Abstract. A method for determination of the limits of noncavitation operation of an electrohydraulic complex with a variable-frequency electric drive is proposed. An analytical expression for determination of the critical value of a relative rotation frequency of an induction motor is obtained. This expression provides a noncavitation operation of a hydrotransport system. A mathematical model of an electrohydraulic complex with a variable-frequency electric drive and a cavitation channel in the pipeline network is given. The obtained curves of pressure variation in the hydraulic system confirm the possibility of provision of cavitation protection of an electrohydraulic complex by means of a variable-frequency electric drive.

Streszczenie. W artykule zaproponowano metodę określania granic działania niekawitacyjnego systemu elektrohydraulicznego z elektrycznym napędem o zmiennej częstotliwości. Wyznaczono analityczną zależność na wyczynanie wartości krytycznej względnej częstotliwości kołowej. Zależność ta opisuje niekawitacyjne działanie systemu transportu cieczy. Utworzono model matematyczny urządzenia elektrohydraulicznego z napędem elektrycznym o zmiennej częstotliwości oraz kanału kawitacyjnego w sieci wodociągowej. Otrzymane zależności zmienności ciśnienia w systemie hydraulicznym potwierdzają możliwość ochrony kawitacyjnej systemu elektrohydraulicznego z napędem elektrycznym ze zmienią częstotliwością. (Ograniczenia niekawitacyjnego działania systemu elektrohydraulicznego z napędem elektrycznym o zmiennej częstotliwości).

Keywords: electrohydraulic complex, cavitation processes, variable-frequency electric drive.

Słowa kluczowe: system elektrohydrauliczny, procesy kawitacyjne, napęd elektryczny ze zmienią częstotliwością

Introduction

Pump units (PU) and electrohydraulic complexes (EHC) on their basis present a united complex system of interacting electromechanical and hydraulic equipment in which regulation of PU process-dependent parameters is carried out by means of variable-frequency electric drive (ED).

When pressure in discharge header drops and temperature condition of fluid motion in the hydrosystem changes, cavitation processes occur. They are accompanied by periodical formation and collapse of cavities filled with steam or gas (cavitation pockets). Presence of cavitation phenomena causes fluctuation of pressure and discharge in the hydraulic network with the amplitude of (6…20)% of the nominal value of these parameters, pipeline choking, growth of unproductive losses of power in EHC, increased wear of hydraulic equipment [1–3]. Although other types of electric motors [4, 5] can also be used in pump units, a squirrel-cage induction motor is the most appropriate one due to its characteristics and robustness.

Practically used methods and technical means of decrease of cavitation are based in most cases on installation of pipeline protection and solve only local problems as they response to factual occurrence of malfunction. Paper [2] demonstrates that decrease of rotation frequency of a pump impeller results in a considerable reduction of losses, amplitude and frequency of EHC pressure fluctuation conditioned by presence of cavitation processes or in their total absence. Thus, use of facilities of variable-frequency ED enables not only regulation of PU parameters but also control of occurrence of cavitation zones in hydraulic network.

Taking the above said into account, the purpose of the research consists in working out a method for determination of boundaries of non-cavitational operation of EHC with a variable-frequency ED when cavitation processes develop in the pipeline.

Research method

Determination of boundaries of non-cavitational operation of EHC is based on the analysis of joint operation of PU with a variable-frequency ED for pipeline network:

\[
\begin{align*}
H &= H_0 \nu^2 - R_b Q^2 + \sum_{i=1}^n H_{sl} + \sum_{i=1}^n R_{net} Q^2
\end{align*}
\]

where: \(H_0\) – head developed by the pump at zero displacement, m; \(\nu\) – relative rotational frequency of the pump impeller, r.u.; \(R_b\) – internal resistance of the pump, \(s^2/m^5\); \(Q\) – displacement of EHC, \(m^3/s\); \(H_{sl}\) – static head in EHC at the \(i\)-th section of the pipeline, m; \(R_{net}\) – hydraulic resistance of the corresponding section of the network, \(s^2/m^5\); \(n\) – number of the pipeline sections, \(i\) – number of the section of the hydraulic network.

Solution to system (1) allowed determination of the head at the input of every section of the pipeline, m:

\[
H_i = \Delta H_1 + \Delta H_{i+1} + \ldots + \Delta H_n + \sum_{i=1}^n H_{sl}
\]

where: \(\Delta H_i = Q^2 R_{net}\) – losses of the head at the \(i\)-th section of the pipeline, m.

