A Novel Prony Approach for Synchronous Generator Parameter Estimation

Abstract. In this paper we propose to use a novel Prony's approach, relying on suitable time-segmentation of current oscillograms and proper model order choice, to estimate transient and sub-transient parameters of synchronous generators associated to sudden three-phase short-circuit tests. Comparison between the proposed approach and a traditional graphical method, described in the IEEE-115 standard, has been performed. The current data used here have been generated from characteristic equations of short-circuit armature current. Based on these results, we conclude that the Prony's algorithm is a suitable alternative to the graphical method for parameter estimation in synchronous machines, providing more accurate estimates than the later approach.


Keywords: Prony algorithm; synchronous generators; parameter estimation; short-circuit test
Słowa kluczowe: algorytm Prony, generator synchroniczny, testy zwarciowe

Introduction

Regarding electrical power systems, the usage of numerical simulations to probe operational efficiency and safety is a common practice. Particularly, transient simulations are quite informative to forecast the power system behavior under the influence of faults or abrupt changes in its topology [1, 2, 3, 4]. The information extracted from these simulations must include parameter values from different machines placed along the power grid. In this context, and taking account the recent competitiveness and deregulated conditions of the energy market, the estimation of the parameters of synchronous machines for distinct stages of a transient system condition is a concern of growing relevance [5, 6, 7]. These parameters are an essential data for the developing of control schemes for power quality as well the adoption of efficient protection procedures.

In literature, one may find out some useful methods to estimate transient parameters for synchronous machines. The standard IEEE-115 [8] suggests the sudden short-circuit test to obtain these parameters through a graphical procedure, based upon the construction of envelopes passing through the phase currents peaks. Unfortunately, this procedure is inaccurate and may lead to unsatisfying results, especially to small rated machines ranging from 30 kVA to 2 MVA with very short time constants [13]. For instance, a subtransient time constant of one cycle or less and with the first peak located at half cycle afterwards the short-circuit instant the model may require few samples and the usage of interpolation techniques. This can result in a large statistical dispersion with regards to the parameter values related to a time-consuming process.

There have been efforts to overcome the aforementioned limitations. In [14] it was proposed an improved online procedure based on a genetic algorithm for synchronous machine parameter estimation. Reference [15] proposed a complete modeling which takes into account the existence of dampers and use the least-squares algorithm to obtain the machine model reactances and time constants. In [16], it was outlined a method weaving the least-squares algorithm, the wavelet transform and the Prony’s method. This last technique was applied only to estimate the sub-transient parameters. This paper proposes a simple identification procedure totally based on the Prony’s modeling of the current signal and proper trace segmentation. This procedure provide all sub-transient, transient and steady-state parameters. It relies on the application of the Prony’s method on three different intervals of the short-circuit current signal. For each interval, the model order is convenient selected and a set of parameters is determined.

This paper is organized as follows. The second section describes the model of a short-circuit stator current for synchronous machine. The third section outlines the traditional graphical method, usually applied for determining the transient parameters. The next section formalize the main equations comprising the Prony’s algorithm. The fifth section explains the implementation of the Prony’s technique for the present problem. The results are presented in the sixth section and the conclusions are drawn in the final section.

Sudden three-phase short-circuit test

Sudden-short-circuit test is usually applied to verify the machine mechanical design is suitable to withstand the ensuing efforts of short-circuits and abnormal operation conditions. These tests, however, are also useful in determining certain machine parameters such as reactances and time constants.

The sudden-short-circuit-test is performed by shorting the synchronous machine terminals and keeping the speed at a constant rate throughout the duration of the evaluation [8]. In such conditions the armature current comprises the following: AC fundamental frequency or symmetric component, including the sub-transient, transient and steady state components; direct-current component, and the second harmonic component, modeled by (1) [15].

\[
I_a(t) = \frac{E \sqrt{2}}{2} \left( \frac{1}{X_d} + \frac{1}{X_q} \right) e^{-\frac{\lambda}{T_d}} \cos\lambda + \frac{E \sqrt{2}}{2} \left( \frac{1}{X_d} - \frac{1}{X_q} \right) e^{-\frac{\lambda}{T_q}} \cos(2\omega t + \lambda) - E \sqrt{2} \left[ \frac{1}{X_d} + \frac{1}{X_q} \right] e^{-\frac{\lambda}{T_d}} \cos(\omega t + \lambda) + \left( \frac{1}{X_d} - \frac{1}{X_q} \right) e^{-\frac{\lambda}{T_q}} \cos(\omega t + \lambda)
\]

