4), (6) - (8), (10), let us form the description of the working space of the robot:

$$(D_1\cap D_2\cap \overline{D_3}\cap D_9\cap \overline{D_7}\cap \overline{D_8})\cup((D_1\cap D_2\cap D_4)\cup \\ \cup(D_1\cap D_2\cap \overline{D_4}\cap D_6))\cap(D_7\cap D_8)=1$$

Description of the workspace, when changing limits of variation of the articulation parameters of the robot manipulator

Let the robot workspace is located in the sector, limited by the angle $\angle MOQ$ (fig. 5) and also performed the condition (9). Then, the logical expression that describes presented working space takes the following form:

$$(D_1 \cap D_2 \cap \overline{D_3} \cap D_9 \cap \overline{D_7} \cap \overline{D_8}) \cup ((D_1 \cap D_2 \cap D_4) \cup \cup (D_1 \cap D_2 \cap \overline{D_4} \cap D_6)) \cap (D_7 \cup D_8) = 1 .$$

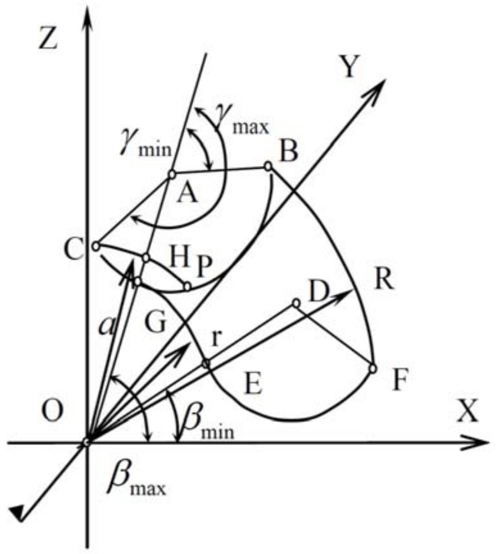

Fig.5. Projection of working space in the plane OXY

Let us change the value of the angle γ_{max}, then the robot's workspace takes the form shown in Fig. 5. In this case, in contrast to fig. 4, the workspace will be added by the area limited by the line *CHPG* appearing in the form of rotating figure, and which is limited by the internal surface of the torus and sphere (radius).By determining the logical variable (5) the value $r = l_1 - l_2$.

In addition, let us introduce the logical variable D_{10} describing the internal space of the sphere radius $a = \sqrt{(l_1 - l_2 Cos(\gamma_{max} - 180^\circ))^2 + l_2^2 Sin^2(\gamma_{max} - 180^\circ)}$:

(12) $$D_{10} = \begin{cases} 1, \text{ if } a^2 - x^2 - y^2 - z^2 \geq 0, \\ 0, \text{ otherwise.} \end{cases}$$

Let the robot workspace is located in the sector, limited by the angle $\angle QOP$ (fig. 3). Then, the logical expression on the basis of the logical variables (1) - (8), (12), describing this workspace, takes the following form:

$$((D_1 \cap D_2 \cap D_3 \cap D_4 \cap D_5) \cup (D_1 \cap D_2 \cap \overline{D_4} \cap D_6) \cup \cup (\overline{D_5} \cap D_{10})) \cap (D_7 \cap D_8) = 1 .$$

Let the robot workspace is located in the sector, limited by the angle $\angle MON$ (fig. 3). Then, the logical expression that describes the given work space takes the following form:

$$((D_1 \cap D_2 \cap D_3 \cap D_4 \cap D_5) \cup (D_1 \cap D_2 \cap \overline{D_4} \cap D_6) \cup \cup (\overline{D_5} \cap D_{10})) \cap (D_7 \cup D_8) = 1 .$$

Let us change the value of the angle β_{max} and get the robot's workspace represented in fig. 6.

