

# Determination of photovoltaic installation nominal power based on electrical energy consumption profile in the context of prosumer policy

**Abstract.** In this paper a novel approach to sizing PV system has been presented which takes into consideration the possibility of resealing the surplus of electrical energy yield. A mathematical model and its application based on real meteorological year and electrical energy consumption data from manufacturing facility.

**Streszczenie.** W poniższym artykule zaprezentowano nowe podejście do sposobu wymiarowania instalacji fotowoltaicznych, uwzględniające możliwość odsprzedaży nadwyżek energii elektrycznej do sieci. Przedstawiono model matematyczny, oraz jego aplikację w oparciu o rzeczywiste dane meteorologiczne. (Wyznaczanie mocy znamionowej instalacji fotowoltaicznej w oparciu o profil konsumpcji energii elektrycznej w kontekście polityki prosumenckiej)

**Słowa kluczowe:** fotowoltaika, model matematyczny, moc znamionowa, prosument  
**Keywords:** photovoltaics, mathematical model, nominal power, prosumer

## Introduction

Over the recent years an increase in electrical capacity installed in solar power plants utilizing photovoltaic effect is being observed. Also in Poland a slow development of photovoltaic marked is discernible however inhibited by an unclear standpoint of legislative body [1,2,3]. It is mainly result of descending trend of photovoltaic system costs and different forms of financial support [3]. One vision of renewable energy development suggests expansion of distributed generation/energetics, based on thousands of micro and small power plants of powers not exceeding 200 [kW]. The concept of distributed generation should not be narrowed only to renewable energy sources [5]. Role of electrical energy producers would be taken by owners of adjacent installations, from which part of electrical energy yield would be used for their own needs and the rest (for adequate price) sold to the mains. On that basis a notion of *prosument*, being a calque from English, has been adopted. It describes the producer and consumer of certain good in the same time [6,7]. Issue related to sizing photovoltaic systems sizing has been described, inter alia, in [8-14] where problems like: reliability aspects of power supply, off-grid photovoltaic systems, selection of energy storage systems or conversion of DC power to AC power have been presented. In hereunder article powered by mathematical model, an approach based on an assumption that, the surpluses in electrical energy generation from given PV system may be sold to the electrical grid.

## Mathematical model

Discussion about nominal power of photovoltaic system may be limited to the task which aims to realize required functions of the objective whilst their fulfilment should be accompanied by matching, determined at the beginning of the problem formulation, constraints. This problem may be depicted in a such a way: known are the initial investments  $N$  for PV system, and annual costs of maintenance  $n_j$ . Calculations are based on known electrical energy consumption  $Z_{i,j,k}$  and possible yield  $P_{i,j,k}$  of electrical energy from PV system. Incomes from PV system are understood as, savings resulting from not bought electrical energy from national grid for known cost  $V_i$  and revenue from resale of electrical energy surplus for known price  $B_i$ . Selection of PV system size should be done in such a way that the annual incomes per unit of installed capacity will be maximal. Hereabove defined problem may be expressed by a mathematical model, which is represented as follows (1):

$$(1) \quad W = N + \sum_{i=1}^{30} n_i - \sum_{i=1}^{30} \sum_{j=1}^{365} \sum_{k=1}^{24} \{ (P_{i,j,k} V_i) x_{i,j,k} + (1 - x_{i,j,k}) [ (-1)(P_{i,j,k} - Z_{i,j,k}) B_i + Z_{i,j,k} V_i ] \}$$

where:  $W$  – final account,  $N$  – initial investments for PV system in year 0,  $n_i$  – cost of service, replacement and maintenance of installation in year  $i$ ,  $B_i$  – price of electrical energy sold to the grid in year  $i$ ,  $V_i$  – price of electrical energy purchase in year  $i$ ,  $Z_{i,j,k}$  – electrical energy consumption in year  $i$  day  $j$  and hour  $k$ ,  $P_{i,j,k}$  – electrical energy yield in year  $i$  day  $j$  and hour  $k$ .

Furthermore, an extra binary variable is introduced to the model  $x_{i,j,k}$  described by formula (2), where all other variables are defined as in formula (1):

$$(2) \quad x_{i,j,k} = \begin{cases} 1 & \text{for } Z_{i,j,k} - P_{i,j,k} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

If the difference from subtraction of variable  $Z_{i,j,k}$  and  $P_{i,j,k}$  is greater than zero, then more electrical energy has been used for own demand than could has been generated from PV system. Equal values of these variables denote that the electrical energy demand has been covered by PV system. On the other hand, when  $P_{i,j,k}$  is greater than  $Z_{i,j,k}$  then PV system has generated more energy than was the demand in given period.

