Qualitative 3D FEM study of B and H distribution in circular isotropic samples for two-dimensional loss measurements

Abstract. This paper investigates the effect of the approximation of an ellipsoidal shape by three laminations. The calculations are carried out with three-dimensional finite-element method for a non-linear isotropic sample with 60 mm diameter and with both lamination thickness and spacing between laminations as 0.5 mm. It is possible to find an optimum configuration for which the uniformity of B and H distribution is improved, but there is no universal configuration because the distribution is affected even by the amplitude of excitation.

Streszczenie. Artykuł opisuje badanie wpływu aproksymacji kształtu epsoidalnego przez trzy warstwy. Obliczenia wykonane są przy użyciu trójwarstwowej metody elementów skończonych dla nieliniowej izotropowej próbki o średnicy 60 mm i grubości odstępach 0.5 mm. Możliwym jest znalezienie optymalnej konfiguracji, dla której jednorodność rozkładu B i H jest ulepszona, ale nie ma uniwersalnej konfiguracji, ponieważ rozkład zmienia się nawet przy zmianie amplitudy magnesowania. (Badanie 3D FEM jednorodności pola magnetycznego w jedno- i trójwarstwowych izotropowych próbkach do pomiarów dwuwymiarowych strat mocy).

Keywords: two-dimensional power loss, rotational power loss, rotational magnetisation, 3D FEM, ellipsoidal sample
Słowa kluczowe: dwuwymiarowe straty mocy, obrotowe straty mocy, magnesowanie obrotowe, 3D FEM, próba elipsoidalna

Introduction
Power loss dissipated under rotational magnetisation in electrical steels remains relevant for design of rotating machines and it is a leading topic for an international conference [1].

Measurements under rotational magnetisation are more difficult to perform than under the more conventional alternating (uni-directional) excitation as standardised by the group of international standards IEC 60404. The absolute reproducibility of rotational measurements are significantly worse between different laboratories and measurement systems [2], so that international standardisation has not been implemented so far. There is a number of effects and parameters which contribute to the increased measurement errors [3, 4].

One of the main difficulties is to maintain sufficient uniformity of excitation over the measurement area, which is a significant problem with two-dimensional and rotational excitation. The non-uniformity is partly introduced by the magnetising yoke, but also the shape of the sample is a significant factor, especially that most magnetising systems use open magnetic circuit (with a significant air gap).

Several researchers suggested that using additional laminations (also referred to as "shields" or "dummy samples") above and/or below the specimen can improve uniformity of magnetisation and reduce measurement errors [5-7]. Two main methods were shown in the literature. In one approach, the additional laminations have the same dimensions as the sample under test. In the other, the additional laminations have smaller diameter so that the overall stack resembles an ellipsoidal shape. An ideally shaped ellipsoid is known to result in uniform magnetisation [6].

However, the multi-layer approach is still only a coarse approximation of an ellipsoid and it is therefore expected that the resulting magnetisation will be non-uniform.

The aim of this paper is to study both approaches and to show their implications on the uniformity of the resulting distribution of flux density $B$ and magnetic field strength $H$.

Such calculations cannot be performed in an analytical way, because each lamination is a cylinder. In the modelled problem the distance between the laminations (cylinders) is comparable to the thickness of the laminations. Therefore, approximation of each lamination with an ellipsoid is not possible as it would result with oversimplification of the problem. This combined with the magnetic non-linear characteristics of the material requires a numerical study, rather than analytical. For this reason a three-dimensional finite-element method (3D FEM) was used in this investigation.

The conditions for simulations presented in this paper were a result of a careful compromise. Many factors had to be taken into account and the main concept was to eliminate as many additional effects as possible. All these considerations and assumptions are listed in detail in the next section.

The simulations are performed mostly in order to illustrate qualitative behaviour (order of magnitude) and their likely implications on the measurements of rotational power loss.

Sample and excitation method
In a magnetic circuit with an air gap the demagnetising factor of the whole circuit changes with the variation of the gap or the shape of the sample. For this reason it is not possible to study the sample anisotropy of the sample if there is a magnetising yoke in the close vicinity, due to the fact that such effect would be a function of the air gap. This would be especially important for the laminations of various diameters. Also, the simulations were performed with a magnetostatic solver in order to exclude any additional dynamic effects.