This equation provides the instantaneous value of the armature current, \( I_a \), which is a function of the effective value of the open-circuit voltage, \( E \), before shorting the machine ter-
minals. The current is also function of the angular frequency, \( \omega \), and the incidence angle, \( \lambda \), which depends on the point in the cycle where the current acquisition started. \( X_d, X_d', \) and \( X_d'' \) are the direct axis reactances in stationary, transient and sub-transient regimes, respectively. \( X_q'' \) is the quadrature axis reactance in sub-transient regime. The steps to be followed consists in obtaining reactances and time constants from the envelope of the fault symmetric trace. The steps to be followed are described below [1]:

Graphical approach

The traditional approach to estimate synchronous machine parameters, from a sudden-short-circuit test, applies the well known technique to determine current trace envelope [8]. Fig. 1 shows an example of a short-circuit current trace with an indication of the transient and steady state amplitudes. This graphic is called "symmetric trace" of the symmetric sudden-short-circuit current.

From the symmetrical trace one can obtained the current envelope, and consequently estimate the reactances, \( X_d, X_d', X_d'' \), and the time constants \( T_d, T_d' \) and \( T_d'' \). The symmetric current envelope is modeled by (2) [8]:

\[
I_{en}(t) = E \sqrt{2} \left\{ \frac{1}{X_d} + \left( \frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-\frac{t}{T_d'}} + \left( \frac{1}{X_d''} - \frac{1}{X_d} \right) e^{-\frac{t}{T_d''}} \right\}
\]

It is observed in this expression that the current consists of a first constant term, which corresponds to the steady state regime and two decaying exponential terms, corresponding to the transient and sub-transient components. The later one decay faster than second. The technique to estimate the parameters follows the IEEE-115 standard, and basically consists in obtaining reactances and time constants from the envelope of the fault symmetric trace. The steps to be followed are described below [1]:

1. The envelopes of the sudden-short-circuit current, \( I''_D \) and lower \( I''_{low} \), is obtained from the trace shown in Fig. 1.
2. The DC component \( I_{DC} \) is estimated using (3), where \( I_a \) is the machine base current.
3. This DC component is then subtracted from the trace and (2) is used as a model for the remain signal.
4. The steady state component \( I_{ss} \) is estimated from the mean of last peaks and then the direct axis synchronous reactance, \( X_d = E/I_{ss} \), is computed using the first term of (2).
5. Once the steady state and DC components have been removed, the remain wave form is then mapping to a semi-logarithmic graph paper, as shown in Fig. 2. Curve B in this figure corresponding to the sub-transient (first cycles) and transient components (last cycles).
6. A straight line \( C \) is drawn and extrapolating to the time \( t = 0 \) in order to obtain the initial transient component, as observed in Fig. 2.
7. Adding the initial transient component to \( I_{ss} \) gives the transient component of the short-circuit current, \( I' \). The transient reactance, \( X_d' = E/I' \), is then calculated.
8. The subtraction of curveB and line C line produces curveA, which is used to estimate the sub-transient component.
9. A straight line \( D \) is drawn and extrapolating to the time \( t = 0 \) in order to obtain the initial sub-transient component.
10. Adding the initial sub-transient component to \( I' \) gives \( I'' \), the total short-circuit current. The transient reactance, \( X_d'' = E/I'' \), is then calculated.
11. Determining the direct axis transient time constant, \( T_d \), which is the time in seconds required for the transient component (line \( C \) reaches 36.8% of \( I'' \) initial amplitude.
12. Determining the direct axis sub-transient time constant, \( T_d'' \), which is the time in seconds required for the sub-transient component (line \( D \) reaches 36.8% of \( I'' \) initial amplitude.
13. The armature time constant, \( T_a \), is obtained by plotting the DC component and determine the time in seconds required for \( I_{DC} \) reaches 36.8% of its initial amplitude.