Let the robot workspace is located in the sector, limited by the angle $\angle NOQ$ (fig. 3) and also performed by condition (9). Then, the logical expression on the basis of the logical variables (1) - (8), (10), (12), describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_3 \cap D_4) \cup \cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_9} \cap D_{10})) \cap (D_7 \cap D_8) = 1$$

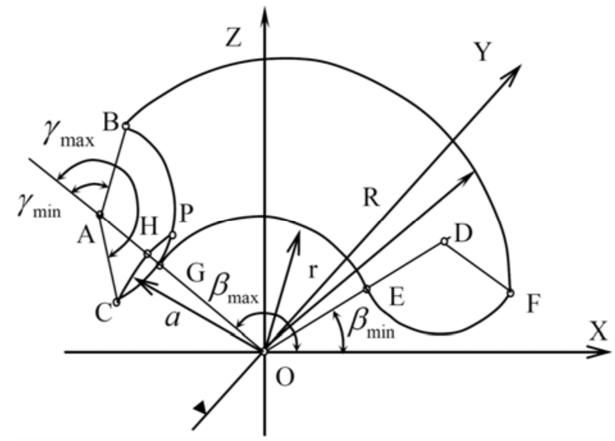

Fig.6. Projection of working space in the plane OXZ

Let the robot workspace is located in the sector, limited by the angle $\angle MON$ (fig. 3) and also performed by condition (9), then the logical expression that describes this working space takes the form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_3 \cap D_4) \cup \cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_9} \cap D_{10})) \cap (D_7 \cup D_8) = 1$$

Let the robot workspace is located in the sector, limited by the angle $\angle NOQ$ (fig. 3) and also performed by condition (11). In this case, the logical expression on the basis of the logical variables (1) - (8), (10), (12), describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_4) \cup \cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_9} \cap D_{10})) \cap (D_7 \cap D_8) = 1$$

Let the robot workspace is located in the sector, limited by the angle $\angle MON$ (fig. 3) and also performed by condition (11), then the logical expression that describes the given working space takes the form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_4) \cup \cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_9} \cap D_{10})) \cap (D_7 \cup D_8) = 1 .$$

In this case, in contrast to fig. 2, the workspace will be added by the area limited by the line *CHPG* appearing in the form of rotating figure is limited by the inner surface of the torus and sphere (radius). By determining the logical variable (5) the value $r = l_1 - l_2$.

Let us change the value of the angle γ_{min}, then the robot's workspace takes the form shown in fig. 7. In this case, in contrast to fig. 2, the workspace will be added by the area limited by the line *BHPG* appearing in the form of rotating figure is limited by the inner surface of the torus and sphere (radius). By determining the logical variable (1) the value $R = l_1 + l_2$.

In addition, let us introduce the logical variable D_{11} describing the external space of the radius sphere $a = \sqrt{(l_1 + l_2 Cos\gamma_{min})^2 + l_2^2 Sin^2 \gamma_{min}}$:

(13) $$D_{11} = \begin{cases} 1, \text{ if } x^2 + y^2 + z^2 - a^2 \geq 0, \\ 0, \text{ otherwise.} \end{cases}$$

Let the robot workspace is located in the sector, limited by the angle $\angle NOQ$ (fig. 3). Then, the logical expression on the basis of the logical variables (1) - (8), (13) that describes the given workspace takes the following form:

$$((D_1 \cap D_2 \cap D_3 \cap D_4 \cap D_5) \cup (D_1 \cap D_2 \cap \overline{D_4} \cap D_6) \cup \cup (\overline{D_5} \cap D_{11})) \cap (D_7 \cap D_8) = 1 .$$

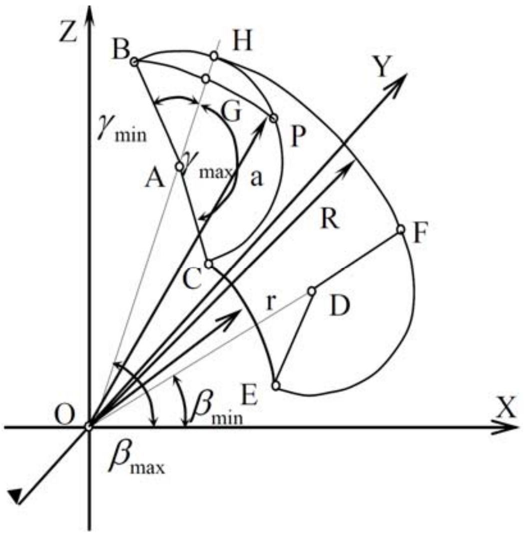

Fig.7. Projection of working space in the plane OXY

Let the robot workspace is located in the sector, limited by the angle $\angle MON$ (fig. 3). Then, the logical expression on the basis of the logical variables (1) - (8), (13) that describes the given workspace takes the following form:

$$((D_1 \cap D_2 \cap D_3 \cap D_4 \cap D_5) \cup (D_1 \cap D_2 \cap \overline{D_4} \cap D_6) \cup \cup (\overline{D_5} \cap D_{11})) \cap (D_7 \cup D_8) = 1 .$$

Let us change the value of the angle $\beta_{\max}$ and get the robot's workspace, represented in fig. 8.