Therefore, it was decided to perform this study without a magnetising yoke, so that the demagnetising effects of just the sample itself were investigated.

Hence, the excitation was modelled as a 1000 mm long, 200 mm in diameter solenoid in order to produce uniform magnetic field at its centre, within a volume much greater than the volume of the specimen. In this way any effects of magnetising yoke on the non-uniformity of the excitation were excluded.

Circular samples are widely used for studies of rotational power loss and were used in this study as well.

The material of the sample was set up as fully isotropic with magnetisation characteristics of non-oriented electrical steel (grade M-27). Isotropic material was chosen in order to further minimise any secondary effects not caused by the size and spacing of the laminations, which was the main focus of this investigation. The simulations were performed with ANSYS Maxwell software [8].

Therefore, only single direction of magnetisation was modelled, as the results would be identical for all directions if the frequency was low (i.e. quasi-static regime).
The main specimen was modelled as 0.5 mm thick (a typical value for non-oriented electrical steel), with a 60 mm diameter. The additional laminations had the same thickness, and their diameter was either 60 mm or 36.75 mm (for simulation of ellipsoid from a three-layer structure).

Each modelled structure was fully symmetrical in each Cartesian plane and the symmetry was used for modelling only half of the assembly (see Fig. 1) in order to reduce the computer memory and computation time requirements. The centre of the sample was placed at the coordinates (0, 0, 0).

Fig.1. Central part of the modelled geometry

The three-lamination stack was set up to be symmetrical so that an upper "shield" was added as well as a bottom one (Fig. 2). Again, such configuration was used in order to minimise any additional effects on non-uniformity, which would undoubtedly happen for an asymmetrical structure.

Fig.2. Modelled configurations: a) cross-section of a single circular sample, b) cross-section of a circular sample with two shields approximating a cylindrical structure, c) overview of a 3-layer approximated ellipsoid, d) and e) cross-section of a 3-layer approximated ellipsoid with alternative ellipse (drawings not to scale)

The laminations were spaced by 0.5 mm (the same as the sample thickness). This value was chosen due to practical requirement of enclosing an H-coil (a typically used sensor of the magnetic field strength \(H\)) between two adjacent laminations. Even for a very thin H-coil there is the combined thickness of the coil substrate, the diameter of the wire of the H-coils and protecting varnish, as well as usually also the thickness of the B-coil wires.

Reduced diameter of the outer laminations for the ellipsoidal approximation was chosen such that the cross-section of the laminations were inscribed in an ellipse, as shown in Fig. 2d. However, the elliptical cross-section could be also fitted in an alternative way, as shown in Fig. 2e. The approach from Fig. 2e results with a slightly "flatter" approximated oblate ellipsoid, even though any other dimensions are exactly the same as in Fig. 2d.

Initially three configurations were modelled: single sample on its own (Fig. 2a), three laminations with identical dimensions approximating a cylindrical stack (Fig. 2b), and three laminations approximating ellipsoidal stack (Fig. 3d or Fig. 3e).

Additionally, each configuration was magnetised with a "low" (in quasi-linear region, below 0.4 T, 1000 A-t) and a "high" excitation (around or above the magnetisation "knee", >1.3 T for non-oriented electrical steel, 15 kA-t).

All the graphs presented in this paper show a total amplitude (combining all three directional components X, Y and Z) of the flux density \(B\) or magnetic field strength \(H\), unless stated otherwise.

Uniformity of applied excitation

The initial calculation was performed without any ferromagnetic laminations in order to verify the uniformity of excitation. Fig. 3 shows distribution of \(B\) and \(H\) plotted for a 1000 A-t (ampere-turns) excitation of the solenoid.

The symbol \(\parallel\) denotes the direction parallel to the applied excitation (see also Fig. 2c) and \(-\) is the direction perpendicular to the applied excitation.

In all the graphs, the inscription sample denotes a line leading from the geometrical centre of the sample towards its edge (remaining within the lamination). Analogically, shield denotes similar line leading from the centre of the shield towards its edge (remaining within the lamination).