\[
I_{DC} = \frac{I''_{up} + I''_{low}}{2\sqrt{2}I_n}
\]

From the symmetrical trace one can obtained the current envelope, and consequently estimate the reactances, \( X_d, X_d', X_d'' \), and the time constants \( T_d, T_d' \) and \( T_d'' \). The symmetric current envelope is modeled by (2) [8]:

\[
I_{en}(t) = E \sqrt{2} \left\{ \frac{1}{X_d} + \left( \frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-\frac{t}{T_d'}} + \left( \frac{1}{X_d''} - \frac{1}{X_d} \right) e^{-\frac{t}{T_d''}} \right\}
\]

It is observed in this expression that the current consists of a first constant term, which corresponds to the steady state regime and two decaying exponential terms, corresponding to the transient and sub-transient components. The later one decay faster than second. The technique to estimate the parameters follows the IEEE-115 standard, and basically consists in obtaining reactances and time constants from the envelope of the fault symmetric trace. The steps to be followed are described below [1]:

1. The envelopes of the sudden-short-circuit current, \( I''_D \) and lower \( I''_{low} \), is obtained from the trace shown in Fig. 1.
2. The DC component \( I_{DC} \) is estimated using (3), where \( I_a \) is the machine base current.
3. This DC component is then subtracted from the trace and (2) is used as a model for the remain signal.
4. The steady state component \( I_{ss} \) is estimated from the mean of last peaks and then the direct axis synchronous reactance, \( X_d = E/I_{ss} \), is computed using the first term of (2).
5. Once the steady state and DC components have been removed, the remain wave form is then mapping to a semi-logarithmic graph paper, as shown in Fig. 2. Curve B in this figure corresponding to the sub-transient (first cycles) and transient components (last cycles).
6. A straight line \( C \) is drawn and extrapolating to the time \( t = 0 \) in order to obtain the initial transient component, as observed in Fig. 2.
7. Adding the initial transient component to \( I_{ss} \) gives the transient component of the short-circuit current, \( I' \). The transient reactance, \( X_d' = E/I' \), is then calculated.
8. The subtraction of curveB and line C line produces curveA, which is used to estimate the sub-transient component.
9. A straight line \( D \) is drawn and extrapolating to the time \( t = 0 \) in order to obtain the initial sub-transient component.
10. Adding the initial sub-transient component to \( I' \) gives \( I'' \), the total short-circuit current. The transient reactance, \( X_d'' = E/I'' \), is then calculated.
11. Determining the direct axis transient time constant, \( T_d \), which is the time in seconds required for the transient component (line \( C \) reaches 36.8% of \( I'' \) initial amplitude.
12. Determining the direct axis sub-transient time constant, \( T_d'' \), which is the time in seconds required for the sub-transient component (line \( D \) reaches 36.8% of \( I'' \) initial amplitude.
13. The armature time constant, \( T_a \), is obtained by plotting the DC component and determine the time in seconds required for \( I_{DC} \) reaches 36.8% of its initial amplitude.

\[
I_{DC} = \frac{I''_{up} + I''_{low}}{2\sqrt{2}I_n}
\]
and

\[ z_k = e^{i(\alpha_k + j2\pi f_k)\Delta T}, \]

is the \( k^{th} \) pole. The parameter \( p \) is the model order, \( A_k \) and \( \theta_k \) are, respectively, the initial amplitude and phase of the \( k^{th} \) damped sinusoid. The parameter \( f_k \) is the \( k^{th} \) frequency, \( \alpha_k \) is the damping factor, and \( \Delta T \) is the signal sampling period.