Cavitational processes are known to develop when current cavitation number \(\gamma\) is equal to its critical value \(\gamma_{kr}\) or smaller that it [3]. In this case the flow rate corresponding to the beginning of the cavitational process at \(i\)-th section of the hydraulic network is, m/s:

\[
u_{kr} = \sqrt{\frac{2(p_i - p_{para})}{\rho_i \gamma_{kr}}}
\]

where: \(p_i, p_{para}\) – pressure of the flow and liquid saturated vapors, respectively, at the \(i\)-th section of the hydraulic network, Pa; \(\rho_i\) – fluid density at the corresponding section of the pipeline, \(kg/m^3\); \(\gamma_{kr}\) – critical value of cavitation at the \(i\)-th section of the pipeline.

Taking into account relations \(H_i = p_i / \rho_i g\) and \(Q_i = \nu_i S_i\), EHC displacement corresponding to the boundary of non-cavitational operation for any section of hydrodynamic network, \(m^3/s\), is:

\[
Q_{kr} = \frac{S_i \sqrt{2(p_i g H_i - p_{para}) / \rho_i \gamma_{kr}}}{\rho_i}
\]

where: \(g\) – gravitational acceleration, \(m/s^2\); \(S_i\) – cross-section area of the \(i\)-th section of hydraulic network, \(m^2\).

Solution to system (1) in relation to \(\nu\) taking into account expression (4), allowed determination of critical value of head...
\[ H_{kr,i} \text{ and relative frequency } \nu_{kr} \text{ of pump impeller rotation,} \]
which corresponds to beginning of development of cavitational processes at the \( i \)-th section of the pipeline:

\[
(5) \quad H_{kr,i} = \frac{\nu_{kr} \cdot H_{st,i} - 2 \nu_{i}^2 \cdot R_{net,i} \cdot p_{parai}}{\nu_{kr} \cdot H_{st,i} - 2 \nu_{i}^2 \cdot R_{net,i} \cdot p_{parai}}.
\]

Then to provide non-cavitational operation of EHC with a variable frequency ED of a PU the condition \( \nu < \nu_{kr} \) is to be true.

Graphic interpretation of head-discharge pump and pipeline characteristics reflecting noncavitation condition of EHC operation is shown in Fig. 1, where: \((H - Q)_{kr,i}, (H - Q)_{net,i}, (H - Q)_{i+1}\) – head at the corresponding sections of the hydraulic system; \((H - Q)_{i+1}, (H - Q)_{net,i+1}\) – curves reflecting variation of the head at the corresponding sections of the hydraulic system; \((H - Q)_{i}\), \((H - Q)_{p}\) – head-discharge characteristics of the pump at \( \nu = 1 \) and \( \nu_{kr} \), respectively; \( H_{i}, H_{i+1}, H_{kr,i}, H_{kr,i+1}\) – head and its critical value at corresponding sections of EHC pipeline at displacement \( Q_{A} \).

The analysis of the curves (Fig. 1) demonstrates that pump assembly current operation mode corresponding to \( Q_{A} \) results in meeting condition \( H_{i+1} < H_{kr,i+1} \), at which cavitation processes develop at the \((i+1)\)-th section of the hydrodynamic system.

Decrease of relative rotation frequency of the PU up to value \( \nu_{kr} \) causes the shift of the pumping assembly operation mode to point B where curves \((H - Q)_{i}\) and \((H - Q)_{net,i}\), cross. In this case condition \( H_{i+1} = H_{kr,i+1}\), corresponding to point B where curves \((H - Q)_{i+1}\) and \((H - Q)_{net,i+1}\) cross and providing the limit of non-cavitational operation of EHC at the \((i+1)\)-th section of the hydrodynamic system is met.

Mathematic modelling

Fig. 2 contains a block diagram of a model of EHC with a variable-frequency ED and a cavitation channel. The model includes: a frequency converter (FC) presented by an aperiodic link of the first order with a quadratic law \( U/\theta^2 = K_{pr} \) of frequency control; a squirrel-cage induction motor (IM) presented in “\( u, v, 0 \)” – coordinates by a model well-known in electromechanics; a pump unit with a gently sloping head-discharge characteristic; a pipeline network with three sections connected in series; a cavitation channel taking into account the cavitation processes at the second section of the hydraulic system.

Physical processes in the pump and in the adjacent section of the pipeline are described by transfer functions of the form:

\[
(7) \quad W_{p1}(p) = \frac{\frac{Q(p)}{H_{p}(p)} - \Delta H_{p}(p)}{\Delta H_{p}(p)} = \frac{k_{u,i}}{T_{\Sigma} p + 1};
\]

\[
(8) \quad W_{p2}(p) = \frac{\Delta H_{p}(p)}{Q(p)} = R_{b};
\]

\[
(9) \quad W_{p3}(p) = \frac{\Delta H_{p}(p)}{Q(p) - Q_{OS}(p)} = \frac{k_{el}}{T_{c} p};
\]

\[
(10) \quad W_{p4}(p) = \frac{\Delta H_{p}(p)}{\Delta H_{p}(p)} = R_{net}\]