![Graphs showing uniformity of excitation](image-url)
In Fig. 3 there are additional limits calculated as +0.1% and -0.1% relative to the value at the centre of the sample (where the sample will be positioned). The uniformity of the excitation is significantly better than ±0.1% over the whole volume of the sample and shields.

The uniformity of the $B$ distribution was plotted along two vertical lines and the results are shown in Fig. 3b. The vertical line centre starts below the bottom shield and passes through the $(0, 0, 0)$ point of the system.

The line $5 \text{ mm}$ is parallel to the line centre but it is moved $5 \text{ mm}$ along the axis at which the excitation is applied as defined in Fig. 2c.

**Single sample**

For a single sample the top and bottom shields were obviously not used, so the resulting $B$ along the line shield was much smaller than in the sample. For this reason only the sample $\perp$ and sample $\parallel$ are shown in Fig. 4a, for "low" excitation.

This is better illustrated by the flux lines depicted in Fig. 5. The $B$ is highest at the centre of the sample (as it is also evident from Fig. 4a) and reduces towards the edges, because more flux lines "leave" the volume of the sample.

As a consequence there is a significant component of both $B$ and $H$ perpendicular (normal) to the surface of the sample.

Therefore, due to the normal component of $B$ the modulus of the $H$ vector can increase abruptly, even though the tangential component cannot change in a discontinuous fashion [9]. Similar behaviour was shown and discussed for instance in [5] and [10], the latter performed with magnetising yoke and dynamic magnetisation.

However, the computations in [5] were carried out in 2D FEM, which is not capable of representing the exact distribution of non-uniformities in a 3D system. On the other hand, the results showed in [10] focus only on $B$ distribution.

The values in Fig. 4 are plotted centrally along the magnetisation axis, so there are only two components – tangential and normal, because the material is isotropic (no crystallographic anisotropy).

It should be noted here that the value changes from $46 \text{ A/m}$ to $415 \text{ A/m}$ (~9x) at the surface of the sample. This means that the normal component is almost completely responsible for the abrupt change at the sample-air interface. The tangential component changes very little, as it is evident from Fig. 4a, because the amplitude of $B$ changes only by around 1%.

Such large values of normal $H$ can contribute to additional measurement errors, as suggested already in [5] and [7]. However, quantification of such effects is not possible for a general case.

Qualitatively, very similar behaviour is found for "high" excitation, but the non-uniformity of $B$ reduces slightly, to remain within the ±1% limit for up to $10 \text{ mm}$ radius, which is not the case for "low" excitation as shown in Fig. 4a.

Similarly for the values of $H$ plotted along vertical lines there is an abrupt change from $485 \text{ A/m}$ to $2005 \text{ A/m}$ (~4x), so that the normal component was still prevailing.

It should be noted that the at the centre of the sample only the tangential component is present, parallel to the applied excitation, as evident from Fig. 4b and Fig. 5.

The corresponding distribution of $B$ for the plane at the centre of the sample (X-Y plane, for $Z = 0$) is shown in Fig. 6. It should be noted that the non-uniformity is worse along the direction of the applied excitation, due to the apparent magnetic poles appearing at the opposing ends of the magnetised sample. For this reason the non-uniformity for the parallel $\parallel$ curves is always worse than for the perpendicular $\perp$ curves.
Three-disc cylinder

A system of three identical discs was modelled as shown in Fig. 2b. The corresponding distribution of $B$ and $H$ is shown in Fig. 7.

Amplitude of $B$ in the shield is greater than that of the sample (roughly double, not shown for brevity).

Within the central sample the uniformity is very similar to that from a single-lamination setup, as can be seen from comparison of Fig. 7a and Fig. 4a. The same excitation was applied in both cases (1000 A-t), but the resulting $B$ amplitude is much lower for the three-lamination case, caused by an increased demagnetising effect.

$$\text{Fig. 6. Top view of } B \text{ distribution for the same conditions as in Fig. 4, (configuration from Fig. 2a)}$$

Similar ratio can be seen for values at the 5 mm vertical line. The amplitude changes abruptly from 19.1 A/m to 111.9 A/m (~6x).

Similar results occur for higher excitation, and although the uniformity improves somewhat the overall behaviour is comparable to the single lamination system.

Therefore, the use of such shielding does not give significant improvement of the uniformity of magnetisation.

