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# Power effective work of PMSM in electric vehicles at the account of magnetic saturation and iron losses

**Abstract.** In this paper the improved model of permanent magnet synchronous machine (PMSM) considering iron losses and magnetic saturation is developed. The optimal dependences between projections of armature current for maximum energy efficiency of PMSM is calculated. To determine the point of transition from the first to the second zone of speed regulation, the information about the references on projections of inverter voltage is used. The simulation model of control system of PMSM for electric vehicle, which operates with maximum power efficiency at wide range of speed and is insensitive to parametric changes of PMSM, is implemented.

**Streszczenie.** W artukóle opracowano polepszone modeli matematyczne i komputerowe maszyny synchronicznej o magnesach trwałych (PMSM) z uwzględnieniem nasycenia magnetycznego oraz strat w żelazie. Obliczono krzywe optymalne składowych prądu twornika dla maksymalnej efektywności energetycznej pracy PMSM. W celu ustalenia punktu przejścia od pierwszej do drugiej strefy sterowania prędkości proponuje się wykorzystać informacje o zadaniu napięcia falownika. Stworzono model komputerowy układu sterowania napędem pojazdów elektrycznych na bazie PMSM, który działa z maksymalną wydajnością w szerokim zakresie prędkości obrotowej oraz jest nieczuły na zmiany parametryczne. (Energooszczędna praca PMSM w pojazdach elektrycznych z uwzględnieniem nasycenia magnetycznego i strat w żelazie)

**Keywords:** permanent magnet synchronous machine, flux weakening, magnetic saturation, iron losses, efficient control. **Slowa kluczowe:** maszyna synchroniczna o magnesach trwałych, osłabienie pola, nasycenie magnetyczne, straty w żelazie, energoefektywne sterowanie.

#### Introduction

At last decades wide application were finding electric drives on the basis of synchronous machines with permanent magnets (PMSM). Such systems have high energy performance and good dynamics at relative simplicity of control.

In different papers optimum from the power point of view operating modes of PMSM were defined with the account or only losses in copper [1], or in copper and in iron [2,3], or in copper taking into account magnetic saturation [4]. However in electric vehicles, where the traction torque and the angular speed of the motor changes largely, an optimum control of PMSM should be carried out taking into account all features of work and providing the minimum total losses in copper and iron of the motor.

# Mathematical modeling of PMSM taking into account magnetic saturation and iron losses

Based on the equivalent circuits of PMSM in rotating reference frame dq orientated along the rotor magnetic flux (Fig. 1), the steady state electromagnetic processes is possible to describe by the following equations [3]:

(1) 
$$\begin{cases} i_{q} = i_{q0} + \frac{\hat{L}_{q}(i_{q0})}{R_{c}} \frac{d}{dt} i_{q0} + i_{d0} \frac{\hat{L}_{d}(i_{d0})\omega}{R_{c}} + \frac{\psi_{pm}\omega}{R_{c}} \\ i_{d} = i_{d0} + \frac{\hat{L}_{d}(i_{d0})}{R_{c}} \frac{d}{dt} i_{d0} - i_{q0} \frac{\hat{L}_{q}(i_{q0})\omega}{R_{c}} \\ \end{cases}$$
(2) 
$$\begin{cases} u_{q} = i_{q0}R + \hat{L}_{q}(i_{q0})k_{r} \frac{d}{dt} i_{q0} + \psi_{iq}(i_{q0})\omega k_{r} + \psi_{pm}\omega k_{r} \\ u_{d} = i_{d0}R + \hat{L}_{d}(i_{d0})k_{r} \frac{d}{dt} i_{d0} - \psi_{id}(i_{d0})\omega k_{r} \end{cases}$$

where  $\omega$  is the angular frequency of armature voltage,  $\psi_{pm}$  is the flax linkage of the armature windings coursing by the permanent magnets of the rotor,  $k_r = (1 + R/R_c)$  is the coefficient characterizing the ratio between the losses in copper and iron losses, all other denominations are clear from Fig. 1.