## Data and model implementation

Depicted by formulas (1, 2) mathematical model maps changes in incomes structure from photovoltaic installation over the 30 years period of it's operation, whilst considering the annual maintenance costs and the difference in income depending on the utilization of electrical energy yield. Such a model may be used as well ex-post as ex-ante. However the analysis looking to the future, is weighted by the error being result of the variability of solar radiation over the years and by the forecast concerning electrical energy consumption and its market prices. Forecasting model based on seasonal indicators wrought on the basis of meteorological data [15] for years 1985 - 2005, consists in 90,2% from seasonality, 1,5% trend and the random factor share is 8,3%. This prediction is weighted by mean absolute percentage error amounting to 19%. In figure 1 a match of forecast to real changes in energy of solar radiation on monthly basis has been presented. On the abscissa axis following months for years 1985 – 2005 have been

presented, starting from July and ending on June. On the ordinate axis we put the amount of solar radiation energy expressed in  $[Wh/m^2]$ .

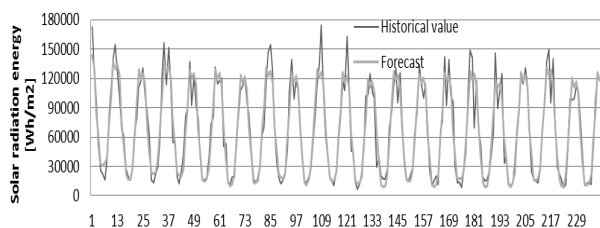


Fig.1. Forecast adjustment to the real values

In implementation of this model a meteorological data and this concerning electrical energy consumption in manufacturing facility for year 2004 has been used. In figure 2 characteristic daily electrical energy consumption for working days in June has been depicted.

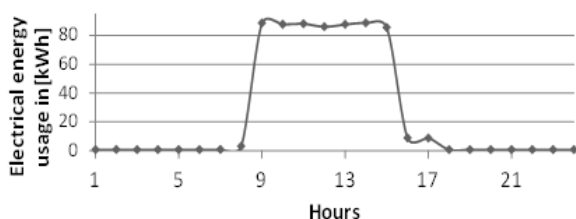


Fig.2. Daily average electrical consumption in June

For the scrutiny of changes in incomings from PV system, systems of nominal power of: 1, 10, 40 and 160 [kW] have been chosen. Results obtained from the model are presented in tables 1 i 2.

Table.1. Total income for different size PV systems [PLN]

month	Nominal power			
	1 kW	10 kW	40 kW	160 kW
I	10,6	95,7	329,7	1236,2
II	16,7	139,3	487,1	1849,2
III	38,6	330,7	1214,2	4507,2
IV	73,8	522,8	1905,8	6897,7
V	94,1	649,9	2369,2	8544,2
VI	86,7	625,3	2265	7803,9
VII	101,1	854,4	3243,2	11350
VIII	74,7	588,6	2204,2	8163,4
IX	60,5	461,3	1703,7	6089,2
X	36,2	280,7	996,6	3700
XI	14,7	129,4	444,3	1669,6
XII	6,6	64,9	214,8	772

Table.2. Percentage share of savings in total incomings during each month

month	Nominal power			
	1 kW	10 kW	40 kW	160 kW
I	100,0%	96,5%	90,6%	87,8%
II	100,0%	93,4%	87,6%	85,2%
III	100,0%	94,4%	90,9%	87,0%
IV	100,0%	86,2%	81,6%	76,2%
V	99,7%	84,6%	79,8%	74,0%
VI	100,0%	86,9%	82,0%	73,8%
VII	100,0%	93,8%	91,7%	85,8%
VIII	100,0%	91,0%	88,1%	84,5%
IX	100,0%	89,6%	86,0%	80,4%
X	100,0%	90,4%	84,9%	81,2%
XI	100,0%	95,6%	89,3%	86,5%
XII	100,0%	99,2%	92,1%	87,5%

As can be observed in table 1 according to expectations maximal incomings from PV system are generated in summer months when the values of solar radiation are highest. However the value of incomings is strongly dependent on the electrical energy consumption profile. Therefore the PV system income is not a linear function of its nominal power, because the price of electrical energy used for domestic demand is much higher than this resold (on behalf of the analysis prices of accordingly 0,6[PLN] and 0,15[PLN] has been adopted).

In table 2 a percentage share of incomings from savings in total incomings has been presented. As can be observed, the smallest system of power of 1 [kW] generates almost 100% of electrical energy only and solely will be consumed to fill the domestic demand. Based on the analysis of table 1 and 2 we can draw out a conclusion that with the increasing nominal power of the installation there is a corresponding drop of savings share in total incomings and as a result the total incomings per unit of installed capacity. It has an effect on the PV installation economics, however not always it is the most important criterion during assessing this kind of projects.

### Defining nominal power

Sizing of PV system nominal power has been based on typical meteorological year and predicted changes in electrical energy demand structure. Those predictions were made on the basis of information obtained from proprietor. In order to solve this optimization problem an available Excel 2007 Solver has been used. Objective function has been determined as:

- maximization of income per unit of installed capacity;
- annual coverage of electrical energy demand equal to 100% - energy balance equal to 0;
- electrical energy demand coverage in months VI-VIII, energy balance equal to 0;
- electrical energy demand coverage in months XII-I-II, energy balance equal to 0.