where: \( k_{u,i} = Q_{nom}/H_{nom} \), \( k_{u,i} = c/gS \) – coefficients of proportionality, respectively, \( m^{2}/s, s/m^{2} \); \( H_{nom} \) – nominal head at the output of the pipeline pump, \( m \), \( Q_{nom} \) – displacement of liquid at the head \( H_{nom} \), \( m^{3}/s \); \( T_{\Sigma} = T_{p} + T_{u} \) – time of the travel of the liquid through the pump and the adjacent pipeline, \( s \); \( T_{u} = lQ_{nom}/gS \) – inertia and capacitance time constants of the pipeline, respectively, \( s \), \( S_{p} = d_{2} - d_{1}/\sqrt{\nu_{kr} \ln(d_{2}/d_{1})} \) – time constant of the pump, \( s \), \( d_{2}, d_{1} \) – input and output diameters of the circular grid of the centrifugal pump, \( m \), \( \nu \) – velocity of the fluid flow in the inter-vane space, \( m/s \), \( z_{p} \) – number of vanes of the circular grid, \( \Delta H_{\Sigma} \) – total head losses in the hydraulic network, \( m \), \( \Delta H_{i} \) – head losses in the adjacent section of the pipeline, \( R_{net} \) – hydrodynamic resistance of the adjacent section of the pipeline, \( s^{2}/m^{3} \).

Free head at the end of the \( i \)-th section of the pipeline:

\[
(11) \quad H_{i+1} = H_{i} - \Delta H_{i} = \Delta H_{i+1} + \ldots + \Delta H_{n}.
\]

An elementary \( i \)-th section of the hydraulic network is described by transfer functions of the form:

\[
(12) \quad W_{i1}(p) = \frac{Q(p)}{H_{i}(p) - \Delta H_{i}(p)} = \frac{k_{u,i}}{T_{i,p} + 1};
\]

\[
(13) \quad W_{i2}(p) = \frac{\Delta H_{i}(p)}{Q(p) - Q_{OS}(p)} = \frac{k_{el}}{T_{c,p}};
\]

\[
(14) \quad W_{i3}(p) = \frac{Q_{OS}(p)}{\Delta H_{i}(p)} = R_{net,i}
\]

where: \( \Delta H_{i} = \sum_{i=1}^{n} \Delta H_{i} + \sum_{i=1}^{n} \Delta H_{St,i} \) – total head losses in the hydraulic network.

Cavitation processes are known to be characterized by periodic occurrence of cavitation pockets whose volume is determined according to expression [3]:

\[
(15) \quad V_{kav} = \frac{1}{l_{k}} \int (Q_{1}(t) - Q_{2}(t)) dt
\]
where: \( T_k = \gamma V_0 / G \) – inertia time constant of the cavitation pocket, \( s \), \( G = pgQ \) – weight flow of the liquid at \( V_0 \), \( N/s \), \( V_0 \) – initial volume of the cavitation pocket, \( m^3 \), \( \gamma = \rho g \) – liquid specific weight, \( N/m^3 \).

During modeling it was assumed that cavitation pocket is of a spherical form with a constant volume, does not move along the pipeline and represents a local narrowing at a section of the network. Then hydraulic resistance conditioned by the presence of cavitation processes in the pipeline, \( s^2/m^5 \).

Critical number

\[
\chi_{\text{cav}} = \frac{1}{\pi} \left[ \left( \frac{d^2}{d_p^2} - 1 \right) \right]^{1/2}
\]

(16)

where: \( \chi_{kav} = \left( \frac{d^2}{d_p^2} - 1 \right)^{1/2} \) – coefficient of local resistance of the flow narrowing caused by cavitation processes in the pipeline, \( \epsilon = 0.57 + 0.043 \left[ 1 - \left( \frac{d_p}{d} \right)^2 \right] \) – coefficient of flow compression, \( d \) – diameter of the pipeline in front of the narrowing, \( m \), \( d_p = d - \frac{3}{2} \frac{6V_{kav}}{\pi} \) – diameter of the flow in the narrow part, \( m \).

Taking into consideration expression (16), hydraulic resistance of the i-th section of the pipeline network:

(17)

\[ R_{\text{net}}(i) = R_{\text{net}}(i-1) + R_{\text{kav}}(i) \]

When a mathematical model of EHC with parameters given in Table 1 was analyzed, curves of time variation of the head (Fig. 3) and pump ED rotation frequency (Fig. 4) were obtained.

Analysis of the curves demonstrated that opening of the stopcock (time moment 4 s) at the pump output causes decrease of head \( H_1 \) in the discharge tube of the turbo mechanism and growth of heads \( H_2 \ldots H_4 \) in the pipeline network (Fig. 3).