Substituting theses parameters in equation (4), yields to:

\[ y[n] = \sum_{k=1}^{p} A_k e^{\alpha_k \Delta T (n-1)} e^{j(2\pi f_k \Delta T (n-1) + \theta_k)}, \]

and by assuming that \( y[n] \) is a real discrete signal, it is mandatory that the terms \( e^{j(2\pi f_k \Delta T (n-1) + \theta_k)} \) occur in conjugated pairs. This allows to rewrite equation (7) as:

\[ y[n] = \sum_{m=0}^{M} C_m e^{\alpha_m \Delta t (n-1)} \cos (\omega_m \Delta t (n-1) + \theta_m), \]

where \( C_0 = A_1 \) is considered here as the DC component amplitude, \( C_m = A_m/2 = A_{2m+1}/2 \) (for \( m = 1, 2 \) and 3) are related to the fundamental amplitude, and \( C_4 = A_8/2 = A_{16}/2 \) to the second harmonic amplitude. One can note in (8) that, for each of \( m \) sinusoids, there are four parameters to be estimated. Therefore, at least \( 2p \) equations are needed to determine all parameters. Moreover, equation (8) is nonlinear with regards to these parameters and imposes a nonlinear technique for determining them. The main idea behind Prony’s method, and also its main advantage is to propose a procedure which allows the parameter estimation into two stages. The first one comprises solutions to a set of linear equations and then a polynomial equation. This implies in determining the poles of equation (4). The second stage is just the solution to another set of linear equations to determine the amplitudes \( A_k \) and the initial phases \( \theta_k \).

To understand the rationale of Prony’s method, consider a window of the signal \( y[n] \) containing \( N \) samples, with \( N \geq 2p \). The equation model (4) gives rise to the following matrix equation:

\[
\begin{bmatrix}
z_0^N & z_0^N & \cdots & z_0^N \\
z_1^N & z_1^N & \cdots & z_1^N \\
\vdots & \vdots & \ddots & \vdots \\
z_{N-1}^N & z_{N-1}^N & \cdots & z_{N-1}^N
\end{bmatrix}
\begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_p
\end{bmatrix}
= \begin{bmatrix}
y[1] \\
y[2] \\
\vdots \\
y[N]
\end{bmatrix},
\]

which can be compactly written as

\[ Zh = y. \]

Now, using the poles of equation (4) to define a polynomial,

\[ \Phi(z) = \sum_{k=1}^{p} a_0 (z - z_k) = \sum_{m=0}^{p} a_m z^{p-m}, \]

where, without loss of generality, \( a_0 \) can be set to one. The \( p+1 \) coefficients of the polynomial can be organized as a \( 1 \times N \) vector, i.e.,

\[ a = [a_p \ a_{p-1} \ \cdots \ \ 1 \ \ 0 \ \ \cdots \ \ 0]. \]

Multiplying \( a \) on both sides of equation (10), yields

\[ aZh = ay = 0, \]

or

\[ a_p y[1] + a_{p-1} y[2] + \cdots + a_1 y[p] + y[p+1] = 0. \]

Right-shifting the polynomial coefficients in vector \( a \) produces a \( N \times 1 \) vector \( a' \), i.e.,

\[ a' = [0 \ a_p \ a_{p-1} \ \cdots \ \ 1 \ \ 0 \ \ \cdots \ \ 0]. \]

Again, multiplying this vector on both sides of equation (10), gives

\[ a_p y[2] + a_{p-1} y[3] + \cdots + a_1 y[p] + y[p+2] = 0. \]

This process can be repeat in order to build the matrix equation

\[
\begin{bmatrix}
y[1] \\
y[2] \\
\vdots \\
y[N-1]
\end{bmatrix}
\begin{bmatrix}
\cdots & \cdots & \cdots & \cdots \\
aj & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \cdots
\end{bmatrix}
= \begin{bmatrix}
y[p+1] \\
y[p+2] \\
\vdots \\
y[N]
\end{bmatrix}.
\]

Once 17 is solved, providing the coefficients \( \alpha_m \), the poles \( z_k \) are obtained from the solution of the polynomial equation (11). Finally, the complex amplitudes \( h_k \), which is equivalent to compute \( A_k \) and \( \theta_k \), are obtained substituting the poles in 9. This algorithm is known as the least-square version of Prony’s method since \( N \geq 2p \), that is, the number of equations is greater than the number of variables.

The poles, \( z_k \), computation by means of a polynomial solution is usually more effective rather than the usage of traditional nonlinear least-squares methods relying on Jacobian matrices. These latter methods usually are excessively dependent on the initial guess and are prone to converge to local optimum [9]. For Prony’s method, there are most efficient methods for polynomial root-finding algorithms [11, 12].