Three-disc ellipsoid

A system of three different discs was modelled as shown in Fig. 2c,d,e. The $B$ distribution within the laminations is shown in Fig. 8a. The amplitude of $B$ in the shields was much lower (not shown for brevity).

$$\text{Fig. 7. Distribution of } B (\text{a}) \text{ and } H (\text{b}) \text{ in the three-lamination system at low excitation, 1000 A-t (configuration from Fig. 2b)}$$

The tangential $H$ component at the centre of the sample was 19.3 A/m. The values increase with distance from the sample, but the gradient is smaller than for the single-lamination system. The value increases to 30.3 A/m at the surface of the shield (0.5 mm distance from the sample surface). This is a small improvement, but the effectiveness of the shielding is rather insignificant.

$$\text{Fig. 8. Distribution of } B (\text{a}) \text{ and } H (\text{b}) \text{ distribution (b) in the three-lamination ellipsoid at low excitation, 1000 A-t (configuration from Fig. 2c,d,e)}$$

Interestingly, the non-uniformity is similar as in previous configurations, but the character is reversed, because the $B$ values increase towards the edge of the sample.

It is evident from the previous case (Fig. 4b) that if the shields have increase diameter (the same diameter as the sample) then the trend is reversed. Therefore, there must be an optimum diameter of the shields which will result in the most uniform magnetisation of the sample under test.

The corresponding distribution of $H$ along vertical lines is shown in Fig. 8b. The value at centre of the sample is 38.1 A/m and this time it reduces towards the shield, at the surface of which is 16.8 A/m (44% of the value).

At the 5 mm vertical there is still an abrupt and significant change at the sample-air interface, from 38.1 A/m to 219 A/m, which is still around ~6x of the tangential $H$.

The situation is very similar for "high" excitation with 15 kA-t (Fig. 9). Again, the uniformity of $B$ distribution improves somewhat, but the effect is not very strong.
However, the abrupt change of $H$ between the laminations is more severe, as compared to the low excitation (Fig. 13). The change is from 1.59 kA/m to 8.0 kA/m (~5x).

This is comparable to the single sample system, so the addition of shields might not necessarily mean an improvement of uniformity from this viewpoint, unless the diameter of the shields is carefully optimised.

However, this optimum diameter will depend on the thickness of the lamination and the distance between the sample and the shields. For this reason an optimum ratio cannot be found for a general case.

Optimised three-disc ellipsoid

The diameter of the shields were optimised by a trial-and-error method using the high excitation as the reference. It was found that the optimum shield diameter was 55 mm, so the configuration was as shown in Fig. 10, rather than in Fig. 2d,e. This is a similar finding as presented in [10].

Also the uniformity of $H$ is greatly improved (Fig. 11b). The sample and the shield are magnetised to similar $B$, so there is little exchange of magnetic flux between the laminations and as a consequence the gradient of normal $H$ is greatly reduced. As can be seen from Fig. 11b the $H$ non-uniformity remains below ±2.5%, as compared to hundreds of percent for all previously discussed configurations.

Such reduction in $H$ gradients would constitute a great improvement in measurement accuracy. However, the effect is not universally applicable. At lower excitation (e.g. 5 kA-t instead of 15 kA-t) the $B$ and $H$ distributions change so that greater non-uniformities are created, as shown in Fig. 12. Again, Fig. 12b shows only a magnification of the area of concern.

The $B$ amplitude in the sample and in the shield are no longer equal, and the difference reaches 5%. This means that there is an increased exchange of magnetic flux between the laminations, a consequence of which is a significantly greater normal component of $H$.

As can be seen from Fig. 12b the magnitude of the $H$ vector increase roughly tenfold (e.g. from 2% to 20%) as compared with the higher excitation case shown in Fig. 11b. It should be stressed here that such changes are caused solely by changing the applied excitation, and these effects arise only from the fact that the whole system is magnetised to a different magnitude of $B$. The non-linearity of the B-H characteristics of the material leads to a different demagnetising effects, and as a consequence there will is no single optimum ratio of diameters for such three-lamination system.

The picture will be complicated further by the crystallographic anisotropy, because during rotation of the excitation the permeability will differ for different directions. So a single configuration or diameters and spacing is unlikely to give an absolute uniformity for all magnetising conditions.