The results of realized objective functions have been presented in table 3 whereby for each solution the income per unit of nominal power has been calculated. For objective function „a” an assumption has been made that, nominal power of the system can not be lesser than 1[kW].

Table.3. Nominal power and income per unit of power

Objective function	a	b	c	d
Nominal power	1	217	166	655
Income [PLN/1kW]	484	305	321	214

As a result of conducted calculations a huge span in area of nominal power of photovoltaic system, which realize different objective functions. Despite the fact that the nominal power of PV system realizing objective function „d” it is almost three Times greater than the system which ensures energy balance equal to 0 (function „b”) the predicted income per unit of power is lesser of not the whole 30%. For better presentation of changes in income per unit of power, those changes have been visualized in figure 3, for greater number of systems.

Considering great variability in terms of solar energy availability over the year, the selection of PV system size is strongly dependent on made assumption which objective function is about to be realized. For objective functions „c” and „d” a nearly quadruple difference in system nominal power has been achieved. What is more, in winter period, photovoltaic system may utterly be out of order as a result of snow layer covering PV modules, which prevents occurring of photovoltaic effect.

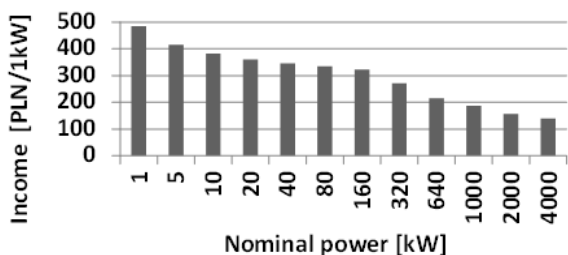


Fig.3. Changes in incomes per unit of power

### Economics of presented solutions

In order to evaluate economic efficiency of presented solutions a pointed hereunder assumptions have been made:

- based on [16] it has been stated that, the overall cost of PV installation can be expressed by formula (3):

$$(3) \quad \text{cost}(p) = 5845p + 3897$$

where:  $\text{cost}(p)$  – cost of PV system in [PLN],  $p$  – nominal power of installation expressed in [kW];

- based on [17] it has been adopted, that according to realistic scenario, the electrical energy increase will be around 3,8% per year;
- from [18] an assumption has been drawn, that it is necessary to replace, every 10 years, some parts of inverters which amounts to 10% of their initial cost;
- the efficiency of PV modules drops about 1% every year;
- the yearly maintenance cost were estimated to be around 0,5% of initial installation cost;
- investment has been realized from own funds and other ways of their usage were not considered;
- the cost of electrical energy purchase is 0,6[PLN/kWh] and sold to the grid is 0,15[PLN/kWh];
- in other scenario a 40% co-financing has been presented.

On the basis on hereabove assumptions, the predicted turnaround time for systems of different nominal power has been calculated. The size of each system is a result of objective functions “a-d” realization. Results have been presented in table 4.

Table.4. Expected payback time

Objective function	a	b	c	d
Payback time [years]	17-18	16-17	16-17	23-24

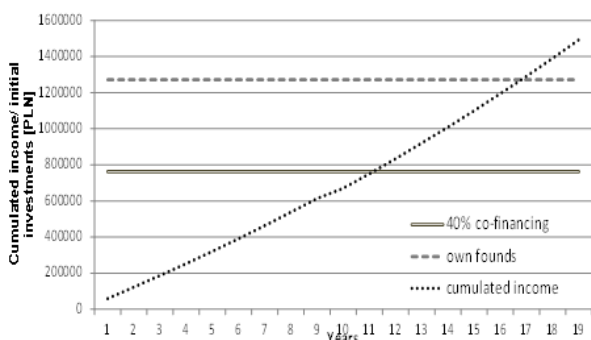


Fig.4. Cumulated income set together with the initial cost of PV system

Under the provisio of financial support from different institutions promoting renewable energy sources, the predicted turnaround time may be shortened proportionally

to the share of external capital. In figure 4 a accumulated income for 217[kW] PV system has been presented.

### Summary

Presented in article mathematical model allows not only estimating predicted incomes from PV installation but also to determine its optimal nominal power, which enables realization of chosen objective functions. The way how the model has been written, causes that its interpretation is easy and legible, and the implementation in MS Excel comes without troubles. Based on conducted calculations, we have demonstrated that predicted turnaround time of PV installations is about 16-24 years in situation when there are no external funds. It is necessary to stress that those results have been achieved for the assumption that electrical energy sold to the grid is four Times cheaper than bought one.

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**Authors:** dr hab. inż., Prof.AGH Jerzy Mikulik, Akademia Górniczo-Hutnicza, Wydział Zarządzania, ul. Antoniego Gramatyka 10, Kraków, E-mail:[jmikulik@zarz.agh.edu.pl](mailto:jmikulik@zarz.agh.edu.pl); mgr inż. Jakub Jurasz, Akademia Górniczo-Hutnicza, Wydział Zarządzania, ul. Antoniego Gramatyka 10, Kraków, E-mail: [jakubkamijurasz@gmail.com](mailto:jakubkamijurasz@gmail.com)