The equations described electromagnetic torque, mechanical part and efficiency of PMSM accordingly are as follows

(3) 
$$T = \frac{3}{2} Z_p \left[ \psi_{pm} i_{q0} + \left( \hat{L}_d (i_{d0}) - \hat{L}_q (i_{q0}) \right) i_{d0} i_{q0} \right]$$

(4) 
$$J\frac{d}{dt}\omega_r + b\omega_r = T - T_c$$

(5) 
$$\eta = \frac{T\omega_r}{\Delta P_{\rm Cu} + \Delta P_{\rm Fe} + T\omega_r}$$

where  $Z_p$  is the number of pairs of the poles of PMSM,  $\omega_r = \omega/Z_p$  is the angular speed of the motor, and the losses in copper and iron can be calculate accordingly to expressions [2]

$$\Delta P_{\rm Cu} = \frac{3}{2} R \left( i_q^2 + i_d^2 \right)$$

(7) 
$$\Delta P_{\text{Fe}} = \frac{\omega^2}{R_c} \left[ \left( \hat{L}_q(i_{q0}) i_{q0} \right)^2 + \left( \hat{L}_d(i_{d0}) i_{d0} + \psi_{pm} \right)^2 \right]$$



Fig. 1. The equivalent circuit of PMSM in rotating reference frame orientated along the rotor magnetic flux

Resistance  $R_c = 1/(K_f + K_h/\omega)$  on the schemes on Fig. 1 and in the expressions (1), (2) and (7) models the iron losses [3], and the coefficients  $K_h$  and  $K_f$  characterize the losses on a hysteresis and eddy currents accordingly.

In contrast to the classical model of PMSM [1], the received mathematical model of the machine taking into account magnetic saturation (1)–(3) contains flux linkages  $\psi_{di}(i_{d0})$ ,  $\psi_{qi}(i_{q0})$  and differential inductances  $\hat{L}_d(i_{d0})$ ,  $\hat{L}_q(i_{q0})$ , which not-linear depend on an armature current. According to the method proposed in [5], flux linkage of armature reaction is determined on expressions

(8) 
$$\begin{cases} \psi_{di}(i_{d0}) = a_1 \arctan(a_2 i_{d0}) + a_3 i_{d0} \\ \psi_{qi}(i_{q0}) = b_1 \arctan(b_2 i_{q0}) + b_3 i_{q0} \end{cases}$$

where  $a_i$  and  $b_i$  are the approximation coefficients of the dependences which can be obtained from computing field research of specific PMSM by finite element method (FEM).

The time derivatives of both components of flux linkage looks like

(9) 
$$\begin{cases} \frac{d}{dt}\psi_{di}(i_{d0}) = \frac{d\psi_{di}(i_{d0})}{di_{d0}} \cdot \frac{di_{d0}}{dt} = \hat{L}_{d}(i_{d0})\frac{d}{dt}i_{d0} \\ \frac{d}{dt}\psi_{qi}(i_{q0}) = \frac{d\psi_{qi}(i_{q0})}{di_{q0}} \cdot \frac{di_{q0}}{dt} = \hat{L}_{q}(i_{q0})\frac{d}{dt}i_{q0} \end{cases}$$

From here the dependences of differential inductances from corresponding currents will be equal

(10) 
$$\hat{L}_d(i_{d0}) = \frac{a_1 a_2}{1 + a_2^2 i_{d0}} + a_3; \quad \hat{L}_q(i_{q0}) = \frac{b_1 b_2}{1 + b_2^2 i_{q0}} + b_3$$



Fig. 2. Diagrams of the basic variables and indicators of the PMSM at rated speed for the cases:

a) account only of losses in copper, b) account of losses in copper and in iron at magnetic saturation

