Identification of scalar Jiles-Atherton hysteresis model parameters with a new particle behaviour based metaheuristic algorithm

Abstract. This paper is about a possible determination process of the scalar Jiles-Atherton hysteresis model parameters. A newly developed particle behaviour based metaheuristic algorithm, the so-called Weighted Attraction Method is used to find an approximation to the model parameters. The paper shortly discuss phenomena of magnetic hysteresis, the equations of the Jiles-Atherton hysteresis model, and the applied optimization method, thereafter the identification process, and the effectiveness of the optimization process is presented.


Keywords: magnetic hysteresis, Jiles-Atherton model, metaheuristic optimization, weighted attraction method.

Introduction
During the development of different electronic devices, that have parts made of ferromagnetic materials (e.g. electric motors and transformers), it is important to not to ignore the magnetic hysteresis loss. In the literature there are several modelling methods to approximate the hysteresis characteristics, and one of the most well-known is the Jiles-Atherton model [1, 2, 3, 4]. The original scalar frequency independent version of the model operates with 5 different parameters. There are several methods to determine the values of these parameters [3, 5], but neither of them is capable to find a parameter set, that perfectly approximates the hysteresis characteristics.

This paper presents an easy method to identify the model parameters of the scalar Jiles-Atherton model. This method is a new metaheuristic optimization algorithm, inspired by the gravitational attraction between particles.

Magnetic hysteresis
The magnetic hysteresis [6, 7] is a nonlinear multivalued relationship (as shown in fig. 1. with \( \Gamma \) as the nonlinear multivalued operator) between the magnetic field intensity \( H \) and magnetization \( M \) as \( M = \Gamma (H) \). This means, that the actual state of the hysteresis loop depends on the previous time course. This behaviour of the ferromagnetic materials causes loss, and should be taken into account during numerical field calculations. The hysteresis loss is proportional to the area of the hysteresis curve, and this causes the warming of a device.

Fig. 1. Representation of a nonlinear multivalued relationship.

Fig. 2. shows a simple scalar hysteresis characteristic, where \( H_c \) is the coercive field, \( M_s \) is the magnetic remanence, \( H_m \) and \( M_m \) are the field values of the loop tip.

Scalar Jiles-Atherton hysteresis model
David Jiles and David Atherton have written a paper in 1984 [1] about a possible way of modelling ferromagnetic hysteresis. Nowadays, this is one of the most widely used method to model hysteresis characteristics. This model is based on the theory of domain wall motions and energy balance [1, 2, 4]. The model supposes that the magnetization is the sum of a reversible and irreversible magnetization component:

\[
M = M_{\text{rev}} + M_{\text{irr}}.
\]

The Jiles-Atherton model is a differential equation system, that deals with the difference of the anhysteretic and hysteretic magnetization. The anhysteretic magnetization \( M_{\text{an}} \) is usually approximated with the so-called Langevin-function. The Langevin-function can be written as:

\[
M_{\text{an}} = M_s \left[ \coth \left( \frac{H_c}{a} \right) - \frac{a}{H_c} \right].
\]
where $M_s$ is the saturation magnetization of the material, $a$ is a model parameter, that quantifies the domain wall density, and $H_e$ is the so-called effective field intensity as:

$$H_e = H + \alpha M,$$

where $a$ is another model parameter, that quantifies the interdomain coupling. Introducing a new parameter $c$ that stands for the magnetization reversibility, (1) can be rewritten as:

$$M = cM_{an} + (1-c)M_{irr}.$$  

The irreversible magnetization can be obtained from the energy equation as:

$$M_{irr} + k\delta \frac{dM_{irr}}{dH_e} = M_{an}. $$

In (5) $k$ is a model parameter for quantifying the average energy required to break pining site, and $\delta = \text{sign}(dH/dt)$. Based on (4) and 5, the differential equation for the direct Jiles-Atherton model can be introduced as:

$$\frac{dM}{dH} = \frac{c\frac{dM_{an}}{dH_e} + (1-c)\frac{dM_{irr}}{dH_e}}{1 - \alpha c \frac{dM_{an}}{dH_e} - \alpha (1-c)\frac{dM_{irr}}{dH_e}}.$$  

Summarizing, the model parameters are: $M_s$, $\alpha$, $a$, $k$ and $c$.

The optical effects of changing the different parameters on the shape of the modelled hysteresis curve can be seen in Fig. 3a-3e.

It is suggested in [1], that the range of the model parameters can be bounded with different measured quantities of the hysteresis characteristics (see in Fig. 2.). These bounds are defined as:

$$M_s \in \{M_m; 1.5M_m\},$$

$$\alpha \in \left\{0.5 \frac{H_c}{M_m}; 0.7 \frac{H_m}{M_m}\right\},$$

$$a \in \{0.5H_c; 5H_c\} ,$$

$$k \in \{0.5H_c; 5H_c\} ,$$

$$c \in \{0; 1\}.$$
Scalar hysteresis measurement

The measurement on the scalar hysteresis characteristics were performed on a toroid shaped C-19 structural steel core (See in Fig. 4.).