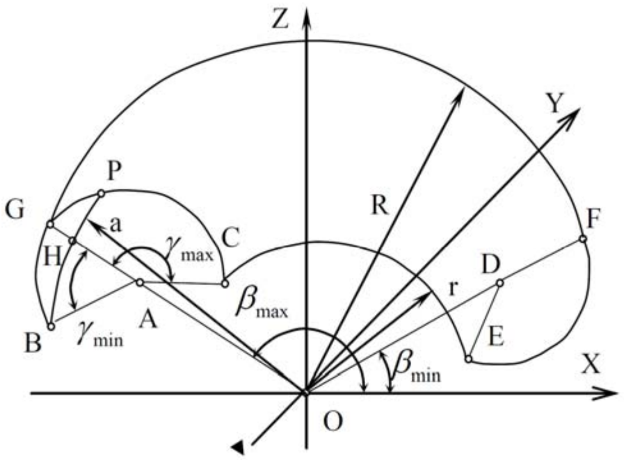

Fig.8. Projection of working space in the plane OXZ

Let the robot workspace is located in the sector, limited by the angle $\angle NOP$ (fig. 3) and also performed by condition (9). Then, the logical expression on the basis of the logical variables (1) - (8), (10), (12), describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_3 \cap D_4) \cup \cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_{10}} \cap D_{11})) \cap (D_7 \cap D_8) = 1 .$$

Let the robot workspace is located in the sector, limited by the angle $\angle MON$ (fig. 3) and also performed by condition (11), in this case, the logical expression describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_3 \cap D_4) \cup \cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_{10}} \cap D_{11})) \cap (D_7 \cup D_8) = 1$$

Let the robot workspace is located in the sector, limited by the angle $\angle NOQ$ (fig. 3) and also performed by condition (9). Then, the logical expression on the basis of the logical variables (1) - (8), (10), (12), describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_4) \cup \cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_{10}} \cap D_{11})) \cap (D_7 \cap D_8) = 1$$

Let the robot workspace is located in the sector, limited by the angle $\angle MON$ (fig. 3) and also performed by condition (11), in this case, the logical expression describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_4) \cup \cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_{10}} \cap D_{11})) \cap (D_7 \cup D_8) = 1$$

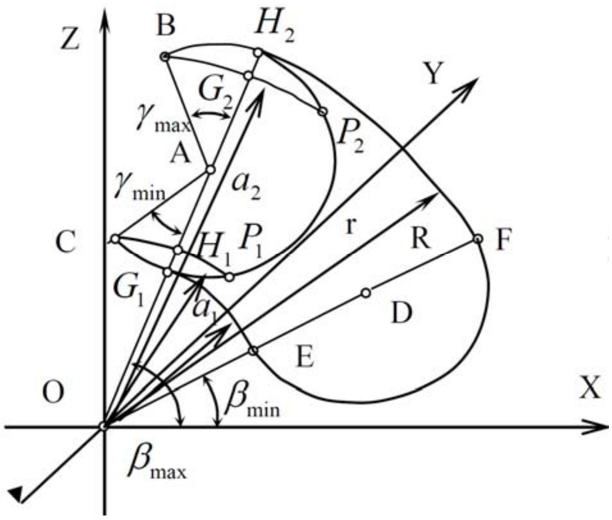

Fig.9. Projection of working space in the plane OXY

Let us change the value of the angle $\gamma_{\min}$ and $\gamma_{\max}$, then the robot workspace takes the form represented in Fig. 9. In this case, in contrast to fig. 4, the workspace will be added by the form of the rotating figure, limited by the torus internal surface and 2 radius sphere, limited by straights $BH_1P_1G_1$ and $CH_2P_2G_2$. By determining the logical variable (4), the value $R = l_1 + l_2$ and by determining the logical variable (5), the value $r = l_1 - l_2$. The magnitude of a for the lower zone $BH_1P_1G_1$ is marked as a_1, for upper zone $CH_2P_2G_2$ as a_2.