In the MathCAD software the program, which carries out the calculations on expressions (1)–(10) for the purpose of power optimization, was developed. The program allows visualizing the results in the form of diagrams in reference frame  $i_q(i_d)$  of the basic variables in relative units at normalizing on their rated values. For example, on Fig. 2 for the PMSM (the basic rated parameters are  $T_n = 300$  Nm,  $\omega_{r.n} = 100 \text{ s}^{-1}$ ,  $Z_p = 2$ ,  $I_n = 200$  A, R = 0.075 Ohm,  $L_{d0} = 0.5$  mH,  $L_{q0} = 1.5$  mH,  $\psi_{pm} = 0.5$  Wb,  $I_{max} = 3I_n$ ,  $K_f = 0.18$  Ohm<sup>-1</sup>,  $K_h = 90$  Ohm<sup>-1</sup>·s<sup>-1</sup>,  $a_1 = 5.2$ ,  $a_2 = 0.005$ ,  $a_3 = -0.0021$ ,  $b_1 = 5.2$ ,  $b_2 = 0.009$ ,  $b_3 = -0.00135$ ) are presented the diagrams received for the rated angular speed and for cases of the account only of loss in copper (Fig. 2a) and of losses in copper and iron at the account of magnetic saturation (Fig. 2b). In diagrams there are the isolines of torque (a scale at the left) and efficiency (lines 4). The curves 3 show the optimal from the power point of view dependences between the currents  $i_{q.opt}(i_{d.opt})$ . Moreover, the arches 1 and 2 reflect the limitations on accordingly

maximum and continued allowable values of rms armature current.

## Optimum curves of efficiency control

For maintenance of an operating mode with constant power in electric drives of vehicles on the basis of PMSM it is necessary to regulate a transverse component of armature current which weakens the field of machine. Besides, for work in the second zone with field weakening it is necessary to take into consideration the current limit and voltage limit

(11) 
$$i_s = \sqrt{i_d^2 + i_q^2} \le I_{\max}$$

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(2) 
$$u_s = \sqrt{u_d^2 + u_q^2} \le U_{DC} / \sqrt{3}$$

Typical diagram of currents, and also curves of the equal torques and angular speeds are represented on Fig. 3. Optimum currents  $i_d$  and  $i_q$  are formed according to the curve OA in the first zone: if loading increase the working point moves from O to A. Work above the point A on curve OA is impossible because of limitations on the maximum current (11). Voltage limitation (12) inactive at rated speed, therefore system work is possible with  $i_{q.opt}(i_{d.opt})$ . If speed increases to  $\omega_2$  the machine emf increases so that the further increase in speed with the maximum torque (work in the point A) is impossible. To provide the further run-up of the machine, it is necessary to form the references signals of currents  $i_d$  and  $i_q$  on the curve AB.



Fig. 3. Diagrams of the basic variables of PMSM

If to compare Fig. 2a and 2b, it can be concluded that the withdrawal from curve OA to curve AB leads to reduction of efficiency of system. However, at work in the second zone at small accelerations or at small static loadings work on a curve  $i_{q.opt}(i_{d.opt})$  is possible. So, for example, for the speed  $\omega_2$  at the necessary electromagnetic torque  $(0..., T_2)$  it is desired a work on site OA'. It will provide the maximum power efficiency. If the electromagnetic torque be more  $T_2$  currents  $i_d$ ,  $i_q$  are necessary for forming on the curve of voltage limit A'C. The point C for the speed  $\omega_2$  corresponds to the maximum possible electromagnetic torque. The further increase of the torque of static loading will lead to decrease in speed of the machine, and currents  $i_d$ ,  $i_q$  will be formed on the curve CA. For the speed  $\omega_3$  a work in the first zone is impossible, and currents  $i_d$ ,  $i_q$  are formed on the curve DE in all range of electromagnetic torque.