Prony algorithm realization

In this paper, the Prony’s method is applied to determine the machine parameters related to the sub-transient, transient and steady state periods of the armature current, acquired during the sudden-short-circuit test. The sub-transient period is considered to occur within the first cycles of current trace. The transient period supposed to happen from the 10th to 20th cycle, and the steady state regime in the last cycles.

The parameter estimation procedure is divided into three steps, accordingly with each period mentioned before, sub-transient, transient and steady state. Each one of them is modeled differently, where the Prony’s model order \( p \) is chosen by considering the dominant damped sinusoids present in each period. The Prony’s parameters are sorted out from DC up to second harmonic, according to (7) and (8). The following subsections describe the number of cycles and model order taken into account in each stage.

**Step 1: sub-transient** - Only the first cycle is considered for estimation purpose and, because in this cycle the current trace comprise all components given by (1), the model order \( p \) is set to 9 (and \( M = 4 \)). Therefore, the Prony’s algorithm should taken into account a model with 9 exponential functions: 4 pairs of complex exponential to represent the damped sinusoids related to the sub-transient, transient and steady state components, and a real exponential to model the decaying DC component. The Prony’s parameters to be estimated are nine amplitudes \( A_k \) and nine dumping factors \( \alpha_k \). Among them, only three amplitudes, DC \( A_1 \), fundamental \( A_8 \) and second harmonic \( A_{16} \), and two dumping factor, DC
\( \alpha_1 \) and fundamental \( \alpha_0 \), are necessary to determine the following machine parameters:

\[
T_a = \frac{1}{\alpha_1},
\]

\[
T_d' = \frac{1}{\alpha_0},
\]

\[
X_{d''} = \frac{2}{\left( \frac{2A_d}{X_d} + \frac{V^2}{X_d} \right)},
\]

\[
X_q'' = \frac{1}{\left( \frac{2A_q}{X_d} - \frac{1}{X_d} \right)},
\]

and

\[
X_{d'} = \frac{1}{\left( \frac{A_d}{X_d} - \frac{1}{X_d} \right)}.
\]

Equations from (18) to (22) are derived comparing terms from (1) to (7).

Step 2: transient - The time interval considered for parameter estimation ranges from the 10th to 20th cycle. The decaying fundamental component is the dominant signal present in the current trace. Therefore, the model order \( p \) is set to 4 and the Prony’s algorithm should consider two pairs of complex exponential to represent the dumped sinusoids related to the transient and steady state components. The Prony’s parameters to be estimated are four amplitudes \( A_k \) and four dumping factors \( \alpha_k \), but only one dumping factor, \( \alpha_4 \), is needed to determine the following machine parameter:

\[
T_d'' = \frac{1}{\alpha_4}
\]

Step 3: steady state - Finally, only the last cycle is considered to estimate two amplitudes and two dumping factors in the 2nd order model used to represent the steady state component present in the trace. From the four Prony’s parameter only one, \( A_1 \), is used to determine the machine reactance:

\[
X_d = \sqrt{\frac{E}{A_1}}
\]

It is well known the sensitivity of Pron’s method to noise. One usual alternative to overcome this issue is to augment the model’s order, \( p \), described in equation (7). Then the terms associated with minor amplitudes can be discarded. In this paper, there is no noise added to the simulation signals and the signal-noise ratio is infinity. Thus, one maintains the order \( p \) equal to nine, with eight poles related to for sinusoids and a real pole related to a DC exponential decaying.

Simulations and results

Simulations for the sudden-short-circuit test in three synchronous generators have been performed applying (1) to produce their armature currents. Fig. 3 shows an example of such simulated current and Table 1 the parameters used in the simulations for each machine. The third line in this table contains parameter values presented in [16] and the other two lines, values chosen by the authors in order to compare results from the Prony method to those derived from the graphical approach presented earlier. The angle \( \lambda \) have been set to zero in all simulations and a sampling frequency of 1.2 kHz has been used to produce the results.