Let the robot workspace is located in the sector, limited by the angle $\angle NOQ$ (fig. 3). Then, the logical expression on the basis of the logical variables (1) - (8), (10), (12), describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap D_3 \cap D_4 \cap D_5) \cup (D_1 \cap D_2 \cap \overline{D_4} \cap D_6) \cup$$
$$\cup (\overline{D_5} \cap D_{10} \cap D_{11})) \cap (D_7 \cap D_8) = 1$$

Let the robot workspace is located in the sector, limited by the angle $\angle MON$ (fig. 3). Then, the logical expression describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap D_3 \cap D_4 \cap D_5) \cup (D_1 \cap D_2 \cap \overline{D_4} \cap D_6) \cup$$
$$\cup (\overline{D_5} \cap D_{10} \cap D_{11})) \cap (D_7 \cup D_8) = 1$$

Let us change the value of the angle $\beta_{\max}$ and get the robot's workspace, represented in Fig. 10.

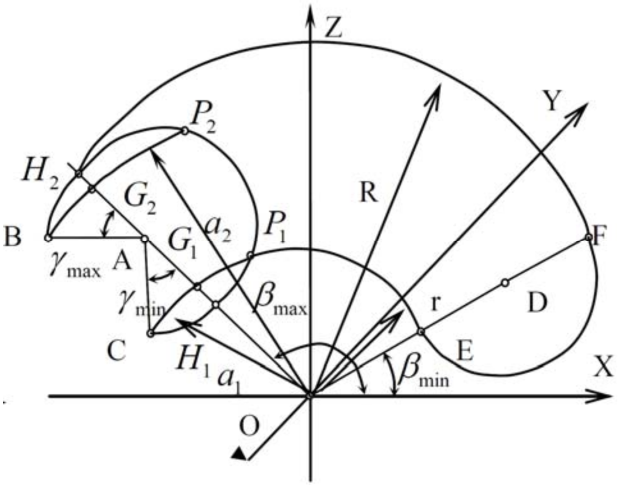

Fig.10. Projection of working space in the plane OXZ

Let the robot workspace is located in the sector, limited by the angle $\angle NOQ$ (fig. 3) and also performed by condition (9). Then, the logical expression on the basis of the logical variables (1) - (8), (10), (12), describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_3 \cap D_4) \cup$$
$$\cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_9} \cap D_{10} \cap D_{11})) \cap (D_7 \cap D_8) = 1$$

Let the robot workspace is located in the sector, limited by the angle $\angle MON$ (fig. 3) and also performed by condition (9), then, the logical expression describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_3 \cap D_4) \cup$$
$$\cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_9} \cap D_{10} \cap D_{11})) \cap (D_7 \cup D_8) = 1$$

Let the robot workspace is located in the sector, limited by the angle $\angle NOQ$ (fig. 3) and also performed by condition (11), then, the logical expression describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_4) \cup$$
$$\cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_9} \cap D_{10} \cap D_{11})) \cap (D_7 \cap D_8) = 1$$

Let the robot workspace is located in the sector, limited by the angle $\angle MON$ (fig. 3) and also performed by condition (11), then the logical expression describing the given workspace takes the following form:

$$((D_1 \cap D_2 \cap \overline{D_3} \cap D_9) \cup (D_1 \cap D_2 \cap D_4) \cup$$
$$\cup (D_1 \cap D_2 \cap D_6 \cap \overline{D_4}) \cup (\overline{D_9} \cap D_{10} \cap D_{11})) \cap (D_7 \cup D_8) = 1$$

Conclusions

As can be seen from the description of the workspace, the following conclusions may be applied:

- the view of the 3-degree manipulator workspace depends on the type of kinematic pairs forming the kinematic chain;
- the type of work space depends on the limits of change of the generalized coordinates by the mobility degrees of the robot manipulator;
- the description of the logical expression depends on the type of working space.

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