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**Fig. 9.** Distribution of $B$ (a) and $H$ (b) in the three-lamination ellipsoid at high excitation, 15 kA-t (configuration from Fig. 2c,d,e).

**Fig. 10.** Cross-section of the optimised three-layer configuration (compare with Fig. 2d,e).

**Fig. 11.** Distribution of $B$ (a) and $H$ (b) in the optimised three-lamination ellipsoid at high excitation, 15 kA-t (configuration from Fig. 10).

The curves of $B$ and $H$ distribution are shown in Fig. 11. For $H$ only the magnification of the area of interest is shown for better clarity.

As can be seen from Fig. 11a the amplitude of $B$ is almost equal in the sample and in the shield, which would be the case for an ideally shaped bulk ellipsoid.

The $B$ uniformity is visibly improved so that the 1% limits are not exceeded for a radius of 20 mm (note the change of scale of the horizontal axis in Fig. 11a).
to 60 mm results in an abrupt increase of the ratio of amplitudes \( R \) for various diameters of shields (Fig. 13). This is assumed for a single H-coil, without extrapolation of the value, \( R \), at the point of 0.5 mm away from the sample surface (located in air), and \( H_0 \) is the value of the air field at the central area of the sample can be lower than 1% and of \( H \) above the sample surface lower than 2.5%.

If the shields have the same diameter as the sample the improvement in uniformity is rather small and there can still be a very significant normal component of \( H \), which is several times greater than the to-be-measured tangential component proportional to the \( H \) in the sample which contributes to the power loss.

**Discussion on optimum configuration**

For the purpose of this paper a figure-of-merit was defined for each configuration. This value was defined as the ratio of amplitudes \( R = H_{0,y}/H_0 \), where \( H_{0,y} \) is the value of \( H \) at the point of 0.5 mm away from the sample surface (located in air), and \( H_0 \) is the value of \( H \) at the surface of the sample (located in the material of the sample).

For such definition, from the measurement viewpoint the ideal value is \( R = 1 \), because the value of \( H \) at the surface of the sample will be the same as at some distance away, as it is assumed for a single H-coil, without extrapolation of the values towards the surface.

These values can be calculated separately for the vertical line centre and 5 mm and they were calculated for four cases with various diameter of shields (Fig. 13). Whereas for diameters smaller than 55 mm increase is slower.

The optimum configuration appears to be quite narrow and it is not generally applicable, because it depends on the level of excitation as it is evident from comparison of Fig. 11 and Fig. 12.

Similar optimisation can be achieved by changing the spacing between the sample and the shields, but the effect is also a function of the excitation level [11].

**Summary**

The FEM simulations presented in this paper show that it is possible to find a three-layer configuration such that the uniformity of \( B \) and \( H \) distribution in an around the sample are greatly improved. However, the effect is not universally applicable as it varies with the level of excitation even for isotropic samples.

Using shielding of incorrect diameter might in fact lead to greater values of normal \( H \) (5-8 times) than it would be without any shielding at all (4 times). Such conditions can be detrimental to the measurement accuracy if the sensor is not immune to such fields.

For an optimum configuration, and specific excitation level, the non-uniformity of \( B \) at the central area of the sample can be lower than 1% and of \( H \) above the sample surface lower than 2.5%.

As can be seen from Fig. 13 there is a quite well defined optimum configuration for a given excitation which results with \( R = 1 \) for both the centre and the 5 mm vertical lines.

However, a fairly small change in diameter from 55 mm to 60 mm results in an abrupt increase of the \( R \) value, whereas for diameters smaller than 55 mm increase is slower.

The optimum configuration appears to be quite narrow and it is not generally applicable, because it depends on the level of excitation as it is evident from comparison of Fig. 11 and Fig. 12.

Similar optimisation can be achieved by changing the spacing between the sample and the shields, but the effect is also a function of the excitation level [11].

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![Fig. 12. Distribution of B (a) and H (b) in the optimised three-lamination ellipsoid at lower excitation, 5 kA-T (configuration from Fig. 10) – compare with Fig. 11](image-url)

![Fig. 13. Ratio R plotted for the two vertical lines for various configurations of a three-layer ellipsoid, high excitation only](image-url)