To verify the accuracy of the estimated parameters, the relative errors (in \%) have been calculated using (25),

\[
e = \left| \frac{x_e - x_t}{x_t} \right| \times 100
\]

where \( x_e \) is the true value used in the simulation and \( x_t \) is its estimate. Table 2, Table 3 and Table 4 show the estimate for all parameters, when applying both approaches to process the current trace from the sudden-short-circuit of G1, G2 and G3 generators, respectively. One can note that the graphical method reactance \( X_{d''} \) estimate is missing in the three tables, once it is not possible to determine it using this approach.

Finally, Table 5 shows a comparison between the Prony’s method and a hybrid method proposed by [16], where the
The authors applied Wavelet transform to pre-process the current trace from sudden-short-circuit tests, before employing the Prony’s algorithm to determine the machine parameters. The results presented in this table indicates that such a procedure is unnecessary, once that errors are in favor to Prony’s method alone.

Conclusions

The use of Prony’s algorithm to estimate synchronous machine parameters from armature current oscillograms, during sudden-short-circuit tests, proved to be an effective technique to this purpose. The traditional graphical approach described in the IEEE-115 standard, and used as a comparison in this paper, not only is a limited procedure, but also it is not capable of rendering accurate estimates. Other works concerned to this subject have been recently published in the scientific literature. One of them is used as comparison in this paper, but also render some inaccurate estimates, even using the Prony’s algorithm. We believe that our success in well estimate the machine parameters is related to the way the algorithm is applied to current oscillograms. Divide the current trace into three time intervals and choose a suitable model order to represent each current segment is crucial to improve the parameter estimate accuracy.

### Table 3. Comparison between the graphical and Prony’s methods - Machine G2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Graphical Method</th>
<th>Prony’s Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>Error (%)</td>
</tr>
<tr>
<td>$X_{d}$</td>
<td>1.1499</td>
<td>0</td>
</tr>
<tr>
<td>$X_{d}^{*}$</td>
<td>0.3089</td>
<td>0.08</td>
</tr>
<tr>
<td>$X_{q}^{*}$</td>
<td>0.1932</td>
<td>6.14</td>
</tr>
<tr>
<td>$T_{d}$</td>
<td>ND</td>
<td>ND</td>
</tr>
<tr>
<td>$T_{d}^{*}$</td>
<td>1.5769</td>
<td>0</td>
</tr>
<tr>
<td>$T_{q}$</td>
<td>0.0255</td>
<td>4.53</td>
</tr>
<tr>
<td>$T_{q}^{*}$</td>
<td>0.0219</td>
<td>12.51</td>
</tr>
</tbody>
</table>

### Table 4. Comparison between the graphical and Prony’s methods - Machine G3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Graphical Method</th>
<th>Prony’s Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>Error (%)</td>
</tr>
<tr>
<td>$X_{d}$</td>
<td>5.341</td>
<td>0</td>
</tr>
<tr>
<td>$X_{d}^{*}$</td>
<td>0.5929</td>
<td>0.81</td>
</tr>
<tr>
<td>$X_{q}^{*}$</td>
<td>0.4526</td>
<td>39.52</td>
</tr>
<tr>
<td>$T_{d}$</td>
<td>ND</td>
<td>ND</td>
</tr>
<tr>
<td>$T_{d}^{*}$</td>
<td>0.182</td>
<td>0</td>
</tr>
<tr>
<td>$T_{q}$</td>
<td>0.0175</td>
<td>112.73</td>
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<tr>
<td>$T_{q}^{*}$</td>
<td>0.0183</td>
<td>12.84</td>
</tr>
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### Table 5. Comparison between the Wavelet and Prony’s methods

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Wavelet Method</th>
<th>Prony’s Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>Error (%)</td>
</tr>
<tr>
<td>$X_{d}$</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>$X_{d}^{*}$</td>
<td>0.5992</td>
<td>0.13</td>
</tr>
<tr>
<td>$X_{q}^{*}$</td>
<td>0.3406</td>
<td>0.32</td>
</tr>
<tr>
<td>$T_{d}$</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>$T_{d}^{*}$</td>
<td>0.183</td>
<td>0.55</td>
</tr>
<tr>
<td>$T_{q}$</td>
<td>0.0082</td>
<td>0</td>
</tr>
<tr>
<td>$T_{q}^{*}$</td>
<td>0.0157</td>
<td>3.08</td>
</tr>
</tbody>
</table>

### REFERENCES


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