As head \( H_2 \) at the second section of the pipeline is lower than the value determined according to expression (5), EHC operation is accompanied by development of cavitation processes in the pipeline. Presence of these processes causes pulsations of the head in the hydrosystem. Their peak-to-peak value changes in the range of 0.7...7 m with frequency 4 Hz (Fig. 3).

![Block diagram of a model of EHC with a cavitation channel](image)

**Table 1. EHC parameters**

<table>
<thead>
<tr>
<th>Parameters of pump Willo MHI 402</th>
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<tbody>
<tr>
<td>Head ( H_p ), m</td>
</tr>
<tr>
<td>Discharge ( Q_p ), m³/h</td>
</tr>
<tr>
<td>Power ( N_p ), W</td>
</tr>
<tr>
<td>Rotation frequency ( \omega_{\text{nom}} ), s⁻¹</td>
</tr>
<tr>
<td>Hydraulic resistance ( R_b ), s²/m⁵</td>
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<tr>
<th>Parameters of pipeline sections</th>
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</thead>
<tbody>
<tr>
<td>Hydraulic resistance ( R_{\text{net}} ), s²/m⁵</td>
</tr>
<tr>
<td>Back pressure ( H_{\text{b}} ), m</td>
</tr>
<tr>
<td>Critical number ( \chi_{\text{cav}} ) of cavitation</td>
</tr>
</tbody>
</table>

Fig. 2. Block diagram of a model of EHC with a cavitation channel

Fig. 3. Curves of head variation in EHC

Fig. 4. Curves of variation of rotation frequency of the pump ED
According to expression (6) for the considered case obtained that decrease of the pump ED rotation frequency (time moment 10 s) by 30% down from the nominal one (Fig. 4) resulted in the absence of cavitation variation of the head in EHC (Fig. 3).

Experimental verification

Experimental verification of EHC characteristics in the presence and absence of cavitation processes is made at a computer-aided laboratory complex (Fig. 5) whose basic parameters are given in Table 1 [6].

With the aim of research of cavitation processes the physical model of EHC is equipped with Venturi tubes of different cross-sections with transparent parts in the area of narrowing, stopcocks at the pump suction connections, a tubular electric heater. In this case one tube is situated in the vertical part of the pipeline and another one in its horizontal part. Pressure at the pump output was controlled by OBEH pressure sensor. A two-channel ultrasonic Ergomer counter 125 was used to measure liquid discharge in EHC. Control of the pump ED rotation frequency was made by a velocity sensor manufactured on the basis of a direct current motor with permanent magnets.

Connection of sensors, measuring EHC technological and mechanical parameters, with the personal computer is provided by an analog-digital converter LCard E440-14.

The analysis of cavitation processes in EHC was made in the presence and in the absence of cavitation processes in EHC. The latter is achieved due to decrease of the pump ED supply voltage frequency up to value $f_s = 36.4$ Hz at which absence of cavitation processes in EHC was observed.

Figs. 6, 7 contain curves of variation of pressure $p_{50}(t)$, $p_{36}(t)$ and rotation frequency $\omega_{50}(t)$, $\omega_{36}(t)$ of the pump in the presence and absence of cavitation processes in the pipeline, respectively.

Analysis of curve $p_{50}(t)$ (Fig. 6, a) enabled singling out sections of periodic growth and decrease of pressure, conditioned by development and further destruction of cavitation pockets in the pipeline. It has been obtained that the peak-to-peak value and the frequency of pressure variations in cavitation mode made $\Delta p_k = 4 \text{ kPa}$ and $f_{kav} = 1.25 \text{ Hz}$, respectively. In this case it should be noted that there are variations of the pump ED rotation frequency in cavitation mode of EHC operation (curve $\omega_{50}(t)$ Fig. 7, a) with peak-to-peak value $\Delta \omega_k = 8...10 \text{ s}^{-1}$. Obviously, their presence causes moment oscillations on the pump ED shaft, which may result in vibration of electric hydraulic equipment.

Pressure curve $p_{36}(t)$ (Fig. 6, b), corresponding to PU operation at rotation frequency that is 30% lower than the nominal one, (Fig. 7, b), demonstrates absence of cavitation processes in EHC. The error of the results of determination of relative critical pump rotation frequency, using expression (6) and the ones obtained by the experimental research, made 2.23%.

Conclusions

A method for determination of the boundaries of noncavitational operation of an electric hydraulic complex with a variable-frequency electric drive has been offered. It is based on the analysis of joint operating conditions of a pump unit for a pipeline network and makes it possible to determine the range of admissible values of variation of rotational frequency of the pump electric drive when the displacement is regulated within the required boundaries.

Obviously, cavitation protection in electric hydraulic complexes can be realized in creation of systems of automatic control by a variable-frequency electric drive in the problems of stabilization of a process-dependent parameter in a pipeline network.

REFERENCES


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