The primary coil was excited with a voltage controlled current source, the induced voltage on the secondary coil was measured with an NI PCI-6251 data acquisition card, and both were controlled with LabVIEW [8] functions (See in Fig 5.).

The magnetic field intensity in the core was expressed with the analytical formula

$$H(t) \approx \frac{N_p \phi(t)}{2 R \pi},$$

where $N_p$ is the turn number of the primary coil, $\phi(t)$ denotes the source current, and $R$ is the mean radius of the core. The magnetization is calculated from the measurement as

$$M = \frac{B}{\mu_0} - H,$$

where

$$B(t) = -\frac{1}{AN_p} \int_0^t i(t) \, dt.$$

In (9), $A$ is the cross section of the core (as seen in Fig. 4), $N_p$ is the turn number of the measuring coil, and $i(t)$ is the measured voltage.

Metaheuristic optimization and the Weighted Attraction Method

Metaheuristic methods [9, 10] are widely used stochastic optimization techniques usually inspired by a natural phenomenon. They are usually called as "black-box optimization methods" [9], because during the optimization process, there is no need to know almost anything about the objective function, that is why these techniques can be easily implemented. These algorithms provide quasi-optimal result with usually fast convergence (but the convergence speed is strongly dependent on the objective function). On the other hand, finding the optimum is not assured, some methods can stuck, and some others deals with slowing convergence speed.

In this paper, a newly developed method is applied to find an approximation of the Jiles-Atherton model parameter set, the weighted attraction method [11]. The main goal was to develop a new algorithm that is capable of global optimization using less number of individuals without increasing the runtime. The idea behind this method is the gravitational attraction between particles. Now, an individual result of the Jiles-Atherton model for any parameter set is called as a particle. All of the particles together are called as a swarm. The magnitudes of the attraction forces between the particles are depending on their fitness function.

This method has four main steps. These are the following in detail:

**Initialization:** Creating initial, randomly placed particle dispersion $x \in \mathbb{R}^n$. The searching particles are placed randomly, but uniformly in the whole search space.

**Calculation of attraction:** Calculating the fitness function $F(x)$, and based on the fitness values of each particle, assigning attraction factor to each one of them. If a result or particle is better than the other, it will gain higher attraction value. One way to do this step is to map the particles to the $[0, 1]$ domain $F \rightarrow F'$, and assign to the mapped particles a weighting coefficient $w(F')$. See an example in Fig 6.

**Moving:** Choosing the next search place for each particle. This moving is based on the actual place and fitness of the particles, and on the previous steps as well. Firstly, calculate the "center of mass" $c$ in the $m$th iteration as:

$$c^{(m)} = \frac{\sum_{j=1}^{k} w_j^{(m)} x_j^{(m)}}{\sum_{j=1}^{k} w_j^{(m)}},$$

where $k$ denotes the number of the searching particles. The direction vectors from each particle pointing to $c$ can be easily calculated as:

$$d_j^{(m+1)} = c^{(m)} - x_j^{(m)}.$$

The new position of the $j$th particle in the $(m+1)$th iteration can be determined with the following formula:

$$x_j^{(m+1)} = x_j^{(m)} + \phi(d_j^{(m+1)} + \beta d_j^{(m)}).$$
In (12), $\phi_a$ and $\phi_b$ are random multipliers.

**Explosion**: Scattering the particles if they are too close to each other, or their fitness values are almost equal. This step prevents the algorithm to be stuck, which is a frequent problem with many metaheuristic optimization methods.

**Parameter identification**

The number of searching particles was set to different values, and it seemed, that 10 initial curve is mostly enough for the optimization. The parameter set was selected in the range of the bounds described by (6). An initial set of curves with the measured characteristics can be seen in Fig. 7.

The fitness function of the $i^{th}$ particle was defined as

$F_i = \left| \frac{M_{\text{measured}} - M_i}{\max(M_{\text{measured}})} \right|.

The attraction function was selected as

$w(F_i) = F_i^3$

to improve the influence of the best particles during the optimization. The best found particle is not allowed to move in any iteration to prevent the algorithm to lose the better places. This makes this optimization some kind of elitist process.

The convergence of the fitness function of a single parameter searching process can be seen in Fig. 8.

Conclusions

A particle behaviour based metaheuristic optimization method was developed previously, and was successfully implemented to determine an approximation of the scalar Jiles-Atherton hysteresis model parameters. For the parameter identification process only 10 searching particle was enough. This optimization task showed, that the Weighted Attraction Method is an attractive technique to decrease the number of function evaluations, helping to speed up different optimization processes.

**REFERENCES**


**Authors**: Gergely Friedl, Assistant lecturer, Széchenyi István University, Department of Automation, E-mail: friedl.gergely@sze.hu; Prof. Dr. Miklós Kuczmann, Full professor, Dean, Head of Department, Széchenyi István University, Department of Automation, E-mail: kuczmann@sze.hu