# Development of control system of PMSM

The function scheme of control system is presented on Fig. 4. On an input the reference signal of speed  $\omega_r^*$  is compared to the valid value  $\omega_r$ . Output of the PI speed regulator (SR) is the reference on armature current  $i^*$ . Output signal of the SR is limited at the maximum current  $I_{\text{max}}$ . The block of field weakening (FW) forms the reference signals of currents  $i_d^*$ ,  $i_q^*$  on the basis of values:  $i^*$ ,  $\omega_r$ ,

and  $u_d^*$ ,  $u_a^*$  – the reference signals of voltage components.

The subsystem of current regulators (CR) is represented on Fig. 5. Using of sliding regulators allows to avoid necessity of use of PMSM mathematical model in comparison with traditional PI-regulators. Outputs of sliding regulators, after dq-ABC transformation, form the reference signals for corresponding phase voltages.



Fig. 4. Function scheme of the control system of electric drive on the basis of PMSM with field weakening



Рис. 5. Subsystem of current regulation CR

To determine of a point of transition from the first zone to the second zone the mathematical model of PMSM, as a rule, is used [1]. In [6] for transition to the second zone the information about conditions of keys of the inverter is used. In this paper, for this purpose using of reference signals of voltage projections  $u_d^*$ ,  $u_q^*$  is proposed.

Fig. 6 shows the algorithm of work of FW block. At the initial stage the correction current on an d axis  $\Delta i_d^*$  is accepted equal 0. Next, on the basis of values of the reference voltage signals  $u_d^*$ ,  $u_q^*$  amplitude of voltage  $u_s^* = \sqrt{u_d^* + u_q^*}$  is calculating. From the received amplitude value is subtracting the maximum possible value  $u_{\lim_{} pu}$ . Their difference  $u_{sig}$  enters on the integrator and forms  $\Delta i_d^*$ , which correcting a part of a negative signal of the reference  $i_d^*$ . If speed is on border of transition from the first to the second zone and according to reference signal

should grow, then signal  $u_{sig} > 0$  and  $\Delta i_d^*$  increases. Thus, near the second zone the signal  $i_d^*$  will gradually increase and will ensure functioning on voltage limit curve from the points A' to C (Fig. 3). If in the second zone the torque decreases, a signal  $u_{sig} < 0$ . As a result, the value  $\Delta i_d^*$  will decrease, and the working point will move from the point C to the point A'. If the torque continues to decrease, working points will be formed on a curve A'O. Since on the curve A'O a correcting part is not necessary, under a condition  $u_{sig} < 0$  and  $\Delta i_d^* < 0$  is accepted  $\Delta i_d^* = 0$ . This condition prevents decrease  $\Delta i_d^*$  and moving to the right from the curve  $i_{q.opt}(i_{d.opt})$ . The reference signal  $i_q^*$  is formed on the optimum curve  $i_{q.opt}(i^*, \omega_r)$  in the first zone and on the voltage limit curve in the second.

### **Results of modeling**

For research of dynamic characteristics, computer modeling of PMSM work was conducted. The machine was run till angular speed 80 s<sup>-1</sup> at the torque of static loading of 200 Nm, and at the moment of time 0.3 s carried out the increase in loading to 600 Nm. At the moment of time 0.5 s reference signal for angular speed increased to 190 s<sup>-1</sup> and at the time 0.57 s PMSM goes to work in a second zone of speed with flux weakening. This is evidenced by the emergence and growth the module of correction current component  $\Delta i_d^*$ . At the moment of time 1.1 s carried out the increase in loading to 400 Nm. Transfer characteristics shown in Fig. 7.



Fig. 6. Algorithm of field weakening



Fig. 7. The time dependences of angular speed, torque, currents and efficiency of PMSM, obtained in simulation

# Conclusion

The conducted researches have shown that for an electric vehicle at its different modes the copper and iron losses and magnetic saturation have significant impacts on the energy efficiency of PMSM. As a result of calculations by the developed program, we will easily obtain the optimum dependences  $i_{d.opt}(i_s, \omega_r)$ ,  $i_{q.opt}(i_s, \omega_r)$ . Implemented method allows to provide transition and work in the second zone with field weakening without use of mathematical model of PMSM.

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